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Interest Rate**

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Imperfect Credibility and the Zero Lower Bound on the Nominal Interest Rate*

Martin Bodenstein, James Hebden, and Ricardo Nunes**

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Abstract

When the nominal interest rate reaches its zero lower bound, credibility is crucial for conducting forward guidance. We determine optimal policy in a New Keynesian model when the central bank has imperfect credibility and cannot set the nominal interest rate below zero. In our model, an announcement of a low interest rate for an extended period does not necessarily reflect high credibility. Even if the central bank does not face a temptation to act discretionarily in the current period, policy commitments should not be postponed. In reality, central banks are often reluctant to allow a recovery path with output and inflation temporarily above target. From the perspective of our model such a policy reflects a low degree of credibility. We find increased forecast uncertainty in inflation and the output gap at the zero lower bound while interest rate uncertainty is reduced. Furthermore, misalignments between announced interest rate paths and market expectations are found to be best explained by lack of credibility.

Keywords: Monetary policy, zero interest rate bound, commitment, liquidity trap

JEL Classification: C61, E31, E52

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1 Introduction

When the nominal interest rate reaches zero, the central bank faces a time inconsistency problem. Initially, a promise to keep the nominal interest rate low for an extended period will raise inflation expectations, lower current and future real interest rates, and thus stimulate current output. However, once the economy has emerged from the recession, honoring the earlier promise of low interest rates overstimulates the economy leading to elevated inflation and a positive output gap. Therefore, a central bank whose announced promises for the interest rate path are perceived as not credible cannot provide effective forward guidance and the economy goes through a deeper recession than otherwise. Whereas commitment always matters for the conduct of monetary policy, the negative effects of low credibility are particularly painful at the zero bound.¹

Academics and policymakers alike deem central bank credibility pivotal for the conduct of monetary policy as documented in Blinder (2000). However, recent episodes of zero (or very low) interest rates suggest that the credibility of central banks in industrialized economies is imperfect. For example in 1999/2000, the Bank of Japan never followed the prescription of the optimal policy under full commitment. After instituting a zero interest rate policy in February 1999, it raised interest rates in March 2000 although the economy was still weak and inflation was negative.

We analyze the optimal monetary policy in a standard New Keynesian model when the central bank has imperfect credibility and the nominal interest rate is constrained to be non-negative. The central bank can promise a policy plan, however, in the future it may obtain the opportunity to discard its earlier promises and re-optimize. This framework captures the fact that in practice central banks have an intermediate degree of credibility. Besides being more realistic than the two extremes of full commitment and full discretion, our model of imperfect credibility allows us to address the following questions. How does the possibility of future renouncements influence current policies and promises? What are the economic consequences

¹Following the seminal papers by Kydland and Prescott (1977) and Barro and Gordon (1983) the literature has taken two different approaches to tackle optimal policy problems – commitment and no commitment.

of policy renouncements? How does the temptation to default evolve over time?

First, as private agents' expectations reflect the possibility of future policy renouncements under imperfect credibility, the central bank resorts to an even more extreme promise about keeping the interest rate at zero than under full commitment. Hence, the mere announcement to keep the interest rate low for an extended period is not necessarily a sign of credibility. Second, re-optimization does not prompt an immediate exit from the zero bound unless the economy has sufficiently recovered. However, the new policy promises to raise the interest rate earlier in order to moderate the post-recession boom. And third, policy renouncements are most valuable when the economy is about to emerge from the zero lower bound regime. At this stage, the benefits from forward guidance have been reaped whereas the welfare costs of elevated inflation and higher output gap have not yet materialized.

Our results support the view expressed in Walsh (2009) that central banks operate under imperfect commitment and have not followed the prescriptions of the optimal monetary policy under full commitment as derived in the New Keynesian model. Federal Reserve officials have stated publicly that promising to keep both inflation and the output gap above target is currently not considered, i.e., they do not intend to follow the optimal policy prescription of the New Keynesian model. While there may be other interpretations of such statements, our model establishes a theoretical link to central bank credibility and suggests that it is low.²

Furthermore, when the Swedish Riksbank lowered its path for the repo-rate in April 2009, market expectations decoupled from the announced repo-rate path, thus raising doubts about the credibility of the Riksbank – see Svensson (2009, 2010). Through the lens of our model, such misalignments between the announced interest rate path and market expectations more likely stem from imperfect credibility rather than increased forecast uncertainty. Whereas uncertainty about inflation and the output gap rises, the forecast for the interest rate becomes less uncertain at the zero bound. Our model also provides insight why central banks, in particular the Federal Reserve System, have come under unprecedented political pressure in the aftermath of the

²Details of statements by Federal Reserve officials are provided in section 4. Admittedly, the reluctance of policymakers to follow the prescriptions of the New Keynesian model may reflect general uncertainty about the right model.

2008 financial crisis. Even without changes in the political environment, the temptation to re-optimize a policy path is substantially larger in a recession with the interest rate at zero than in stable economic times.

Our work is related to Adam and Billi (2006, 2007) and Nakov (2008) who analyze stochastic economies imposing the zero lower bound as an occasionally binding constraint. These authors focus on the extreme cases of full commitment and full discretion. Schaumburg and Tambalotti (2007) examine imperfect commitment but abstract from the zero lower bound.³ Since the zero lower bound introduces pronounced non-linearities, closed form solutions as described in Schaumburg and Tambalotti (2007) are not available. Instead, we use the imperfect credibility setting with global methods proposed in Debortoli and Nunes (2010).

Benhabib et al. (2001, 2002) show that once the zero lower bound on the nominal interest rate is taken into account, active interest rate feedback rules can lead to liquidity traps and multiple equilibria. Schmitt-Grohé and Uribe (2007) follow a different strategy by analyzing simple rules in stochastic economies and checking ex-post that the zero lower bound constraint is not violated too often. In this paper, we do not focus on interest rate rules nor on multiple equilibria.

Other authors opted to examine the zero lower bound constraint in a perfect foresight setting, e.g., Eggertsson and Woodford (2003), Jung et al. (2005), Levin et al. (2009) and Bodenstein et al. (2009).⁴ This modeling approach allows for analyzing more complex models. However, under perfect foresight agents do not take into account the possibility of future shocks when forming their expectations. In our setting of imperfect credibility, the uncertainty about future shocks and the possibility of policy renouncements crucially shape agents' expectations and impact the policy plans of the central bank. Thus, global methods that can handle occasionally binding constraints need to be employed.

³The imperfect commitment setting with stochastic replanning was first proposed in Roberds (1987).

⁴Lindé et al. (2010) propose a different method to impose the zero lower bound in perfect foresight simulations by adding anticipated shocks to the instrument rule.

2 The Model

The model consists of two building blocks: the private sector and the monetary authority. The behavior of the private sector is given by a standard New Keynesian model as described among others in Yun (1996), Clarida et al. (1999), and Woodford (2003). The central bank attempts to maximize the welfare of the representative household but faces two limitations. First, the central bank's single tool is the nominal interest rate on one-period, non-contingent debt which cannot fall below zero (zero lower bound). Second, the central bank's commitment to earlier plans is revoked with a known and fixed probability (imperfect credibility).

2.1 Private Sector

The optimization problems of households and firms imply the well-known linear aggregate demand and supply relationships

$$\pi_t = \kappa y_t + \beta E_t \pi_{t+1} + u_t \quad (1)$$

$$y_t = E_t y_{t+1} - \sigma (i_t - E_t \pi_{t+1}) + g_t \quad (2)$$

where u_t is a cost push shock

$$u_t = \rho_u u_{t-1} + \varepsilon_{u,t} \quad (3)$$

and g_t is a demand shock

$$g_t = \rho_g g_{t-1} + \varepsilon_{g,t}. \quad (4)$$

π_t denotes the inflation rate, y_t is the output gap, and i_t is the nominal interest rate on one period non-contingent debt.⁵ Policymakers do not tax money holdings and thus the nominal

⁵The representative household in this economy chooses consumption, leisure, money and bond holdings subject to her budget constraint. Firms are monopolistic competitors and set nominal prices. The nominal price contracts are modeled as in Calvo (1983) and Yun (1996), i.e., each period only a fixed fraction of firms is allowed to re-optimize their prices. We follow the literature in using the linear equations that are obtained from log-linearizing the nonlinear equations around the model's deterministic steady state. Although such an approach removes possibly interesting nonlinearities, it facilitates our computations and the comparisons with earlier work on the zero bound constraint and/or optimal monetary policy. See also the discussion in Section 2.1 in Adam and Billi (2006).

interest rate is bounded from below.⁶ As i_t is expressed in deviation from the steady state interest rate (with zero steady state inflation), this condition reads as follows

$$i_t \geq -r^* \quad (5)$$

where r^* is the value of the nominal interest rate in steady state. For later use, we also define the level of the nominal interest rate $\tilde{i}_t \equiv i_t + r^*$.

The discount factor is denoted by $\beta \in (0, 1)$ and the slope of the Phillips curve is $\kappa = \frac{(1-v)(1-v\beta)}{v} \frac{\sigma^{-1} + \omega}{1 + \omega\theta}$. v is the probability with which a firm cannot adjust its price, σ is the intertemporal elasticity of substitution of the household, ω measures the elasticity of a firm's real marginal cost with respect to its own output level, and θ is the elasticity of substitution between the varieties produced by the monopolistic competitors.

2.2 Monetary Policy

The monetary authority minimizes the present discounted value of its period objective function subject to the constraints (1) – (5) using the one-period nominal interest rate as its sole policy instrument. The period utility function U_t is of the quadratic form

$$U_t = -\pi_t^2 - \lambda y_t^2. \quad (6)$$

Following the derivations in Woodford (2003), we assume $\lambda = \frac{\kappa}{\theta}$.

As in Roberds (1987), Schaumburg and Tambalotti (2007), and Debortoli and Nunes (2010), the policymaker has imperfect credibility. At the beginning of each time period, a realization of the random variable X is drawn with $X = \{C, D\}$ and the probability distribution

$$p(x) = \begin{cases} \eta & \text{if } x = C \\ 1 - \eta & \text{if } x = D \end{cases}$$

with $\eta \in [0, 1]$.

⁶Goodfriend (2000) proposes three options to overcome the zero bound on interest rate policy: a carry tax on money, open market operations in long bonds, and monetary transfers. Our choice to not include into our analysis these and other policies, such as fiscal stimulus and credit easing policies, should not be interpreted as us passing judgement on the effectiveness of these policies. The purpose of our paper is to shed light on the effectiveness of forward guidance through short term interest rates.

In the event $x = C$, the policymaker follows her previously announced policy path, whereas she reneges on her earlier promises if $x = D$. Thus, a policymaker's promises made in time t about the future path of the nominal interest rate will be implemented in period $t + s$ with probability η^s .

If $\eta = 1$, policy promises are always kept and the policymaker is referred to as fully committed or perfectly credible. If $\eta = 0$, the policymaker acts under full discretion. Adam and Billi (2006, 2007) analyze the optimal monetary policy under the zero bound constraint in a fully stochastic environment for these two cases.⁷ The intermediate cases, with $\eta \in [0, 1]$, correspond to imperfect credibility and have not been previously analyzed.

Assuming the default event to be stochastic rather than an endogenous decision is a simplification analogous to the Calvo pricing model. This setup allows us to solve a nonlinear stochastic model with global methods, which may become infeasible with more complex imperfect credibility settings. More importantly, the credibility setting modeled here allows us to address our goals: examining the interaction between the anticipation of future policy re-optimizations and commitments, and analyzing the effects of policy renouncements and the welfare effects of doing so. These issues cannot be addressed in either the full commitment or the discretion framework.

Under imperfect credibility, the optimization problem of the policymaker is stated as

$$\begin{aligned}
V(u_t, g_t) &= \max_{\{y_t, \pi_t, i_t\}} E_t \sum_{t=0}^{\infty} (\beta\eta)^t \{-\pi_t^2 - \lambda y_t^2 + \beta(1-\eta)E_t V^D(u_{t+1}, g_{t+1})\} \\
s.t. \pi_t &= \kappa y_t + \beta\eta E_t \pi_{t+1} + \beta(1-\eta) E_t \pi_{t+1}^D + u_t \\
y_t &= \eta E_t y_{t+1} + (1-\eta) E_t y_{t+1}^D - \sigma (i_t - \eta E_t \pi_{t+1} - (1-\eta) E_t \pi_{t+1}^D) + g_t \\
i_t &\geq -r^* \\
u_t &= \rho_u u_{t-1} + \varepsilon_{u,t} \\
g_t &= \rho_g g_{t-1} + \varepsilon_{g,t}
\end{aligned} \tag{7}$$

where variables evaluated under default carry the superscript D.

⁷Nakov (2008) performs a similar analysis but includes instrument and targeting rules.

The objective function contains two parts. The first term in the summation refers to future nodes in which current promises are kept. The possibility of future re-optimizations causes such histories to be discounted at the rate $\beta\eta$. Second, at any point in time, current promises are discarded with probability $1 - \eta$ and a new policy is formulated. The value obtained by the monetary authority in that case is summarized in the function V^D . The expectation terms in the constraints also reflect the uncertainty about future policy renouncements.

2.3 Equilibrium and Solution

The current problem can be recast into the recursive formulation of Marimon (2009). After rearranging terms, the problem can be written as

$$\begin{aligned}
V(u_t, g_t, \mu_t^1, \mu_t^2) &= \min_{\{\gamma_t^1, \gamma_t^2\}} \max_{\{y_t, \pi_t, i_t\}} h(y_t, \pi_t, i_t, \gamma_t^1, \gamma_t^2, \mu_t^1, \mu_t^2, u_t, g_t) \\
&\quad + \beta\eta E_t V(u_{t+1}, g_{t+1}, \mu_{t+1}^1, \mu_{t+1}^2) \\
&\quad + \beta(1 - \eta) E_t V^D(u_{t+1}, g_{t+1}) \\
s.t. \quad &i_t \geq -r^* \\
u_t &= \rho_u u_{t-1} + \varepsilon_{u,t} \\
g_t &= \rho_g g_{t-1} + \varepsilon_{g,t} \\
\mu_{t+1}^1 &= \gamma_t^1, \mu_0^1 = 0 \\
\mu_{t+1}^2 &= \gamma_t^2, \mu_0^2 = 0,
\end{aligned} \tag{8}$$

where (γ_t^1, γ_t^2) are the Lagrange multipliers associated with the aggregate supply and demand constraints,

$$\begin{aligned}
h(y_t, \pi_t, i_t, \gamma_t^1, \gamma_t^2, \mu_t^1, \mu_t^2, u_t, g_t) &\equiv -\pi_t^2 - \lambda y_t^2 \\
&\quad + \gamma_t^1 (\pi_t - \kappa y_t - \beta (1 - \eta) E_t \pi_{t+1}^D - u_t) \\
&\quad + \gamma_t^2 (-y_t + (1 - \eta) E_t y_{t+1}^D - \sigma (i_t - (1 - \eta) E_t \pi_{t+1}^D) + g_t) \\
&\quad - I_\eta \mu_t^1 \pi_t + I_\eta \frac{1}{\beta} \mu_t^2 (y_t + \sigma \pi_t),
\end{aligned} \tag{9}$$

and I_η is an indicator function satisfying

$$I_\eta = \begin{cases} 0 & \text{if } \eta = 0 \\ 1 & \text{if } \eta \neq 0. \end{cases}$$

It follows from Marcer and Marimon (2009) and Debortoli and Nunes (2010) that the optimal policy and the value functions are time invariant if the state space is enlarged to contain the lagged Lagrange multipliers (μ_t^1, μ_t^2) . The multipliers summarize previous promises, with the case of non-binding promises corresponding to the multipliers being zero.⁸ Since the multipliers are not physical state variables, only commitment impedes the monetary authority from ignoring previous promises, reflecting the time inconsistent nature of the problem. In fact, resetting the Lagrange multipliers to zero is optimal, and occurs in equilibrium when the monetary authority is allowed to do so. Definition 1 specifies the equilibrium concept.

Definition 1 *The equilibrium with imperfect commitment satisfies the following conditions:*

1. Given $\{y_t^D, \pi_t^D\}_{t=0}^\infty$ and the value V^D , the path $\{y_t, \pi_t, i_t\}_{t=0}^\infty$ solves problem (7).
2. The value function V^D is such that $V^D(u_t, g_t) = V(u_t, g_t, \mu_t^1 = 0, \mu_t^2 = 0)$ and V is defined by equation (8).
3. Denote the optimal policy functions as $(y_t, \pi_t) = \psi(u_t, g_t, \mu_t^1, \mu_t^2)$. The pair (y_t^D, π_t^D) satisfies the condition $(y_t^D, \pi_t^D) = \psi(u_t, g_t, 0, 0)$.

First, definition requires optimality given the constraints. The second part defines the value of default V^D to be the continuation value without binding promises, i.e., the lagged Lagrange multipliers are at zero. The two value functions would not coincide if policy objectives are not consensual.⁹ The third part requires the policy functions that private agents expect to be implemented under default (y_t^D, π_t^D) to be consistent with the optimal policy functions implemented when there are no promises to be honored.

⁸In the case of full discretion, the lagged Lagrange multipliers are neither zero nor a state variable. This case is covered in the general formulation by the inclusion of the indicator function I_η in equation (9).

⁹This case is analyzed in Debortoli and Nunes (2006) who consider two political parties with different utility functions. In such a situation, the continuation value function depends on which party gains power.

The solution of equation (8) is not standard, as both the value function V^D and the policy functions under default are unknown. In addition, the solution requires maximizing with respect to the controls and minimizing with respect to the Lagrange multipliers. In the numerical algorithm, we approximate the value function directly. As in Debortoli and Nunes (2010), we make use of Definition 1 to employ a numerical procedure with one fixed point only. The numerical algorithm is described in detail in Appendix A.2.

The two model features that dictate the use of global methods are the presence of the occasionally binding zero bound constraint and the possibility of policy renouncements. Most of the literature on monetary policy at the zero bound assumes perfect foresight. Whereas this approach allows for a simplified treatment of the binding constraint, it inhibits studying the link between monetary policy and household expectations about future policy renouncements. By construction, households do not anticipate policy changes under perfect foresight.

3 Results

The parameterization of our model is summarized in Table 1. To facilitate comparison with the work of Adam and Billi (2006, 2007) and Woodford (2003), we opted not to deviate from these authors' parameter choices. The one parameter in our analysis for which there is almost no guidance provided in the literature is η , the probability with which the policymaker will honor her previous promises. One can also measure the credibility level by the expected time before a policy renouncement occurs $\alpha = 1/(1-\eta)$. Throughout our discussion, we show results for the cases described in Table 2.

Depending on the long-run inflation target, previous research suggests that the nominal interest rate reaches the zero bound with low probability.¹⁰ However, once the economy is at the zero bound, additional shocks have amplified consequences on economic activity. In the New

¹⁰See Reifschneider and Williams (2000), Schmitt-Grohé and Uribe (2007), Billi (2009), and most recently Williams (2010).

Table 1: Parameterization

parameter	value	economic meaning
β	0.9913	discount factor
v	0.66	prob. of no price change
σ	6.25	interest rate sensitivity consumption
ω	0.47	elasticity of firms' marginal cost
θ	7.66	price elasticity of demand
κ	0.024	slope of Phillips curve
λ	0.003	weight on output in utility function
ρ_u	0	persistence cost push shock
σ_u	0.154	std. cost push shock
ρ_g	0.8	persistence demand shock
σ_g	1.524	std. demand shock

Keynesian model, demand shocks are considered to be more volatile and persistent than cost-push shocks and therefore play a more important role when focusing on zero bound issues.¹¹ Furthermore, for a large demand shock that pushes the interest rate to zero, the economy experiences key characteristics of a liquidity trap with both output and inflation dropping. By contrast, for a cost-push shock to lower the interest rate, inflation drops but output rises.

Table 2: Imperfect credibility cases

$p(x = C) = \eta$	$\alpha = \frac{1}{1-\eta}$	economic meaning
0	1	full discretion
0.5	2	loose commitment
0.75	4	loose commitment
1	∞	full commitment

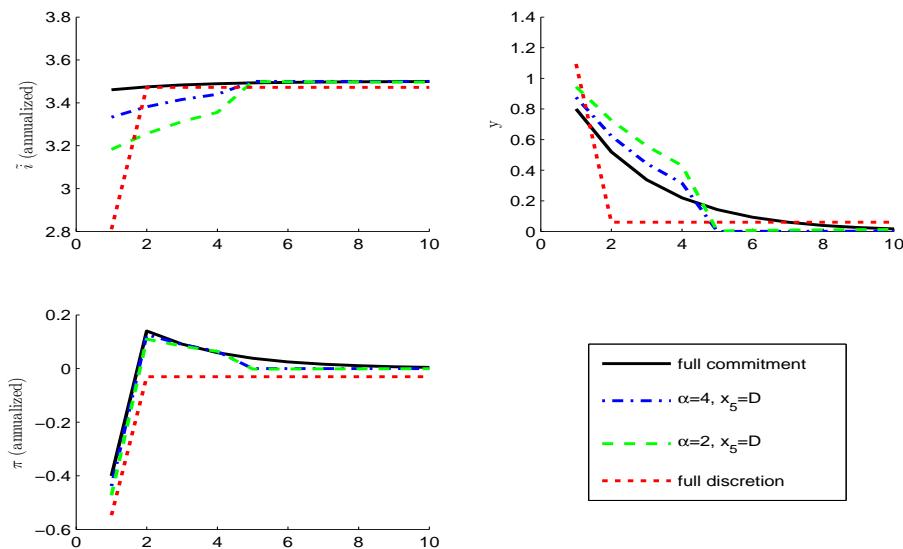
3.1 Credibility and cost-push shocks

To illustrate the effects of imperfect credibility in a simple example, we first examine the familiar response to a cost-push shock. Figure 1 plots the responses of the nominal interest rate in levels \tilde{i}_t , the output gap y_t , and the inflation rate π_t to a negative i.i.d. cost push shock (i.e., $u_1 = -\sigma_u$, $g_1 = 0$, and $(\varepsilon_{u,t}, \varepsilon_{g,t}) = 0 \forall t \geq 2$). Under full discretion, the economy

¹¹In our framework, small aggregate demand shocks can be completely offset by adjusting the nominal interest rate, as long as the interest rate is sufficiently positive.

experiences price deflation and an expansion in output. In the second period, the cost-push shock vanishes and inflation and output are back to target.¹² This is not the case with full commitment. The monetary authority promises to keep the interest rate below its long-run value, thus causing persistent inflation and elevated output. Consequently, the initial impact of the shock is dampened.

Figure 1: Cost-push Shock under Imperfect Credibility



Notes: the figure plots the response to a negative cost-push shock. The shocks are initialized at $\varepsilon_{u,1} = -\sigma_u, \varepsilon_{g,1} = 0$, and $(\varepsilon_{u,t}, \varepsilon_{g,t}) = 0 \forall t \geq 2$. For the imperfect commitment cases ($\alpha = 2$ and $\alpha = 4$), the figure plots the specific history with $x_5 = D, x_t = C \forall t \neq 5$.

For the imperfect credibility settings with $\alpha = 2$ and $\alpha = 4$, Figure 1 considers the specific history in which the monetary authority reneges in period 5 only ($x_5 = D, x_t = C \forall t \neq 5$). Until period 4, the output gap and inflation are kept above target, honoring the monetary authority's past promises. In period 5, policy is re-optimized and it is feasible to bring output and inflation back to target. Since private agents are aware of the possibility of such future policy renouncements, the task of the central bank is adversely affected: in the first period the central bank does not stabilize the economy as effectively as under full commitment.

¹²As we solve for the model using global methods rather than linear methods, the long-run responses do not return to zero in the presence of an occasionally binding constraint due to precautionary motives.

3.2 Credibility at the zero lower bound

When the nominal interest rate reaches zero in response to a large contractionary demand shock, monetary policy faces a dilemma. To mitigate the impact of the shock the central bank has to promise to keep the interest rate low for an extended period in order to allow inflation and the output gap to rise above their target values in the future. However, once the economy enters the phase of elevated inflation and a positive output gap, the central bank faces the temptation to renege on its earlier promise and to raise the interest rate. Under imperfect credibility, the monetary authority will not deliver on its earlier promises and will raise the interest rate path if it obtains the chance to do so. Thus, the ability of such a central bank to ease the recession may be greatly diminished.

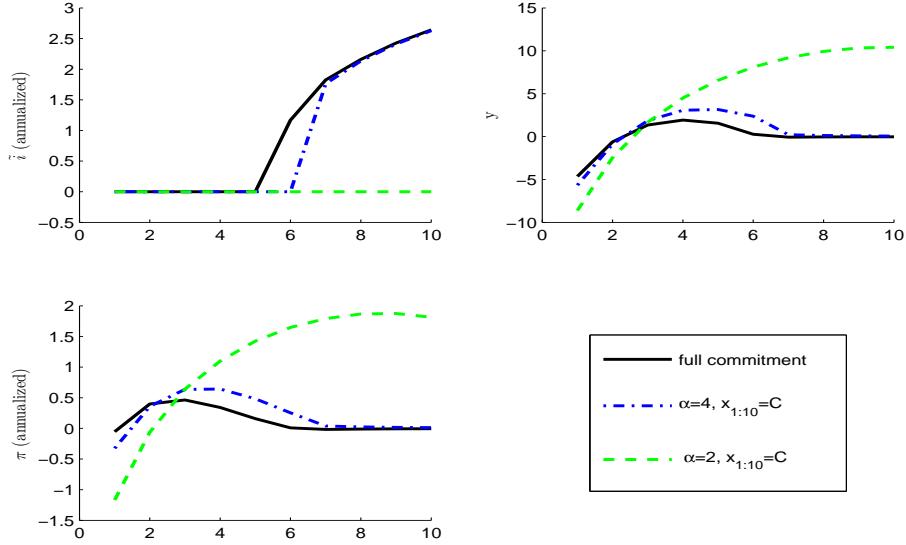
To illustrate how policy promises and renouncements chosen under limited credibility affect the path of the economy, we analyze the impulse response functions after a large and persistent contraction in aggregate demand that pushes the nominal interest rate to zero. More specifically, across experiments we set $g_1 = -10$, $u_1 = 0$, and $(\varepsilon_{u,t}, \varepsilon_{g,t}) = 0 \forall t \geq 2$. However, we consider different specific histories of x_t . Initially, the lagged Lagrange multipliers are at zero, i.e., $\mu_1^1 = \mu_1^2 = 0$.

3.2.1 Promises

If the central bank has limited credibility, private agents correctly incorporate the possibility of future re-optimizations when forming expectations. The monetary authority tries to influence the expectations of the private sector by adjusting its announced policy. Figure 2 plots the optimal plan announced by the central bank without policy renouncements ($x_t = C \forall t \geq 1$) for different levels of credibility ($\alpha = 2$, $\alpha = 4$, and full commitment). The lower the credibility of the monetary authority is, the more extreme its promises are. The policymaker with $\alpha = 2$ promises to keep the interest rate at zero over the entire horizon plotted, whereas the policymaker with $\alpha = 4$ promises to keep the interest rate at zero for 6 periods. Hence, the announcement that the interest rate is planned to be at zero for a long period should not be necessarily understood as a signal of high credibility. In a purely forward looking model, such

inference is correct only when comparing the extreme cases of full commitment and discretion.¹³

Figure 2: Imperfect Credibility and the Zero Lower Bound – Extreme Promises



Notes: the figure plots the transition dynamics in response to a negative and large demand shock causing the interest rate to reach its zero lower bound. The shocks are initialized at $u_1 = 0$, $g_1 = -10$, and $(\varepsilon_{u,t}, \varepsilon_{g,t}) = 0 \forall t \geq 2$. For the imperfect commitment cases ($\alpha = 2$ and $\alpha = 4$), the figure plots the specific history with $x_t = C \forall t \geq 1$.

However, the central bank may have to deliver on its promises. At the optimum, the policymaker equates the expected benefits from a promise to keep the interest rate low with the expected costs of having to deliver on these promises (the cost of not defaulting). If a policymaker does not get a chance to revisit an extreme promise, the low interest rate fuels a sizable and persistent boom with increased inflation, as seen in particular for the case of $\alpha = 2$.

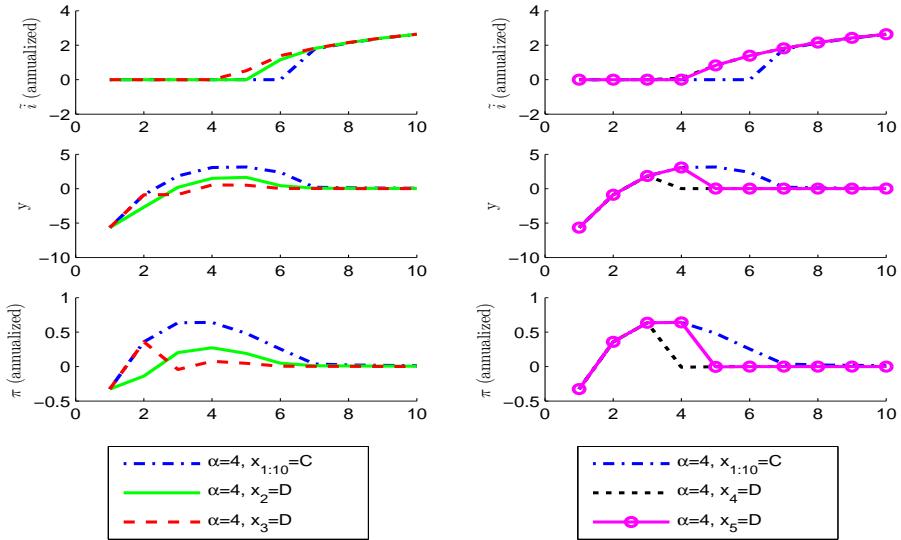
3.2.2 Renouncements

While a policymaker in default ignores all past promises, the new policies depend on the state of the economy at the time of re-optimization. If the economy is still sufficiently deep in recession, the central bank will keep the interest rate at zero under its new policy. The left panel in Figure 3 confirms this claim for $\alpha = 4$ when a policy renouncement occurs in period

¹³As shown in Section 5, the observed interest rate may be lower under full discretion than under full commitment for a given demand shock if inflation is not purely forward looking due to price indexation.

2 ($x_2 = D$ and $x_t = C \forall t \neq 2$) or period 3 ($x_3 = D$ and $x_t = C \forall t \neq 3$). One could therefore infer that in times of a deep recession, policy commitments are not relevant: the interest rate will be at zero irrespective of policy renouncements. However, such inference is misguided. The promise of a low interest rate is made because of its positive effects throughout the entire recession. If the interest rate policy is reformulated in the midst of the recession, the incentive to keep the interest rate low is reduced. In fact, if policy renouncements occur in the early phase of the recession, the economy exits the zero bound earlier. Commitment is important well before the central bank faces the temptation to start economic tightening.

Figure 3: Imperfect Credibility and the Zero Lower Bound – Reneging on Promises

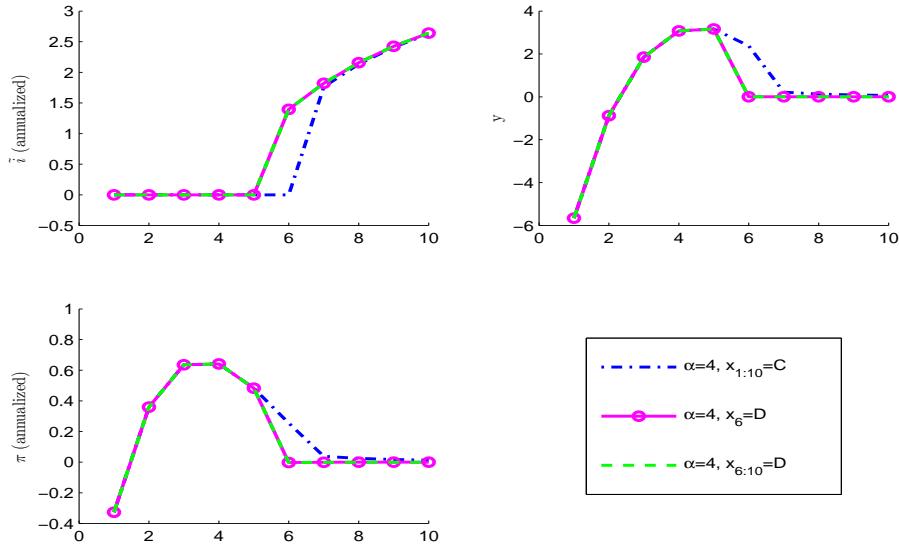


Notes: the figure plots the transition dynamics in response to a negative and large demand shock causing the interest rate to reach its zero lower bound. The shocks are initialized at $u_1 = 0, g_1 = -10$, and $(\varepsilon_{u,t}, \varepsilon_{g,t}) = 0 \forall t \geq 2$. All cases refer to imperfect credibility with $\alpha = 4$. The dot-dashed line in both panels refers to the specific history $x_t = C \forall t$. The remaining lines in the figure plot a specific history $x_t = C \forall t \neq m$, with $m \in \{2, 3, 4, 5\}$.

While the economy is on its recovery path, the interest rate may still be zero due to earlier commitments. As depicted in the right panel of Figure 3, the central bank raises the interest rate immediately if a default occurs in this phase. For defaults in period 4 ($x_4 = D, x_t = C \forall t \neq 4$) or 5 ($x_5 = D, x_t = C \forall t \neq 5$), such action puts an end to the policy of elevated inflation and output gap previously promised and brings both variables closer to target.

Once the economy has exited from the zero bound, it is immaterial whether new commitments are made after a policy renouncement. After exiting from the zero lower bound, commitments hardly have additional value with respect to the demand shocks. Figure 4 shows that after a default in period 6, the projected path with $x_t = C \forall t \geq 7$ and the path with $x_t = D \forall t \geq 7$ overlap.¹⁴

Figure 4: Imperfect Credibility and the Zero Lower Bound – After Reneging on Promises



Notes: the figure plots the transition dynamics in response to a negative and large demand shock causing the interest rate to reach its zero lower bound. The shocks are initialized at $u_1 = 0, g_1 = -10$, and $(\varepsilon_{u,t}, \varepsilon_{g,t}) = 0 \forall t \geq 2$. All cases refer to imperfect credibility with $\alpha = 4$. The first line plots the specific history $x_t = C \forall t$; the second line plots the specific history $x_t = C \forall t \neq 6$; the third line plots the specific history $x_t = C \forall t < 6$.

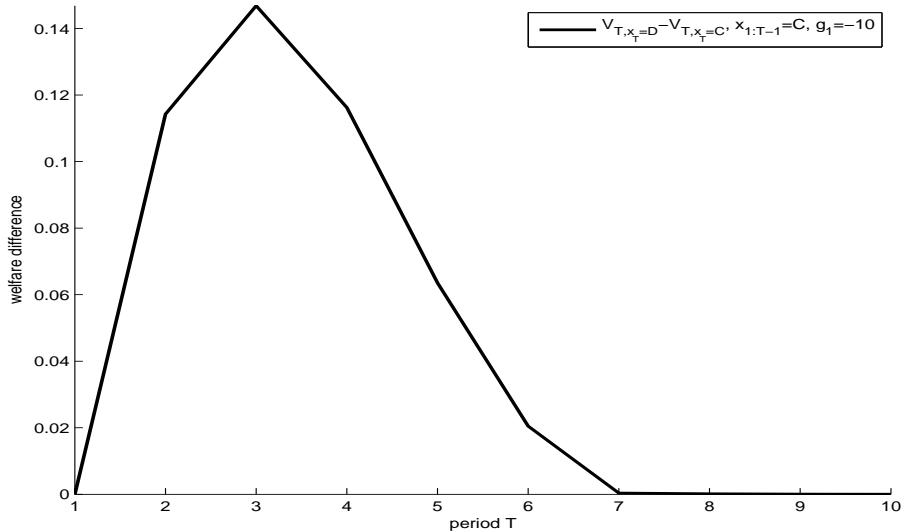
3.2.3 Temptation and Welfare

The exact nature of the time inconsistency problem faced by the central bank is illustrated by a welfare analysis in Figure 5. We plot the welfare gain from a random default in period T relative to staying committed, $V_{T,x_T=D} - V_{T,x_T=C}$. V_{T,x_T} is the present discounted value of

¹⁴In creating Figure 4, we set $x_t = C \forall t \leq 5$. Furthermore, no shocks to demand and costs realize after period 1. If the economy experiences additional shocks after exiting the zero bound, new commitments can be important.

the central bank's utility from T onwards with realization x_T .¹⁵ In period 1, the economy experiences a large contractionary demand shock, $g_1 = -10$, that pushes the interest rate down to zero. Afterwards no cost-push or demand shocks realize. In all periods prior to period T , the policymaker is assumed to honor her promises.

Figure 5: Welfare Gains from Default



Notes: the figure plots the period T expectation of the gain in discounted welfare due to a default in period T , for the case of imperfect credibility with $\alpha = 4$ and the specific histories $x_t = C \forall t < T$. The shocks are initialized at $u_1 = 0, g_1 = -10$, and $(\varepsilon_{u,t}, \varepsilon_{g,t}) = 0 \forall t < T$.

The temptation to renege is the highest in period 3. Revisiting Figure 3 reveals that defaulting in period 3 avoids the costly overshooting of inflation and the output gap. The economy is stabilized more effectively if the opportunity to re-optimize occurs in this period rather than any other.

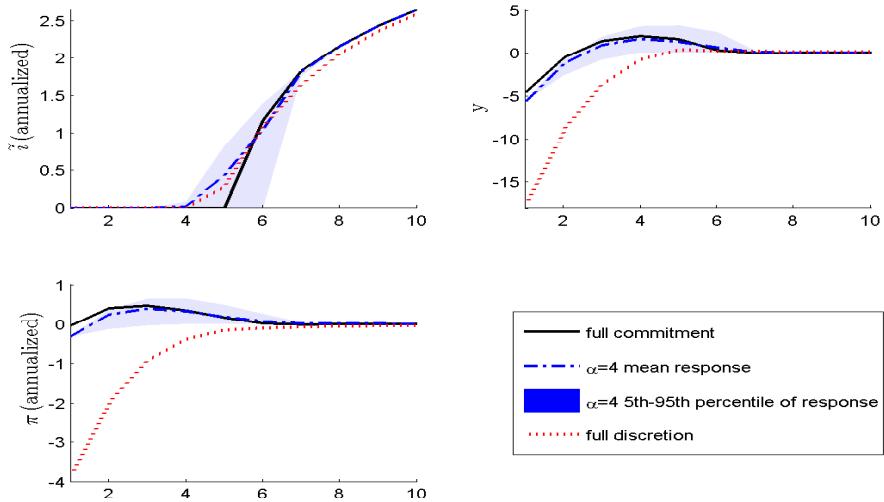
¹⁵For a fixed default probability η , the central bank achieves higher utility in a given period when re-optimization occurs. In equation (9) the last two terms make the problem recursive but are not welfare relevant. In the computations we consider only the welfare relevant terms, which would be equivalent to computing the discounted sum of the period utility function in equation (6).

3.3 Distribution of Responses

To highlight specific features of our model, the impulse response functions shown so far have been based on specific histories for x_t , $\varepsilon_{g,t}$, and $\varepsilon_{u,t}$. For the next two experiments, we analyze more generally the distribution of impulse responses at the zero bound.

First Experiment We first restrict attention to histories with $g_1 = -10$ and $\varepsilon_{g,t} = 0 \forall t \geq 2$, and $u_1 = 0$ and $\varepsilon_{u,t} = 0 \forall t \geq 2$, but allow for all possible histories of x_t . Figure 6 shows the mean impulse response under imperfect credibility with $\alpha = 4$ surrounded by the impulse response functions lying between the 5th and 95th percentiles, the shaded area. The figure also displays the impulse responses when the policymaker has full commitment or acts under full discretion, respectively. As we still condition on a specific history for the demand and cost-push shocks, there is no uncertainty about the impulse responses under full commitment and full discretion.

Figure 6: Distribution of Impulse Response Functions – Default Uncertainty Only



Notes: the figure plots the transition dynamics in response to a negative and large demand shock causing the interest rate to reach its zero lower bound. The shocks are initialized at $u_1 = 0, g_1 = -10$, and $(\varepsilon_{u,t}, \varepsilon_{g,t}) = 0 \forall t \geq 2$. The mean and selected percentiles are computed from simulated histories of the commitment shock (i.e. $x_t \in \{C, D\}$). For the full commitment and full discretion cases, there is no uncertainty with regard to x_t shocks, and therefore any percentile coincides with the mean.

For the mean response under imperfect credibility, the economy exits the zero bound earlier than both under full commitment and full discretion.¹⁶ Nevertheless, the output gap and inflation rise above target before returning to it for the mean response under limited credibility as is the case under full commitment. However, this mean response is surrounded by considerable uncertainty. The shaded area spanned by the 5th and 95th percentile responses covers the special cases depicted in Figures 2 through 4. These scenarios showed that imperfect credibility may lead to large and persistent positive or negative deviations of the output gap and inflation from their long-run target values.

For histories with many default events, the impulse response functions replicate the qualitative features of the economy under full discretion. However, for our benchmark calibration of $\alpha = 4$, the economy does not perform as poorly as the economy under full discretion, in which the policymaker re-optimizes every period. Thus, being somewhat committed but defaulting in many periods by chance, is different from being known to have no commitment at all.

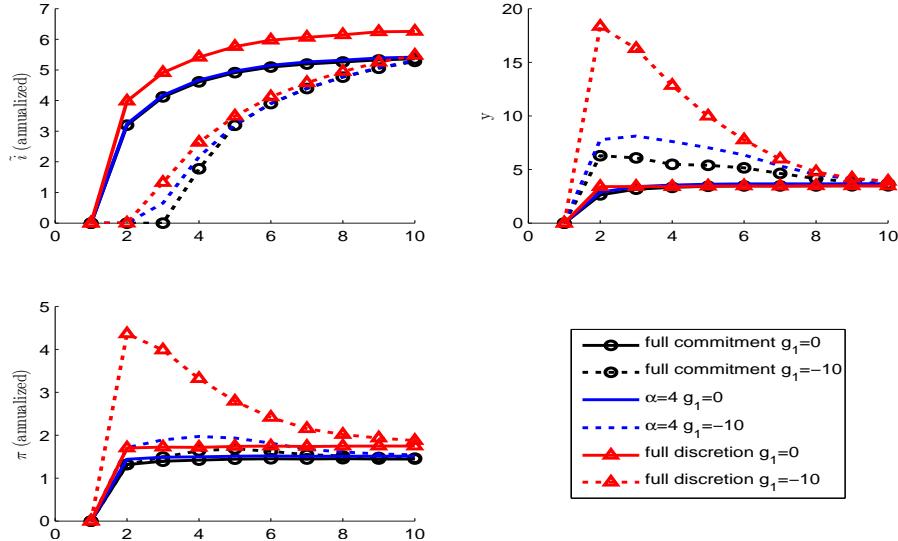
Second Experiment We consider all possible histories for x_t , $\varepsilon_{g,t}$, and $\varepsilon_{u,t}$ for $t > 1$. For the initial period, $t = 1$, however, we distinguish two cases. In the first case, the economy experiences a large negative demand shock that pushes the nominal interest rate to zero, i.e., $g_1 = -10$, $u_1 = 0$, whereas for the second case we initialize the economy at $g_1 = 0$, $u_1 = 0$.

Figure 7 plots the interquantile range between the 95th percentile response and the 5th percentile response when policymakers act under full commitment, imperfect credibility ($\alpha = 4$), and full discretion. The dashed lines depict the interquantile ranges when the economy is initially at the zero bound ($g_1 = -10$) and the solid lines depict the case when the economy is initialized at $g_1 = 0$.

The economy experiences higher uncertainty about the future paths of the output gap and the inflation rate as measured by the interquantile range when the economy is initially at the zero lower bound. Monetary policy can no longer counteract shocks as effectively as it can when the interest rate is positive. The resulting increase in the economy's sensitivity to shocks translates into (temporarily) increased uncertainty about the output gap and the inflation rate.

¹⁶Our results remain largely unaffected when replacing the mean by the median of the impulse responses.

Figure 7: Forecast Uncertainty – Interquantile Range



Notes: the figure plots the difference between the 95th and 5th percentiles in several scenarios. In all simulations, the shocks $(\varepsilon_{u,t}, \varepsilon_{g,t}, x_t)$ are drawn from their respective distributions. For the cases reported with the solid lines, the simulations are initialized at $u_1 = 0, g_1 = 0$. For the cases reported with the dashed lines, the simulations are initialized at $u_1 = 0, g_1 = -10$, which causes the interest rate to reach its zero lower bound. For the full commitment and full discretion cases, there is no uncertainty with regard to x_t shocks.

As the nominal interest rate is the predominant shock absorber when the interest rate is positive, the uncertainty about the future path of the nominal interest rate is lower when the economy is initially at the zero bound.¹⁷ For periods further in the future, the interquantile ranges for the two cases ($g_1 = -10$ and $g_1 = 0$) converge as it is less likely for the zero bound to remain binding for the case with $g_1 = -10$ once the initial demand shock has sufficiently receded.

Policy commitment is especially important when the economy is at the zero bound. For an interest rate of zero, the interquantile ranges for inflation and the output gap are substantially higher in an economy with imperfect credibility compared to an economy with full commitment. Doubts about the commitment of the central bank impinge upon the ability to use interest

¹⁷Among others, Eggertsson (2006) and Bodenstein et al. (2009) have pointed out the increased sensitivity of the economy to additional shocks once the policy interest rate has reached zero. However, as most authors have abstained from employing global methods in solving their models, earlier work does not show the rise in uncertainty explicitly.

rate announcements for forward guidance.¹⁸ By contrast, for $\alpha = 4$, there is little difference regarding the interquantile ranges for the two policy regimes away from the zero bound. In the 2008/2009 recession, markets have experienced increased volatility and uncertainty in the inflation and output outlook. Our findings suggest that part of this uncertainty is a consequence of the zero lower bound and low credibility.

4 Credibility and Times of Crisis

Actions by politicians, central bank officials, and financial markets during the sharp worldwide downturn in economic activity 2008/2009 have created doubt about how much credibility central banks actually have. In this section, we provide examples of such actions and interpret them through the lens of our model.

4.1 Credibility and Increased Uncertainty

A central bank's interest rate forecast can be misaligned with market expectations either because of credibility problems or because of large forecast uncertainty. Our framework can distinguish between these two competing explanations. The recent experience of the Swedish Riksbank provides an interesting application of our model along this dimension. Since February 2007, the Riksbank has published its intended path for the repo-rate, the short-term interest rate used by the Riksbank to achieve its policy goals.¹⁹

Figure 8 plots the repo-rate path and market expectations before and after the February 11 and April 21 2009 meetings. On February 11, the interest rate was reduced by 100 basis points to 1%. At this date, the new repo-rate path and market expectations were aligned. At the April 21 meeting, the repo-rate was reduced further and approached the effective lower bound.²⁰ This time, the repo-rate path and market expectations decoupled, as market expectations suggested

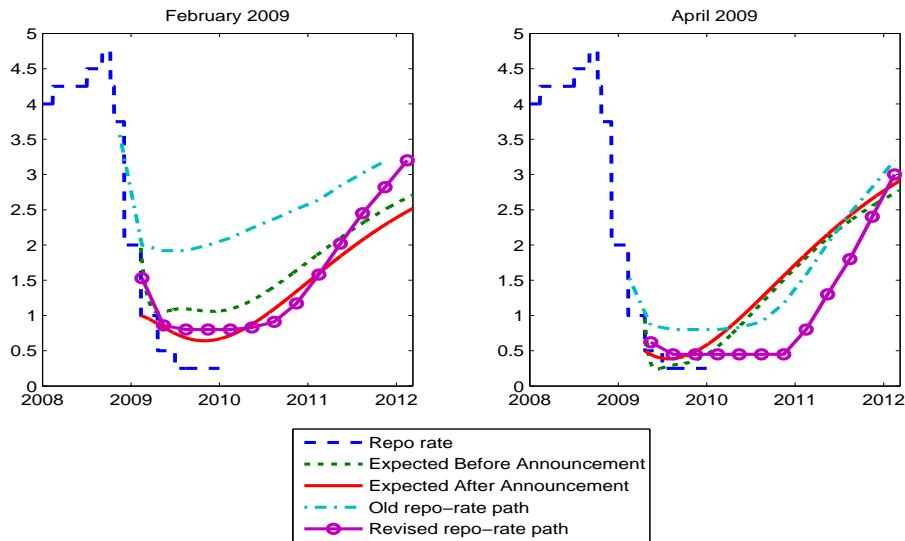
¹⁸Under full discretion the increase in uncertainty is most pronounced. As the monetary authority re-optimizes every period, no forward guidance can be provided.

¹⁹Svensson (2009, 2010) provides a discussion of this data and draws connections to credibility and forecast targeting.

²⁰For reasons not considered in the model, central banks are often reluctant to set interest rates exactly at zero and operate with a positive lower bound.

much faster tightening. These patterns are also present at other dates (not displayed); market expectations are not aligned with the repo-rate path only in periods of very low interest rates.

Figure 8: Low Credibility and Market Expectations



Notes: the figure plots the Riksbank's revised forecast of the repo-rate path as announced in February and April 2009 in the left and right panels, respectively, along with the old forecast, the historical path, and the market expectation of the path both preceding and following each announcement.

Market expectations and the repo-rate path could differ significantly because of increased forecast uncertainty about the interest rate. However, as highlighted in our discussion of Figure 7, the zero lower bound does not introduce additional uncertainty in the interest rate path. From the perspective of our model, the explanation seems to be imperfect credibility.

As discussed earlier, if the central bank reneges on past promises, the interest rate is raised earlier. Furthermore, a central bank with imperfect credibility attempts to correct agents' expectations by making even more extreme promises of low interest rates. The dynamics of market expectations and the repo-rate path in Figure 8 are largely consistent with the case in which the interest rate path announced by the central bank ignores policy renouncements while market expectations do not.²¹

²¹While central banks may not take into account policy renouncements in their forecasts, it is arguable whether in practice central banks change their forecasts to more extreme paths in order to correct expectations. Our

4.2 Central Bank Statements

Kohn (2009b) states: “To be sure, we have not followed the theoretical prescription of promising to keep rates low enough for long enough to create a period of above-normal inflation.” Similar statements can be found in Kohn (2009a) and Bernanke (2009).²² The intention of U.S. policymakers not to follow the recommendations of the optimal policy under full commitment, casts doubt on the credibility of the Federal Reserve under the assumption that the New Keynesian model applies.

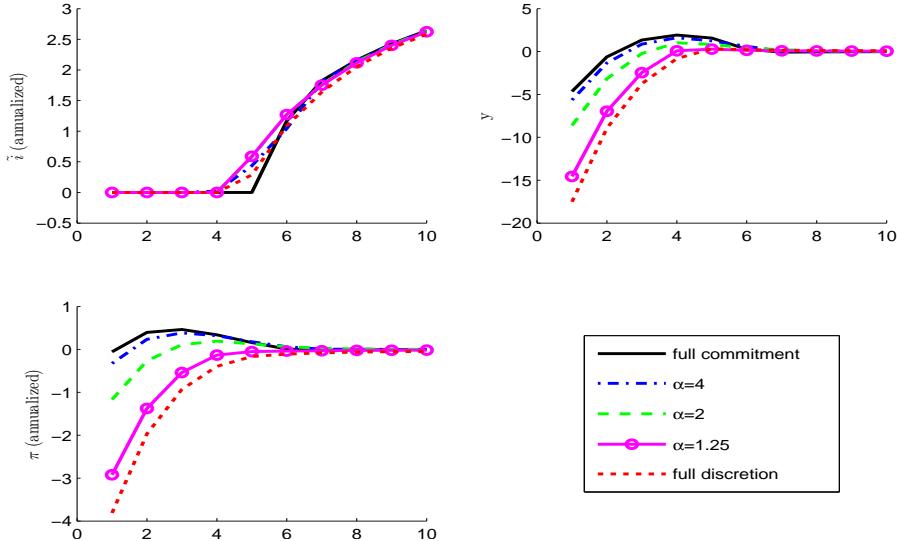
Current Federal Reserve policy simultaneously promises to keep the interest rate low and to prevent inflation from rising. As pointed out by Walsh (2009), such policies are inconsistent with the optimal policy under full commitment. However, these two policies are not mutually exclusive under imperfect credibility. It is because of its imperfect credibility that the central bank makes an even more extreme promise to leave the interest rate low (see Figure 2). At the same time, the policymaker knows that with positive probability the policy path will be re-optimized to avoid the rise in inflation (see Figure 3).

Rather than by appealing to specific histories, we could interpret the above statements by referring to the average path of the economy. In this case, a policy that avoids an expected rise of inflation above target as suggested in Kohn (2009b) indicates very low commitment. Figure 9 plots the mean impulse responses to a large contractionary demand shock that pushes the interest rate to the zero bound for different degrees of policy credibility. The mean response does not entail a significant rise of the output gap and inflation above their long-run target levels for $\alpha < 1.25$.

model identifies the incentive to make a more extreme promise, where the only cost is that such promises may need to be fulfilled. Other costs that may occur in practice affect our results quantitatively but not qualitatively.

²²Presenting the semiannual monetary policy report in July 2009, Federal Reserve Chairman Bernanke stated: “In light of the substantial economic slack and limited inflation pressures, monetary policy remains focused on fostering economic recovery. Accordingly, as I mentioned earlier, the FOMC believes that a highly accommodative stance of monetary policy will be appropriate for an extended period. However, we also believe that it is important to assure the public and the markets that the extraordinary policy measures we have taken in response to the financial crisis and the recession can be withdrawn in a smooth and timely manner as needed, thereby avoiding the risk that policy stimulus could lead to a future rise in inflation. The FOMC has been devoting considerable attention to issues relating to its exit strategy, and we are confident that we have the necessary tools to implement that strategy when appropriate.”

Figure 9: Average Recovery Path for Different Commitment Levels



Notes: the figure plots the mean transition dynamics in response to a negative and large demand shock causing the interest rate to reach its zero lower bound. The shocks are initialized at $u_1 = 0$, $g_1 = -10$, and $(\varepsilon_{u,t}, \varepsilon_{g,t}) = 0 \forall t \geq 2$. All histories regarding the commitment shock are considered (i.e. $x_t \in \{C, D\}$) for the computation of the means. For the full commitment and full discretion cases, there is no uncertainty with regard to x_t shocks.

Kohn (2009b) continues: “The arguments in favor of such a policy hinge on a clear understanding on the part of the public that the central bank will tolerate increased inflation only temporarily – say, for a few years once the economy has recovered – before returning to the original inflation target in the long term. In standard theoretical model environments, long-run inflation expectations are perfectly anchored. In reality, however, the anchoring of inflation expectations has been a hard-won achievement of monetary policy over the past few decades, and we should not take this stability for granted. Models are by their nature only a stylized representation of reality, and a policy of achieving “temporarily” higher inflation over the medium term would run the risk of altering inflation expectations beyond the horizon that is desirable. Were that to happen, the costs of bringing expectations back to their current anchored state might be quite high.” Thus, policymakers seem to be very much aware how fragile their credibility is.²³ To justify their opposition against the theoretical recommendation derived from

²³See also former Fed Governor Mishkin in Mishkin (2009).

the New Keynesian model, policymakers seem to emphasize fear of losing control over inflation expectations. However, such fears can also be rationalized in our model. Although long-run inflation expectations are well-anchored in our model for any degree of credibility, inflation can be hard to control in the medium-run if credibility is low.

A related possibility is that central banks favor models in which a large fraction of agents are backward looking or employ learning mechanisms. Nevertheless insofar as some agents are forward looking, such features dampen the post-recession inflation but do not eliminate it. However, central bank statements dismiss the idea of above normal inflation and output altogether rather than claiming that these effects are of small magnitude. Furthermore, the private sector choosing to dismiss announcements and engaging in backward looking behavior may be a consequence of low central bank credibility.²⁴

4.3 Political Pressure

Since the beginning of the 2008/2009 recession, the Federal Reserve System has encountered unprecedented attacks on its independence. Federal Reserve Chairman Ben Bernanke himself said in a Time Magazine article honoring him as “Person of the Year 2009”: “It is true that the Federal Reserve faces a lot of political pressure and is unpopular in many circles.” Suggestions for how to curb the power of the Fed have ranged from stripping it of its regulatory powers, changing the selection procedures of the president of the New York Fed, to having its monetary policy decisions audited by the Government Accountability Office.²⁵ In reaction to these developments, Goodfriend (2010) has laid out a detailed plan to ensure that the Federal Reserve remains a strongly independent institution.

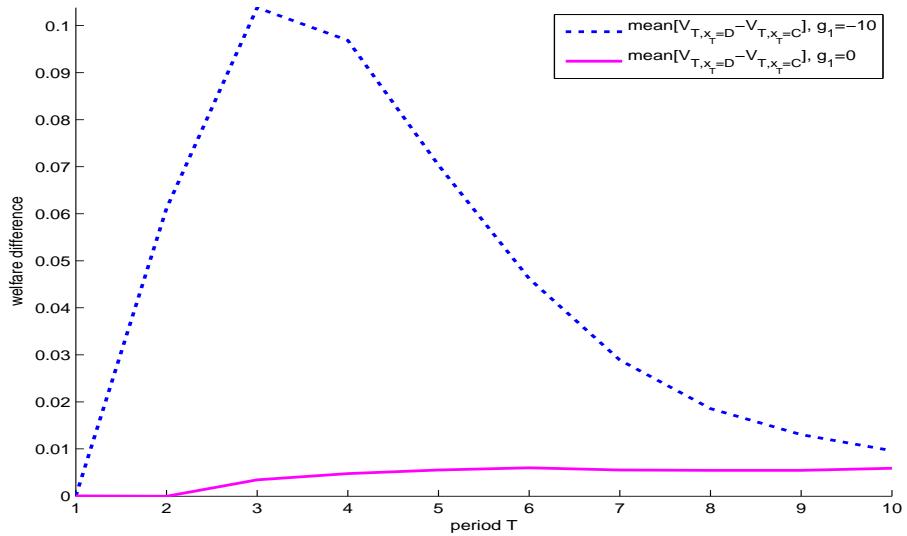
Why has the political pressure on the Federal Reserve intensified so much during this crisis,

²⁴Evans and Ramey (1992), Brock and Hommes (1997) and the literature that followed model private agents as choosing among different predictors based on their performance and availability. If the central bank disseminates and produces credible forecasts, then a larger fraction of private agents is likely to form rational expectations (see the discussions in Tobin (1972) and Sargent et al. (1973)).

²⁵Under a draft bill released by the Chairman of the Senate Committee on Banking, Housing, and Urban Affairs, Chris Dodd, future presidents of the New York Fed would be appointed by the White House and approved by the Senate. The following web site provides links to news stories that report on these attacks, see http://freerisk.org/wiki/index.php/Reform_of_the_Federal_Reserve.

although credibility and central bank independence are perceived to be important building blocks in mitigating the effects of the downturn, e.g., see Walsh (2009)? One possible answer is provided in Figure 10. We plot the difference between the value of staying committed in period T , $V_{T,x_T=C}$, and the value of defaulting in period T , $V_{T,x_T=D}$, both for an economy that has been hit by a large negative demand shock ($g_1 = -10$) and an economy that is initialized at $g_1 = 0$.²⁶ Although commitment is extremely important in times of crisis, this is also the time when commitment is under attack. In Figure 10, the temptation to default on earlier promises is significantly higher at every point in time when the economy is in a severe recession compared to the case of $g_1 = 0$.

Figure 10: Pressure to Default



Notes: the figure plots the mean period T expectation of the gain in discounted welfare due to a default in period T , in simulations of imperfect credibility with $\alpha = 4$. In all simulations, the shocks $(\varepsilon_{u,t}, \varepsilon_{g,t}, x_t)$ are drawn from their respective distributions. For the cases represented by the dotted line and the solid line, simulations are initialized at $u_1 = 0, g_1 = -10$ and $u_1 = 0, g_1 = 0$, respectively.

Admittedly, much of the public criticism of the Federal Reserve has focused on issues other than the future path of the Federal Funds rate. Nevertheless, many legislative suggestions coming out of Congress could directly or indirectly have an impact on the conduct of monetary

²⁶In Figure 10, the two groups of economies experience the same innovations and default shocks $(\varepsilon_{u,t}, \varepsilon_{g,t}, x_t)$.

policy in the future.²⁷

5 Sensitivity Analysis

In this section, we conduct two robustness exercises with respect to partial price indexation and the intertemporal elasticity of substitution.

5.1 Indexation

Galí and Gertler (1999) and several other empirical studies have found that inflation dynamics are also affected by backward looking behavior. Hence, this section considers a hybrid Phillips curve with partial indexation. A firm j that is not allowed to optimally reset its price in the current period adjusts the price mechanically by:

$$\log p_t(j) = \log p_{t-1}(j) + \zeta \pi_{t-1}. \quad (10)$$

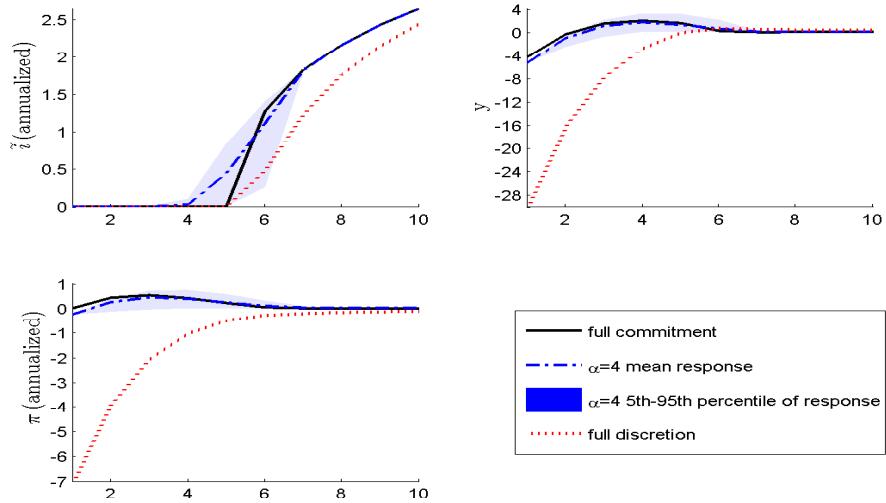
Appendix A.1 describes the Phillips curve, the objective function, and the solution method with indexation. Introducing indexation significantly complicates the solution method. First, lagged inflation increases the number of state variables to 5. Second, inflation can now be used strategically to influence future decisions. The central bank anticipates future decisions and understands that, although it cannot commit to future actions, it can affect future decisions through the inflation process. In the solution procedure, this strategic interaction means that both the level and the derivative of the policy function need to be accounted for.

Figure 11 repeats the exercise shown in Figure 6 for an indexation parameter ζ equal to 0.14.²⁸ The qualitative features pointed out earlier remain unchanged. Quantitatively, price indexation reinforces the problems posed by the zero bound constraint. Indexation is particularly painful for a discretionary central bank because the central bank already cannot control expectations and now price indexation adds momentum.

²⁷Additional political pressure stems from the large fiscal deficit that has accumulated since 2009. Financial market commentators have pushed the notion that higher interest rates may be needed soon to curb inflation fears.

²⁸For values of $\zeta \gtrsim 0.14$ the solution algorithm does not converge for all cases. For high values of ζ and low credibility the recession and the deflation can be quite severe, creating convergence problems in the algorithm. Adam and Billi (2007) also report similar issues.

Figure 11: Distribution of Impulse Response Functions – Default Uncertainty Only - Indexation



Notes: see notes to figure 6. The indexation parameter is $\zeta = 0.14$

Both with and without price indexation, a central bank with high credibility promises and implements an interest rate path that supports a post-recession boom with elevated inflation and a positive output gap. A monetary authority that acts under full discretion does not allow inflation and the output gap to exceed their target levels. In contrast to Figure 6, the observed interest rate under full discretion never lies above the interest rate under full commitment when prices are partially indexed. Thus, a low observed interest rate does not reveal high credibility. The committed policymaker can avoid a deep recession and still implement a higher path of the interest rate because her commitments are conditional on future economic outcomes. If an additional shock occurred, the central bank would adjust the promised interest rate path in accordance with its credible and fully state contingent plan.²⁹ Unable to make highly credible state contingent promises, a monetary authority with low credibility falls into a deep recession to which partial indexation adds momentum. As a result, the observed interest rate path remains below the interest rate path shown under full commitment.

²⁹This reasoning is supported by the fact that under perfect foresight and partial indexation, a fully committed central bank implements an interest rate path that lies below the path implemented by a policymaker acting with full discretion.

The incentives and effects of renouncements under imperfect credibility cannot be inferred by simply comparing the full commitment and full discretion case. Although the interest rate path under full discretion is kept below the path under full commitment, a renouncement under imperfect credibility still implies higher interest rates than promised earlier.

5.2 Intertemporal elasticity of substitution

Our baseline value for the intertemporal elasticity of substitution (σ) has been 6.25. In line with the literature, a high value of σ is supposed to capture interest rate sensitive demand for investment that is not modeled explicitly. However, a literal interpretation of our model requires a lower value of σ . Figure 12 plots the distribution of impulse response functions with default uncertainty only for $\sigma = 5.00$ and $\sigma = 6.25$.³⁰ As stated in equation (2), the demand shock g affects aggregate demand equally irrespective of the value of σ .³¹

For lower values of σ the uncertainty stemming from imperfect credibility increases considerably. In period 4, the interquantile range for inflation roughly doubles. Furthermore, the central bank does not manage to stabilize the economy as effectively. The recession is more pronounced and both inflation and output rise higher above target during the post-recession boom. With a lower value of σ , the nominal interest rate must be adjusted more aggressively in order to affect the path of the economy.

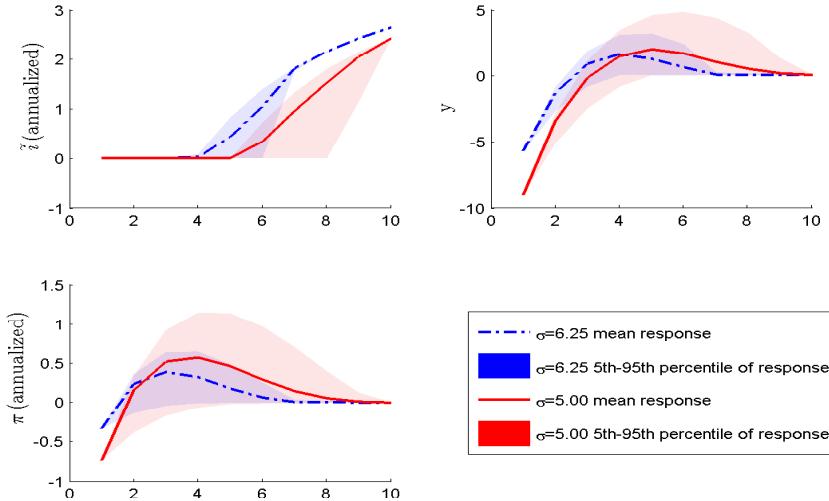
6 Conclusion

When the interest rate reaches zero, central bank credibility and the public's perception thereof are crucial for the conduct of monetary policy. Whereas credibility matters always for monetary policy, its importance is augmented at the zero lower bound. In reality, central banks have some credibility, but they typically do not operate under full commitment. Central banks

³⁰For values of $\sigma \lesssim 5$ the solution algorithm does not converge for all cases. For low values of σ , the nominal interest rate may stay at zero for a very extended period in a deep recession, preventing our algorithm from converging. To circumvent this problem, one could reduce σ_g and consider alternative calibrations. We opted to leave all the other parameters unchanged to facilitate comparisons.

³¹If g was multiplied by σ , the effective shock to the economy would be reduced for a lower values of σ ; doing so would also imply re-calibrating σ_g and the initial shock g_1 .

Figure 12: Impulse Response Functions – Default Uncertainty Only - sensitivity to σ



Notes: see notes to figure 6. The intertemporal elasticity of substitution (σ) is set to 6.25 and 5.00.

announce and describe future policies to influence current economic decisions, but markets realize that not all promises will necessarily be delivered. To address these issues, we analyzed the optimal monetary policy at the zero bound in a setting that allows for imperfect credibility.

Our work could be extended in several dimensions. Considering more complex models and imperfect credibility settings is desirable. For instance, central banks have also used unconventional measures to affect the economy and interest rate spreads at the zero lower bound. While incorporating such features is certainly interesting, we believe it is necessary to first understand the effects of imperfect credibility on conventional monetary policy. In addition, the solution of our model is already intricate and incorporating additional features may be infeasible.

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A Appendix: Model Solution

This appendix provides the mathematical details behind our solution approach and discusses our computational implementation.

A.1 The model with Indexation

The Phillips curve with indexation is given by

$$\pi_t = \frac{1}{1 + \beta\zeta} (\kappa y_t + \beta E_t \pi_{t+1} + \zeta \pi_{t-1} + u_t) \quad (11)$$

while the aggregate demand curve remains unchanged. As in Woodford (2003), the per period utility function is assumed to be

$$U_t = -(\pi_t - \zeta \pi_{t-1})^2 - \lambda y_t^2. \quad (12)$$

Following Marcer and Marimon (2009), the problem of the monetary authority written in its Lagrangian form is given by

$$V(\pi_{t-1}, u_t, g_t) \quad (13)$$

$$\begin{aligned} &= \min_{\{\gamma_t^1, \gamma_t^2\}} \max_{\{y_t, \pi_t, i_t\}} E_t \sum_{t=0}^{\infty} (\beta\eta)^t \{ -(\pi_t - \zeta \pi_{t-1})^2 - \lambda y_t^2 + \beta(1-\eta) E_t V(\pi_t, u_{t+1}, g_{t+1}) \\ &\quad + \gamma_t^1 \left(\pi_t - \frac{1}{1 + \beta\zeta} (\kappa y_t + \beta \eta E_t \pi_{t+1} + \beta(1-\eta) E_t \Psi^1(\pi_t) + \zeta \pi_{t-1} + u_t) \right) \\ &\quad + \gamma_t^2 (-y_t + \eta E_t y_{t+1} + (1-\eta) E_t \Psi^2(\pi_t) - \sigma (i_t - \eta E_t \pi_{t+1} - (1-\eta) E_t \Psi^1(\pi_t)) + g_t) \} \end{aligned}$$

$$s.t. i_t \geq -r^*$$

$$u_t = \rho_u u_{t-1} + \varepsilon_{u,t}$$

$$g_t = \rho_g g_{t-1} + \varepsilon_{g,t}$$

Note, the value $V(\pi_{t-1}, u_t, g_t)$ depends on the state of the shocks and past inflation. The policy functions under discretion are:

$$\pi_{t+1}^D \equiv \Psi^1(\pi_t, u_{t+1}, g_{t+1}, \mu_t^1 = 0, \mu_t^2 = 0) \quad (14)$$

$$y_{t+1}^D \equiv \Psi^2(\pi_t, u_{t+1}, g_{t+1}, \mu_t^1 = 0, \mu_t^2 = 0). \quad (15)$$

For convenience we adopt the short notation:

$$\pi_{t+1}^D \equiv \Psi^1(\pi_t) \quad (16)$$

$$y_{t+1}^D \equiv \Psi^2(\pi_t). \quad (17)$$

Without (partial) indexation, the value V and policy functions (Ψ^1, Ψ^2) do not depend on past inflation.

Rewriting the Lagrangean

$$V(\pi_{t-1}, u_t, g_t) \quad (18)$$

$$\begin{aligned} &= \min_{\{\gamma_t^1, \gamma_t^2\}} \max_{\{y_t, \pi_t, i_t\}} E_t \sum_{t=0}^{\infty} (\beta\eta)^t \left\{ -(\pi_t - \zeta\pi_{t-1})^2 - \lambda y_t^2 + \beta(1-\eta)E_t V(\pi_t, u_{t+1}, g_{t+1}) \right. \\ &\quad + \gamma_t^1 \left(\pi_t - \frac{1}{1+\beta\zeta} (\kappa y_t + \beta(1-\eta)E_t \Psi^1(\pi_t) + \zeta\pi_{t-1} + u_t) \right) - I_\eta \frac{1}{1+\beta\zeta} \mu_t^1 \pi_t \\ &\quad \left. + \gamma_t^2 (-y_t + (1-\eta)E_t \Psi^2(\pi_t) - \sigma(i_t - (1-\eta)E_t \Psi^1(\pi_t)) + g_t) + I_\eta \frac{1}{\beta} \mu_t^2 (y_t + \sigma\pi_t) \right\} \end{aligned}$$

$$s.t. i_t \geq -r^*$$

$$u_t = \rho_u u_{t-1} + \varepsilon_{u,t}$$

$$g_t = \rho_g g_{t-1} + \varepsilon_{g,t}$$

$$\mu_t^1 = \gamma_{t-1}^1, \mu_0^1 = 0$$

$$\mu_t^2 = \gamma_{t-1}^2, \mu_0^2 = 0$$

where $I_\eta = 0$ if $\eta = 0$, and $I_\eta = 1$ if $\eta \neq 0$. The value function can be written recursively as:

$$\begin{aligned} V(\pi_{t-1}, u_t, g_t, \mu_t^1, \mu_t^2) &= \min_{\{\gamma_t^1, \gamma_t^2\}} \max_{\{y_t, \pi_t, i_t\}} h(\pi_{t-1}, y_t, \pi_t, i_t, \gamma_t^1, \gamma_t^2, \mu_t^1, \mu_t^2, u_t, g_t) \\ &\quad + \beta\eta E_t V(\pi_t, u_{t+1}, g_{t+1}, \mu_{t+1}^1, \mu_{t+1}^2) \\ &\quad + \beta(1-\eta) E_t V(\pi_t, u_{t+1}, g_{t+1}, 0, 0) \end{aligned} \quad (19)$$

where

$$\begin{aligned}
& h(\pi_{t-1}, y_t, \pi_t, i_t, \gamma_t^1, \gamma_t^2, \mu_t^1, \mu_t^2, u_t, g_t) \\
& \equiv -(\pi_t - \zeta \pi_{t-1})^2 - \lambda y_t^2 \\
& + \gamma_t^1 \left(\pi_t - \frac{1}{1+\beta\zeta} (\kappa y_t + \beta(1-\eta) E_t \Psi^1(\pi_t) + \zeta \pi_{t-1} + u_t) \right) \\
& + \gamma_t^2 (-y_t + (1-\eta) E_t \Psi^2(\pi_t) - \sigma(i_t - (1-\eta) E_t \Psi^1(\pi_t)) + g_t) \\
& - I_\eta \frac{1}{1+\beta\zeta} \mu_t^1 \pi_t + I_\eta \frac{1}{\beta} \mu_t^2 (y_t + \sigma \pi_t)
\end{aligned} \tag{20}$$

A.2 Solution Algorithm

We use value function iteration to solve the Bellman equation (19). The number of collocation nodes that span the state space $(\pi_{t-1}, u_t, g_t, \mu_t^1, \mu_t^2) \subset \mathbb{R}^5$ is $N = 50000$. The solution algorithm is as follows:

1. Make initial guesses for V , Ψ^1 , and Ψ^2 at the collocation nodes. The solutions of models that do not impose the zero lower bound or do not have indexation can be used as the starting guesses for models with these features.
2. Construct multivariate tensor product spline approximations of V , Ψ^1 , and Ψ^2 . Update V to V' by solving the *min max* problem at each collocation node. In practice, we solve a minimization problem over all state variables subject to nonlinear constraints that correspond to the first order conditions of the maximization problem. Expectations of V , Ψ^1 , and Ψ^2 are approximated using 9 Gauss-Hermite quadrature nodes.
3. Update Ψ^1 and Ψ^2 to $\Psi^{1'}$ and $\Psi^{2'}$ using the arguments π_t and y_t of the *min max* problem, respectively.
4. Repeat steps 2 and 3 until $\max [|V' - V|_{\max}, |\Psi^{1'} - \Psi^1|_{\max}, |\Psi^{2'} - \Psi^2|_{\max}] < \epsilon_{tol}$ where $\epsilon_{tol} > 0$ and $\|\cdot\|_{\max}$ indicates the maximum absolute norm.

The choice of collocation nodes requires some experimentation. We concentrate the nodes where the splines exhibit more curvature. Additionally, the location of the nodes is guided by

the distribution of values $(\pi_{t-1}, u_t, g_t, \mu_t^1, \mu_t^2)$ that are encountered in simulations, such that we do not evaluate the functions far outside of the grid of collocation nodes.