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Abstract

This paper uses a DSGE model to examine the effects of an expansion in government spending in a liquidity trap. If the liquidity trap is very prolonged, the spending multiplier can be much larger than in normal circumstances, and the budgetary costs minimal. But given this "fiscal free lunch," it is unclear why policymakers would want to limit the size of fiscal expansion. Our paper addresses this question in a model environment in which the duration of the liquidity trap is determined endogenously, and depends on the size of the fiscal stimulus. We show that even if the multiplier is high for small increases in government spending, it may decrease substantially at higher spending levels; thus, it is crucial to distinguish between the marginal and average responses of output and government debt.

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1. Introduction

During the past two decades, a voluminous empirical literature has attempted to gauge the effects of fiscal policy shocks. This literature has been instrumental in identifying the channels through which fiscal policy affects the economy, and, in principle, would seem a natural guidepost for policymakers seeking to assess how alternative fiscal policy actions could mitigate business cycle fluctuations.

However, it is unclear whether estimates of the effects of fiscal policy from this empirical literature – which focuses almost exclusively on the postwar period – should be regarded as applicable under conditions of a recession-induced liquidity trap.[1] Keynes (1933, 1936) argued in support of aggressive fiscal expansion during the Great Depression exactly on the grounds that the fiscal multiplier was likely to be much larger during a severe economic downturn than in normal times, and the burden of financing it correspondingly lighter.

In this paper, we use a New-Keynesian DSGE modeling framework to examine the implications of an increase in government spending for output and the government budget when monetary policy faces a liquidity trap. A key advantage of the DSGE framework is that it allows explicit consideration of how the conduct of monetary policy – and, in particular, the zero bound constraint on nominal interest rates – affects the multiplier.

We begin by showing that the government spending multiplier can be amplified substantially in the presence of a prolonged liquidity trap. This corroborates analysis by Eggertson (2008) and Davig and Leeper (2009), which shows that government spending can have outsized effects when monetary policy allows real interest rates to fall, and recent work by Christiano, Eichenbaum and Rebelo (2009) in a model with endogenous capital accumulation.[2] While our workhorse model is a variant of the Christiano, Eichenbaum and Evans (2005) and Smets-Wouters (2007) models, we show that the spending multiplier is even larger in versions that embed hand-to-mouth agents (as in

[1] The bulk of research suggests a government spending multiplier in the range of 0.5 to slightly above unity. One strand of the literature – originating with Barro (1981, 1990) – has estimated the multiplier by examining the response of output to changes in military spending. This approach has typically yielded multipliers in the range of 0.5-1.0, including in recent work by Hall (2009), although Ramey (2009) estimated a somewhat higher multiplier of 1.2. As emphasized by Hall, estimates based on this approach hinge critically on the relationship between output and spending during WWII and the Korean War, and may be somewhat downward-biased due to the "command-economy" features prevalent in WWII, and because taxes were raised markedly during the Korean War. An alternative approach involves identifying the government spending multiplier using a structural VAR – as in Blanchard and Perotti (2002), and Gali, Lopez-Salido, and Valles (2007). These studies report a government spending multiplier of unity or somewhat higher (after 1-2 years), though the cross-county evidence of Perotti (2007) and Mountford and Uhlig (2008) is suggestive of a lower multiplier.

[2] In contrast, Cogan et al. (2009) analyze the effects of government spending shocks in the Smets-Wouters model, and conclude that the multiplier is only slightly amplified under the range of liquidity trap durations that they consider, which extend between 4 and 8 quarters. Mertens and Ravn (2010) develop a stylized model which rationalizes a low and possibly negative spending multiplier in a liquidity trap in an environment with multiple equilibria driven by expectational shocks. In their model, an increase in fiscal spending confirms and reinforces the pessimistic expectations of the private sector.

Galí, López-Salido, and Vallés 2007) and financial frictions (as in Bernanke, Gertler, and Gilchrist 1999, and Christiano, Motto, and Rostagno 2007). Moreover, an increase in government spending against the backdrop of a deep liquidity trap puts less upward pressure on public debt than under normal circumstances, reflecting that the larger output response translates into much higher tax revenues.

At first blush, these results seem highly supportive of Keynes' argument for fiscal expansion in response to a recession-induced liquidity trap – the benefits are extremely high, and the budgetary expense to achieve it very low. But this raises the important question of why policymakers would want to limit the magnitude of fiscal expansion, and thus pass up on what appears to be a "fiscal free lunch."

Our paper addresses this question by showing that the spending multiplier in a liquidity trap decreases with the level of government spending. The novel feature of our approach is to allow the economy's exit from a liquidity trap – and return to conventional monetary policy – to be determined *endogenously,* with the consequence that the multiplier depends on the size of the fiscal response. Quite intuitively, a large fiscal response pushes the economy out of a liquidity trap more quickly. Because the multiplier is smaller upon exiting the liquidity trap – reflecting that monetary policy reacts by raising real interest rates – the marginal impact of a given-sized increase in government spending on output decreases with the magnitude of the spending hike. This dependence of the government spending multiplier on the scale of fiscal expansion evidently contrasts with a standard linear framework in which the multiplier is invariant to the size of the spending shock.[3]

The implication that the multiplier declines in the level of spending provides a potentially important rationale for limiting the size of fiscal spending packages in a liquidity trap. If so, it becomes crucial to characterize the marginal response of output and public debt to higher government spending to make informed choices about the appropriate scale of fiscal intervention in a liquidity trap. A major focus of our paper consists of providing such a quantitative characterization in an array of nested DSGE models.

Section 2 analyzes the effects of government spending shocks in a simple three equation New Keynesian model in which policy rates are constrained by the zero lower bound. Similar to previous research (e.g., Eggertson 2008), the liquidity trap is generated by an adverse taste shock

[3] Bodenstein, Erceg, and Guerrieri (2009) show in the context of simulations of a large-scale open economy model that the contractionary effect of foreign shocks on the domestic economy increases nonlinearly in the size of the shock when the domestic economy is constrained by the zero bound.

that sharply depresses the potential real interest rate. A key result of our analysis is that the government spending multiplier – measured as the contemporaneous impact on output of a very small increment in government spending – is a step function in the level of government spending. If the level of spending is sufficiently small, higher government spending does not affect the economy's exit date from the liquidity trap, and the multiplier is constant at a value that is higher than in a normal situation in which monetary policy would raise real interest rates. However, as spending rises to higher levels, the economy emerges from the liquidity trap more quickly, and the multiplier drops. The multiplier continues to drop discretely as government spending rises further – reflecting a progressive shortening of the liquidity trap – until spending is high enough to keep the economy from falling into a liquidity trap. Beyond this level of spending, the multiplier levels out at a value equal to that under normal conditions in which policy rates are unconstrained.

The simple New Keynesian model is a convenient tool for illustrating the salient role of inflation expectations in determining how the multiplier varies with the level of government spending. If prices are fairly responsive to marginal cost – as implied by relatively short-lived price contracts – the multiplier is extremely high for small increments to government spending, but drops quickly at higher spending levels. Thus, the large multipliers that apply to small fiscal expansion should not be inferred to carry over to much larger fiscal expansions, and it is particularly important to take account of the endogeneity of the multiplier under such conditions. By contrast, the multiplier function is much flatter if the slope of the Phillips Curve is lower, and even at low spending levels it isn't dramatically different than in normal times.

The simple model is also convenient tool for assessing other empirically relevant factors that may affect the multiplier, including implementation lags in spending. Implementation lags may dampen the multiplier significantly, and even cause the multiplier to be negative if the lags are sufficiently long. Thus, echoing Friedman (1953), the efficacy of fiscal policy in macroeconomic stabilization – even in a liquidity trap – can be hampered by "long and variable lags."

The implications of the stylized model prove useful in interpreting the behavior of the government spending multiplier in more empirically-realistic models. In Section 3, we analyze a workhorse model that is very similar to the estimated models of Christiano, Eichenbaum and Evans (2005) and Smets and Wouters (2007). Section 4 extends the workhorse model by including both "Keynesian" hand-to-mouth agents and financial frictions. These additional features boost the multiplier by amplifying the effects of government spending shocks on the potential real interest rate; moreover, as argued by Galí, López-Salido, and Vallés (2007), the inclusion of Keynesian households can help account for the positive response of private consumption to a government spending shock documented in structural VAR studies by e.g., Blanchard and Perotti (2002) and Perotti (2007).[4] Given the relevance of initial conditions that determine the duration and depth of the liquidity trap for the spending multiplier, we analyze the multiplier against the backdrop of a "severe recession scenario" that attempts to capture some of the features of the U.S. experience during the recent financial crisis. For each model variant, such a scenario is constructed by a sequence of adverse consumption demand shocks that depress output by about 8 percent relative to steady state, and that generate a liquidity trap lasting eight quarters.

In our workhorse model, the multiplier implied by a standard-sized 1 percent of GDP increase in government spending is 1.0 in the first four quarters following the shock The multiplier is 1.6 in the augmented model when the share of "Keynesian" households – those that consume their entire after-tax income – equals 50 percent, which would seem at the upper end of the plausible range. Under either model variant, the multiplier is considerably larger and more persistent that under normal conditions in which monetary policy would rates interest rates immediately, and the outsized output effects imply a smaller rise in government debt. Even so, the multiplier declines noticeably as the fiscal spending package exceeds 2-3 percent of GDP, reflecting that stimulus of that magnitude is sufficient to reduce the duration of the liquidity trap by a couple of quarters; for example, the multiplier drops from 1.0 to 0.7 in the workhorse model. Moreover, implementation lags can markedly reduce the multiplier. Thus, against the backdrop of an eight quarter liquidity trap, there may be substantial benefits of increasing some forms of spending that have short implementation lags; but arguments favoring such programs based on an outsized multiplier would seem to apply only for spending packages fairly modest in scale.

The government spending multiplier under our benchmark calibration increases markedly as the liquidity trap duration extends much beyond two years, mainly reflecting that expected inflation becomes much more responsive to shocks. With the high multiplier, government debt falls sharply, so that the fiscal expansion more than pays for itself. However, while such conditions of deep recession provide a strong rationale for fiscal activism, our analysis shows that the multiplier under these conditions tends to drop sharply in the level of government spending. For example, in the workhorse model with an 11 quarter liquidity trap, the multiplier is 3.3 for small increases in

[4] As discussed in recent papers by Leeper, Walker and Yang (2009) and Ramey (2009), identified VARs can produce misleading results if some of the fiscal expansion is anticipated. Accordingly, Fisher and Peters (2009) identify government spending shocks with statistical innovations to the accumulated excess returns of large U.S. military contractors, and find that positive spending shocks are associated with an output multiplier above unity and increases in hours and consumption.

government spending, but declines to 1.2 as spending rises above 2 percent of GDP. Intuitively, just as small adverse shocks can exert a large contractionary impact in a long-lived liquidity trap and deep recession, small fiscal expansions may be highly effective in mitigating the recession; but with a shallower recession, the benefits of additional stimulus drop substantially.

We conduct extensive sensitivity analysis to assess how the relationship between the multiplier and level of government spending is affected by the slopes of the price- and wage-setting schedules. Our benchmark calibration implies a fairly sluggish price and wage adjustment, with the slopes of the price- and wage-setting schedules at the lower end of empirical estimates: for example, price contracts have an effective duration of ten quarters.[5] If *both* prices and wages were more responsive, the multiplier could be very high even for a liquidity trap lasting under two years; under such conditions, fiscal policy would be a very potent tool for reversing the sizeable deflationary pressures that would occur in the wake of recession.[6] However, the multiplier also drops abruptly with the size of the fiscal stimulus, as greater stimulus shortens the duration of the liquidity trap. Thus, with four quarter price and wage contracts, the multiplier exceeds 10 in our workhorse model for a very small increase in spending, but drops to unity when government spending is boosted more than 1 percent of GDP. Given the resilence of short-run expected inflation during the past recession, we are somewhat skeptical of calibrations that imply such extreme variations in the multiplier across spending levels. But at the least, our analysis underscores that the multiplier tends to decline sharply in the level of government spending under exactly the same conditions that are favorable to a high multiplier, i.e., when expected inflation is highly responsive.

Taken together, our results suggest a somewhat nuanced view of the role of fiscal policy in a liquidity trap. For an economy facing a deep recession that appears likely to keep monetary policy constrained by the zero bound for well over two years, there is a strong argument for increasing government spending on a temporary basis. Consistent with the views originally espoused by Keynes, this temporary boost can have much larger effects than under usual conditions, and comes at a low cost to the Treasury. But as the multiplier can drop quickly with the level of fiscal spending, larger spending programs may suffer from sharply diminishing returns, and may increase government debt at the margin. Against the backdrop of a shorter-lived liquidity trap of less than two years, the multiplier is probably only slightly above unity even in the ideal situation in

[5] We stress that the implied slope of the Phillips Curve is consistent with empirical estimates (albeit at the lower end). In this paper, we find it convenient to simply map a given slope of the Phillips curve into an effective contract duration under the assumption that marginal costs are identical across firms. However, the same slope of the Phillips curve can be consistent with a much shorter price contract duration if capital is firm-specific, as shown by e.g. Galí, Gertler, and López-Salido (2001) and Altig, Christiano, Eichenbaum and Lindé (2010).

[6] As shown below, the multiplier isn't nearly as large if only price contracts are shorter.

which fiscal stimulus can be implemented immediately. Such an environment clearly presents more risks that a fiscal spending program will fall short of the objective of producing a large output gain at minimal public cost, especially if the stimulus plan is large in scale and has substantial implementation lags.

2. A stylized New Keynesian model

As in Eggertsson and Woodford (2003), we use a standard log-linearized version of the New Keynesian model that imposes a zero bound constraint on interest rates. Our framework allows exit from the liquidity trap to be determined endogenously, rather than fixed arbitrarily, an innovation that is crucial in showing how the multiplier varies with the level of fiscal spending.

2.1. The Model

The key equations of the model are:

$$x_t = x_{t+1|t} - \hat{\sigma}(i_t - \pi_{t+1|t} - r_t^{pot}), \tag{1}$$

$$\pi_t = \beta\pi_{t+1|t} + \kappa_p x_t, \tag{2}$$

$$i_t = \max\left(-i, \gamma_\pi \pi_t + \gamma_x x_t\right), \tag{3}$$

$$r_t^{pot} = \frac{1}{\hat{\sigma}}\left(1 - \frac{1}{\phi_{mc}\hat{\sigma}}\right)\left[g_y(g_t - g_{t+1|t}) + (1-g_y)\nu_c(\nu_t - \nu_{t+1|t})\right] \tag{4}$$

where $\hat{\sigma}$, κ_p, and ϕ_{mc} are composite parameters defined as:

$$\hat{\sigma} = \sigma(1-g_y)(1-\nu_c) \tag{5}$$

$$\kappa_p = \frac{(1-\xi_p)(1-\beta\xi_p)}{\xi_p}\phi_{mc} \tag{6}$$

$$\phi_{mc} = \frac{\chi}{1-\alpha} + \frac{1}{\hat{\sigma}} + \frac{\alpha}{1-\alpha} \tag{7}$$

and where x_t is the output gap, π_t is the inflation rate, i_t is the short-term nominal interest rate, and r_t^{pot} is the potential (or "natural") real interest rate. All variables are measured as percent or percentage point deviations from their steady state level.[7]

[7] We use the notation $y_{t+j|t}$ to denote the conditional expectation of a variable y at period $t+j$ based on information available at t, i.e., $y_{t+j|t} = \mathrm{E}_t y_{t+j}$. The superscript 'pot' denotes the level of a variable that would prevail under completely flexible prices, e.g., y_t^{pot} is potential output.

Equation (1) expresses the "New Keynesian" IS curve in terms of the output and real interest rate gaps. Thus, the output gap x_t depends inversely on the deviation of the real interest rate $(i_t - \pi_{t+1|t})$ from its potential rate r_t^{pot}, as well as directly on the expected output gap in the following period. The parameter $\hat{\sigma}$ determines the sensitivity of the output gap to the real interest rate; as indicated by (5), it depends on the household's intertemporal elasticity of substitution in consumption σ, the steady state government spending share of output g_y, and a (small) adjustment factor ν_c which scales the consumption taste shock ν_t. The price-setting equation (2) specifies current inflation to depend on expected inflation and the output gap, where the sensitivity to the latter is determined by the composite parameter κ_p. Given the Calvo-Yun contract structure, equation (6) implies that κ_p varies directly with the sensitivity of marginal cost to the output gap ϕ_{mc}, and inversely with the mean contract duration $(\frac{1}{1-\xi_p})$. The marginal cost sensitivity equals the sum of the absolute value of the slopes of the labor supply and labor demand schedules that would prevail under flexible prices: accordingly, as seen in equation (7), ϕ_{mc} varies inversely with the Frisch elasticity of labor supply $\frac{1}{\chi}$, the composite parameter $\hat{\sigma}$ determining the interest-sensitivity of aggregate demand, and the labor share in production $(1 - \alpha)$.

Equation (4) indicates that the potential real interest rate is driven by two exogenous shocks, including a consumption taste shock ν_t and government spending shock g_t. Each shock, if positive, raises the marginal utility of consumption associated with any given output level, which puts upward pressure on the real interest rate if the shock is front-loaded.[8] The consumption taste shock and government spending shock are assumed to follow an AR(1) process with the same persistence parameter $(1 - \rho_\nu)$, e.g., the taste shock follows:

$$\nu_t = (1 - \rho_\nu)\nu_{t-1} + \varepsilon_{\nu,t}, \tag{9}$$

Given the same stochastic structure for the shocks, it is evident from equation (4) that these shocks affect the potential real interest rate in an identical way.

The log-linearized equation for the stock of government debt – assuming for simplicity that the steady state government debt stock is zero – is given by:

$$b_t = (1 + r)b_{t-1} + g_y(g_t - l_t - \zeta_t) - \tau_t, \tag{10}$$

[8] The effect of each shock on the marginal utility of consumption λ_{ct} can be expressed:

$$\lambda_{c,t} = -\frac{1}{\hat{\sigma}}c_t + \frac{\nu_c(1 - g_y)}{\hat{\sigma}}\nu_t = \frac{1}{\hat{\sigma}}\left[\frac{(g_y g_t - y_t)}{1 - g_y} + \nu_c(1 - g_y)\nu_t\right] \tag{8}$$

where c_t is consumption, y_t output, and g_t government spending.

where l_t is labor hours, ζ_t is the real wage, b_t is end-of-period government debt, and τ_t is a lump-sum tax (with both b_t and τ_t expressed as a share of steady state GDP). The government derives tax revenue from a fixed tax on labor income τ_L, and from the time-varying lump-sum tax τ_t. The tax rate τ_L is set so that government spending is financed exclusively by the distortionary labor tax in the steady state. Lump-sum taxes adjust according to the reaction function:

$$\tau_t = \varphi_\tau \tau_{t-1} + \varphi_b b_{t-1}, \tag{11}$$

Given that agents are Ricardian and that only lump-sum taxes adjust dynamically, the fiscal rule only affects the evolution of the stock of debt and lump-sum taxes, with no effect on other macro variables. In Section 3, we consider the implications of rules in which distortionary taxes adjust dynamically.

Our benchmark calibration is fairly standard. The model is calibrated at a quarterly frequency. We set the discount factor $\beta = 0.995$, and the steady state net inflation $\pi = .005$; this implies a steady state interest rate of $i = .01$ (one percent at a quarterly rate, or four percent at an annualized rate). We set the intertemporal substitution elasticity $\sigma = 1$ (i.e., logarithmic period utility), the capital share parameter $\alpha = 0.3$, the Frisch elasticity of labor supply $\frac{1}{\chi} = 0.4$, the government share of steady state output $g_y = 0.2$, and the scale parameter on the consumption taste shock $\nu_c = 0.01$. We examine a range of values of the price contract duration parameter ξ_p to highlight the senstivity of the fiscal multiplier to the Phillips Curve slope κ_p. It is convenient to assume that monetary policy would completely stabilize output and inflation in the absence of a zero bound constraint, which can be regarded as a limiting case in which the coefficient on inflation γ_π in the interest rate reaction function becomes arbitrarily large. The parameters of the tax rule are set so that $\varphi_b = .01$, and $\varphi_\tau = .98$. Importantly, the very small value of the coefficient φ_b implies that the contribution of lump-sum taxes to the response of government debt is extremely small in the first couple of years following a shock (so that almost all variation in tax revenue comes from fluctuations in labor tax revenues). Finally, the preference and government spending shocks are assumed to follow the AR(1) process in equation (9) with persistence of 0.9, so that $\rho_\nu = 0.1$.

2.2. Impulse Responses to a Front-Loaded Rise in Government Spending

The effects of fiscal policy in a liquidity trap depend on agents' perceptions about how long the liquidity trap would last in the absence of fiscal stimulus, as well as the severity of the associated recession.[9] Thus, we must first specify a shock(s) that pushes the economy into a liquidity trap, and then consider how fiscal policy operates against the backdrop of these initial conditions. Clearly, a number of different shocks could cause the "notional" interest rate – the level of the interest rate if monetary policy were unconstrained – to fall persistently below zero, and consequently generate a liquidity trap.

For simplicity, we follow the recent literature – including Eggertson and Woodford (2003), and Eggertson (2009), and Adam and Billi (2008) – by assuming that the liquidity trap is caused by an adverse taste shock ν_t that sharply depresses the potential real interest rate r_t^{pot}. The response of r_t^{pot} to the adverse taste shock is depicted by the solid line in Figure 1a. Given the assumption that monetary policy would fully stabilize inflation and the output gap if feasible, the nominal interest rate i_t simply tracks r_t^{pot} provided that the implied nominal rate is non-negative (i.e., $i_t = r_t^{pot}$, recalling that both variables are measured as percentage point deviations from baseline). The concurrence of the nominal and potential real interest rate is apparent in Figure 1a beginning in period T_n, which is the first period in which r_t^{pot} exceeds $-i = -1$ percent (the figure shows the annualized interest rate, so -4 percent). However, because $r_t^{pot} < -i$ prior to T_n, equations (1)-(3) imply that the nominal interest rate must equal its lower bound of $-i$. The taste shock is scaled so that the liquidity trap lasts for $T_n = 8$ quarters.

The solid lines in Figure 2 shows the effects of the taste shock on the output gap, inflation, the real interest rate, and government debt (relative to baseline GDP). To highlight the role of expected inflation in amplifying the effects of the shock, it is useful to begin by illustrating the effects in a limiting case in which inflation (and hence expected inflation) is constant. This is achieved by assuming that the average duration of price contracts is arbitrarily long, so that κ_p (in equation 2) is close to zero.[10] The left column of Figure 2 shows this limiting case. The real interest rate declines in lockstep with the nominal interest rate (i.e., by $i = 4$ percent). However, because the potential interest rate declines by more, and remains persistently below $-i$, output falls persistently below potential. The government debt/GDP ratio increases substantially, mainly because revenue from the labor income tax declines in response to lower labor demand and falling real wages.

We next consider the effect of a one percentage point of (steady state) GDP rise in government

[9] In the more empirically realistic models considered in Sections 3 and 4, the underlying shock process is chosen to roughly match the decline in U.S. GDP during the recent recession. However, in this section we simply scale the taste shock to generate a liquidity trap of the same duration (8 quarters) as in the models considered subsequently.

[10] The parameter ξ_p is set equal to .9995, implying a mean duration of price contracts of 2000 quarters, and a value of κ_p of .0000028.

spending against this backdrop. As seen in Figure 1a, the higher government spending simply offsets some of the decline in r_t^{pot} induced by the negative taste shock, so that the path of r_t^{pot} shifts upward in a proportional manner. Because the government spending hike is too small to affect the duration of the liquidity trap, monetary policy continues to hold the nominal interest rate unchanged for $T_n = 8$ quarters. As seen in the left column of Figure 2, this invariance of the nominal rate implies that the higher government spending has no effect on the path of the real interest rate (dashed lines) over this period. Accordingly, the output gap is less negative in response to the combined shocks, since the real interest rate remains unchanged even though the potential real interest rate path is higher. The (partial) effect of the government spending rise – the difference between the response to the combined shocks and taste shock alone – is depicted by the dash-dotted line(s) in the figure. Recalling that government spending has no effect on the output gap outside of the liquidity trap, and only affects potential output, the spending multiplier is clearly larger in a liquidity trap. The higher government spending has virtually no effect on the path of government debt: with the outsized multiplier, tax revenues rise enough to finance the fiscal expansion.

The government spending multiplier $\frac{1}{g_y}\frac{dy_t}{dg_t}$, i.e., the percentage increase in output in response to a one percent of baseline GDP rise in government spending, is about 0.7 in this case. The multiplier is the sum of the output gap response of nearly 0.5 shown in the figure, plus the effect on potential output (not shown). The multiplier is amplified substantially more when expected inflation responds to shocks, as illustrated in the right column of Figure 2 for a calibration implying a mean duration of price contracts of 5 quarters ($\xi_p = 0.8$). In this case, the negative output gap due to the taste shock causes inflation to fall persistently (solid lines). With expected inflation falling more than the nominal interest rate, the real interest rate rises, which reinforces the contraction in output. Thus, the same-sized fall in r_t^{pot} has much larger adverse effects on output when expected inflation reacts. Conversely, in addition to boosting r_t^{pot}, higher government spending causes expected inflation to rise, and hence exerts a more stimulative effect on output than when expected inflation remains constant. The peak output gap response of 1.8 seen in the figure implies a spending multiplier of 2.1, and translates into a substantial and persistent reduction in government debt.

2.3. The Multiplier and the Size of Fiscal Spending

In the log-linearized model that ignores the zero bound constraint, the government spending multiplier is invariant to the size of the change in spending. By contrast – as we next proceed to show

– the multiplier in a liquidity trap *declines* in the level of government spending. Intuitively, this behavior reflects that the multiplier varies positively with the duration of the liquidity trap, and that the duration shortens as the level of spending rises.

Because government spending and taste shocks have the same linear effects on r_t^{pot}, and only r_t^{pot} matters for the output gap and inflation response, it is convenient to simply analyze how r_t^{pot} affects those variables in a liquidity trap. Solving the IS curve forward yields:

$$x_t = -\hat{\sigma} \sum_{j=0}^{T_n - 1} (-i - r_{t+j|t}^{pot}) + \hat{\sigma} \sum_{j=1}^{T_n} \pi_{t+j|t} + x_{T_t|t} \tag{12}$$

The output gap x_t in the current period depends on four terms. First, it depends on the *cumulative* gap between the nominal interest rate $-i$ and the potential real interest rate over the interval in which the economy remains in a liquidity trap. This cumulative interest rate gap $\sum_{j=0}^{T_n-1}(-i-r_{t+j|t}^{pot})$ can be interpreted as indicating how shocks to the potential real interest rate would affect the output gap *if expected inflation remained constant*. Second, the output gap depends on cumulative expected inflation over the liquidity trap (or equivalently, the log change in the price level $log(P_{Tn})-log(P_t)$); as indicated above, the effects of shocks to the potential real rate on the output gap can be amplified through changes in expected inflation. Third, the current output gap also depends on the expected output gap $x_{T_n|t}$ when the economy exits the liquidity trap, though both the terminal output gap and inflation terms drop under the assumption that monetary policy completely stabilizes the economy ($x_{T_{n|t}} = \pi_{Tn|t} = 0$). Finally, the exit date T_n is determined endogenously as the first period in which the expected potential real interest rate exceeds $-i$. Thus:

$$T_n = \min_j (r_{t+j|t}^{pot} > -i) \tag{13}$$

In general, this exit date depends both on the size and persistence of the shocks to r_t^{pot}. The relation between the exit date and r_t^{pot} under our baseline calibration – in which both the taste and government spending shocks follow an AR(1) with persistence equal to $(1 - \rho_\nu) = 0.9$ – is shown in Figure 1b. Because the exit date is only affected as r_t^{pot} exceeds certain threshold values, it is a step function in the level of r_t^{pot} (rising as r_t^{pot} assumes more negative values). Thus, a slightly larger adverse taste shock that caused r_t^{pot} to drop more than shown in Figure 1a would leave the duration of the liquidity trap unchanged at 8 quarters; but a large enough adverse shock would extend the duration of the trap, and a sufficiently smaller shock would shorten it.

In the limiting case in which expected inflation remains constant, we can derive a simple closed form solution for the multiplier. Because r_t^{pot} follows an AR(1) with persistence parameter 1-ρ_v, equation (12) implies that the output gap x_t equals:

$$x_t = -\hat{\sigma} \sum_{j=0}^{T_n - 1} (-i - (1-\rho_v)^j r_t^{pot}) = -\hat{\sigma} i T_n + \hat{\sigma} r_t^{pot} \frac{1-(1-\rho_v)^{T_n}}{\rho_v} < 0 \tag{14}$$

For changes in government spending that are small enough to keep the liquidity trap duration unchanged at T_n periods, the multiplier $\frac{1}{g_y}\frac{dy_t}{dg_t}$ is derived by differentiating equation (14) with respect to g_t, and adding the effect on potential output: $(\frac{dy_t^{pot}}{dg_t})$

$$\frac{1}{g_y}\frac{dy_t}{dg_t} = \frac{1}{g_y}\frac{d(y_t - y_t^{pot} + y_t^{pot})}{dg_t} = \frac{1}{g_y}(\frac{dx_t}{dg_t} + \frac{dy_t^{pot}}{dg_t}) = \hat{\sigma}\frac{1-(1-\rho_v)^{T_n}}{\rho_v}\frac{1}{g_y}\frac{dr_t^{pot}}{dg_t} + \frac{1}{g_y}\frac{dy_t^{pot}}{dg_t} \tag{15}$$

The first term – the output gap component – is positive. It varies directly with the duration of the underlying liquidity trap T_n (induced by the taste shock), reflecting that fiscal policy can only affect the output gap over the period in which the economy remains in the trap. The second term $\frac{1}{g_y}\frac{dy_t^{pot}}{dg_t}$ is equal to the spending multiplier in the flexible price equilibrium, as well as during normal times given our assumption that monetary policy, if unconstrained, keeps output at potential. The latter may be expressed as $\frac{1}{g_y}\frac{dy_t^{pot}}{dg_t} = \frac{1}{\phi_{mc}\hat{\sigma}} < 1$ (since $\phi_{mc}\hat{\sigma} = 1 + \frac{(\alpha+\chi)\hat{\sigma}}{1-\alpha} > 1$).

Substituting $\frac{1}{g_y}\frac{dr_t^{pot}}{dg_t} = \frac{1}{\hat{\sigma}}(1 - \frac{1}{\phi_{mc}\hat{\sigma}})\rho_v$ into equation (15), the multiplier can be expressed in the simple form:

$$\frac{1}{g_y}\frac{dy_t}{dg_t} = 1 - (1 - \frac{1}{\phi_{mc}\hat{\sigma}})(1-\rho_v)^{T_n} \tag{16}$$

The solid lines in the upper left panel of Figure 3 show how the marginal multiplier varies with the duration of the liquidity trap, where the latter is indicated by the tick marks along the upper axis. The multiplier associated with a tiny increment to government spending in an 8 quarter liquidity trap is about 0.7, but rises to about 0.8 against the backdrop of an 11 quarter liquidity trap (caused by a larger contractionary taste shock than in Figure 1a). The multiplier increases monotonically with the duration of the trap, but in a concave manner; and importantly, the multiplier remains less than or equal to unity provided that the liquidity trap is of finite duration, however long.

These results provide a key stepping stone for understanding how the multiplier varies with the size of the increase in government spending. While the foregoing analysis examined the effects of tiny increments to government spending against the backdrop of different initial conditions (i.e., associated with liquidity traps of varying length), we now take "initial conditions" – summarized by a given-sized taste shock – as fixed, and assess how increases in government spending affect the multiplier by reducing the duration of the liquidity trap. For a liquidity trap of duration T_n induced by the taste shock, the government spending multiplier remains constant at the value implied by equation (15) until government spending exceeds a threshold level $g_t(0)$ that boosts the potential real interest rate just enough to shorten the liquidity trap by one period (with this threshold determined by equation 13). The multiplier then jumps down to the level implied by a T_{n-1} period trap, where it remains constant for sufficiently small additional increments to spending.

In this vein, the upper left panel of Figure 3 can be reinterpeted as showing how the multiplier varies with alternative levels of government spending. For concreteness, we assume that the liquidity trap is generated by the same adverse taste shock shown in Figure 1a, so that the "0" government spending level on the lower horizontal axis implies an 8 quarter liquidity trap (shown by the tick mark on the upper horizontal axis). For a government spending hike of less than 1.2 percent of GDP, the duration of the liquidity trap remains unchanged at $T_n = 8$ quarters, and the multiplier of about 0.7 equals to the impact multiplier shown in Figure 2. If government spending increases more than 1.2 percent, but less than 3.1 percent, the liquidity trap is shortened by one period, and the multiplier falls discontinuously (to the value implied by equation (15) with $T_n = 7$). The multiplier continues to decline in a step-wise fashion – with equation (13) implicitly determining the threshold levels of spending at which the multiplier drops discontinuously – until leveling off at a constant value of $\frac{1}{g_y}\frac{dy_t^{pot}}{dg_t}$ corresponding to a spending level high enough to keep the economy from entering a liquidity trap.[11] Given that the multiplier declines with spending, the average change in output per unit increase in government spending $\frac{1}{g_y}\frac{\Delta y_t}{\Delta g_t}$ lies well above the marginal response $\frac{1}{g_y}\frac{dy_t}{dg_t}$; to differentiate between these concepts in the figure, the former is labeled the "average multiplier," and the latter the "marginal multiplier" (in a slight abuse of terminology, since the multiplier is inherently a marginal concept).

The relationship between the multiplier and level of government spending can be given an alternative graphical interpretation using Figure 1a. Recall that absent any fiscal response, the adverse taste shock would depress the path of the potential real interest rate as shown by the solid line (labeled "taste shock only"). The effect on the output gap is proportional to $\sum_{j=0}^{T_n-1}(-i - r_{t+j|t}^{pot})$, which is simply the sum of the bold vertical line segments between $-i$ and the path of r_{t+j}^{pot} (the "interest rate gaps") implied by the taste shock through period T_{n-1}. Our assumption that the economy would gradually recover even in the absence of a fiscal response is reflected in the substantial narrowing of the interest rate gap at longer horizons, which in turn allows modest fiscal

[11] As seen in the figure, cuts in government spending exert a progressively more negative marginal impact as they become large enough to extend the duration of the liquidity trap.

stimulus to shrink the duration of the trap. Even so, the 1 percent of GDP rise in government spending shown by the dashed line leaves the liquidity trap duration unchanged, implying that the the higher government spending narrows the gap between between $-i$ and r_{t+j}^{pot} over a full $T_n = 8$ periods. The quantitative effect on the output gap of incremental spending is equal to $\hat{\sigma}\frac{1-(1-\rho_v)^8}{\rho_v}\frac{dr_t^{pot}}{dg_t}dg_t > 0$. But as government spending rises above the threshold of 1.2 percent of GDP, the potential interest rate at T_{n-1} rises above $-i$, and the liquidity trap duration shortens to 7 quarters. Thus, increments to spending in the range of 2 percent of GDP (the dash-dotted line) have no effect on the interest rate gap at T_{n-1}, as the increase in the potential real rate due to the spending increment is completely offset by monetary policy. Accordingly, the fiscal impulse only shrinks the interest rate gaps for 7 periods, and the incremental effect on the output gap falls to $\hat{\sigma}\frac{1-(1-\rho_v)^7}{\rho_v}\frac{dr_t^{pot}}{dg_t}dg_t$.

The "outsized" multiplier in a liquidity trap implies that small increases in government spending have essentially no impact on government debt. This is shown in the upper right panel of Figure 3, which plots the response of government debt after four quarters (relative to baseline GDP) as a function of the level of government spending. However, larger spending increments clearly boost government debt, reflecting that the multiplier falls as spending rises.

The variation in the multiplier with the level of spending is more pronounced in the plausible case in which the Phillips Curve is upward-sloping. When expected inflation responds, movements in the potential real interest rate r_t^{pot} have larger effects on the output gap than implied by equation (14), so that the same taste shock has a larger contractionary effect, and higher government spending has a more stimulative effect. To see how the effects of variation in r_t^{pot} are magnified, equations (1) and (2) can be solved forward (imposing the zero bound constraint that $i_t = -i$) to express inflation in terms of current and future interest rate gaps:

$$\pi_t = -\hat{\sigma}\kappa_p \sum_{j=0}^{T_n-1} \psi(j)(-i - r_{t+j|t}^{pot}), \tag{17}$$

where the weighting function $\psi(j)$ is given by

$$\psi(j) = \lambda_1\psi(j-1) + \lambda_2^j, \tag{18}$$

with the initial condition $\psi(0) = 1$, and where λ_1 and λ_2 are determined as:

$$\lambda_1 + \lambda_2 = 1 + \beta + \hat{\sigma}\kappa_p, \tag{19}$$

$$\lambda_1\lambda_2 = \beta. \tag{20}$$

Given that $\kappa_p > 0$, the coefficients $\psi(j)$ premultiplying the interest rate gap grow exponentially with the duration of the liquidity trap T_n. Moreover, the contour is extremely sensitive to κ_p, as illustrated in Figure 4a for several values of κ_p associated with price contraction durations ranging from four to ten quarters.

The convex pattern of weights reflects that deflationary pressure associated with any given-sized interest rate gap ($-i - r^{pot}_{t+j|t}$) is compounded as the liquidity trap lengthens by the interaction between the response of the output gap and expected inflation. A small interest rate gap of ε at T_{n-1} would reduce the output gap $x_{T_{n-1}}$ by $-\hat{\sigma}\varepsilon$, as inflation would be expected to return to baseline at T_n. But because the lower output gap at T_{n-1} also reduces inflation by $\kappa_p\hat{\sigma}\varepsilon$, the same-sized interest rate gap of ϵ at T_{n-2} would imply a fall in x_{T_n-2} of $2\hat{\sigma}\varepsilon + \beta\kappa_p\hat{\sigma}^2\varepsilon$, with $\beta\kappa_p\hat{\sigma}^2\varepsilon$ reflecting the contribution from lower expected inflation. This nonlinear effect on the output gap arising from the expected inflation channel contributes to a self-reinforcing spiral as the duration of the liquidity trap lengthens.

Thus, consistent with recent analysis by Eggertson (2008 and 2009), Christiano, Eichenbaum, and Rebelo (2009), and Woodford (2010), the multiplier can be amplified substantially relative to normal circumstances in a long-lived liquidity trap, and to the extent that the Phillips Curve slope is relatively high. To illustrate this in our model, the lower left panel of Figure 3 plots the impact government spending multiplier under alternative specifications of the parameter ξ_p implying price contracts with a mean duration between 4 and 10 quarters. With short enough price contracts, the multiplier increases in a convex manner with the duration of the liquidity trap, in contrast to the concave relation when expected inflation is less responsive. For example, in a liquidity trap lasting 11 quarters (see the tick marks on the upper axis), the multiplier is about 6 with five quarter contracts, and 17 with four quarter contracts.

However, under precisely the same conditions in which the government spending multiplier is very large – a long-lived trap, and shorter-lived price contracts – the multiplier drops quickly as government spending increases. Intuitively, the multiplier is large under these conditions because fiscal stimulus helps reverse the strong deflationary pressure arising from from the adverse taste shock: recalling Figure 4a, this deflationary pressure can increase dramatically as the liquidity trap duration lengthens. But insofar as the duration of the recession is abbreviated by the stimulus, the deflationary pressure abates, and the benefits of additional stimulus diminish substantially. The lower left panel of Figure 3 shows how the impact marginal multiplier varies with the level of government spending assuming that the taste shock induces an eight quarter liquidity trap.[12] The multiplier associated with four quarter price contracts drops from about 3-1/2 for a spending level of 1 percent of GDP to about 1-1/2 for spending increments above 3 percent of GDP. The dropoff is even more precipitous when the initial liquidity trap is longer-lived. By contrast, the marginal multiplier for the case of 10 quarter price contracts is relatively flat, decreasing only gradually in the level of spending.

It is important to emphasize that the precise relationship between the multiplier and level of government spending depends on the characteristics of the underlying preference shock: recalling equation (13), the duration of the liquidity trap depends on all of the shocks affecting the potential real interest rate. As discussed above in the context of Figure 1a, the economy is expected to recover eventually even in the absence of any fiscal impetus, and real interest rate gap to narrow as a result. Given this eventual improvement, even fairly modest increases in government spending can boost the potential real interest rate enough to shorten the duration of the liquidity trap, which helps account for the rapid dropoff in the multiplier when expected inflation is highly responsive.[13]

The lower right panel shows the implications for government debt. Small increments to government spending can reduce the government debt substantially when price contracts are shorter-lived. But the policy response reduces the benefit of additional stimulus, causing the marginal impact on government debt to turn positive as spending rises. With longer-lived price contracts (the figure shows 10 quarter contracts), government debt rises even for relatively low levels of spending.

2.4. Effects of Implementation Lags

We conclude this section by examining the implications of lags between the announcement of higher fiscal spending and its implementation. In particular, we assume that the government announces a new stimulus plan immediately in response to the adverse preference shock, but that it takes some time for spending to peak. To capture such delays, we assume that government spending follows an AR(2) as in Uhlig (2009):

$$g_t - g_{t-1} = \rho_{g_1}(g_{t-1} - g_{t-2}) - \rho_{g_2} g_{t-1} + \varepsilon_{g,t}, \tag{21}$$

[12] As in the upper left panel, the "0" spending level on the lower horizontal axis implies an 8 quarter liquidity trap (denoted by the tick marks on the upper horizontal axis).

[13] The two-state Markov framework adopted by Eggertson (2008 and 2009), Christiano, Eichenbaum, and Rebelo (2009), and Woodford (2010) provides a great deal of clarity in identifying factors that can potentially account for a high multiplier, which is the focus of their analysis. But given that the depth of the recession – and associated fall in the potential real interest rate – is assumed to be constant in the liquidity trap state, the multiplier also turns out to be constant in a liquidity trap irrespective of the level of spending (i.e., until spending rises enough to snap the economy out of the liquidity trap entirely).

This representation makes clear that there is some persistence in the growth rate of government spending, even though the level is stationary due to the "error correction" term ρ_{g2}.

The solid lines in Figure 4b show the effects of a rise in government spending that peaks after eight quarters (achieved by setting $\rho_{g1} = .90$ and $\rho_{g2} = 0.025$) against the backdrop of the same adverse preference shock considered previously (again depicted by the dashed lines).[14] Given the implementation lag, the higher spending depresses r_t^{pot} over the entire period in which the economy is in the liquidity trap, while leaving the duration of the trap unchanged at 8 quarters. As seen by equation (4), the expectation that government spending will grow in the future depresses the potential real interest rate r_t^{pot} by encouraging saving. Interestingly, the response of output is significantly *negative*, reflecting that aggregate demand is weaker over the entire period in which the economy is in the liquidity trap. The negative output response induces a larger deterioration of the fiscal balance, and consequent boost in the government debt/GDP ratio.

The rather dramatic consequences for the multiplier shown in the figure are dependent on the monetary policy specification; it can be shown that the output response is in fact uniformly positive under a less aggressive monetary policy rule. Even so, implementation lags may have substantial implications for the multiplier, with the spending multiplier shrinking considerably if the bulk of the higher spending occurs after monetary policy is no longer constrained by the zero bound.

3. An Empirically-Validated New Keynesian Model with Capital

In this section, we present a fully-fledged model with endogenous capital accumulation. Our objectives are to assess whether the factors identified as playing a major role in influencing the multiplier in the simple New Keynesian model continue to be important in a more empirically realistic framework, as well as to provide a more reasonable quantitative assessment of the multiplier.

Our "workhorse" model is a slightly simplified variant of the models developed and estimated by Christiano, Eichenbaum and Evans (2005), and Smets and Wouters (2003, 2007). Christiano, Eichenbaum and Evans (2005) show that their model can account well for the dynamic effects of a monetary policy innovation during the post-war period. Smets and Wouters (2003, 2007) consider a much broader set of shocks, and estimate their model using Bayesian methods. They argue that it is able to fit many key features of U.S. and euro area-business cycles.

[14] The mean duration of price contracts is set to 5 quarters, as in Figure 2.

3.1. The Model

As outlined below, our model incorporates nominal rigidities by assuming that labor and product markets exhibit monopolistic competition, and that wages and prices are determined by staggered nominal contracts of random duration (following Calvo (1983) and Yun (1996)). The model includes an array of real rigidities, including habit persistence in consumption, and costs of changing the rate of investment. Monetary policy follows a Taylor rule, and fiscal policy specifies that taxes respond to government debt.

3.1.1. Firms and Price Setting

We assume that a single final output good Y_t is produced using a continuum of differentiated intermediate goods $Y_t(f)$. The technology for transforming these intermediate goods into the final output good is constant returns to scale, and is of the Dixit-Stiglitz form:

$$Y_t = \left[\int_0^1 Y_t\left(f\right)^{\frac{1}{1+\theta_p}} df \right]^{1+\theta_p} \tag{22}$$

where $\theta_p > 0$.

Firms that produce the final output good are perfectly competitive in both product and factor markets. Thus, final goods producers minimize the cost of producing a given quantity of the output index Y_t, taking as given the price $P_t\left(f\right)$ of each intermediate good $Y_t(f)$. Moreover, final goods producers sell units of the final output good at a price P_t that can be interpreted as the aggregate price index:

$$P_t = \left[\int_0^1 P_t\left(f\right)^{\frac{-1}{\theta_p}} df \right]^{-\theta_p} \tag{23}$$

The intermediate goods $Y_t(f)$ for $f \in [0,1]$ are assumed to be produced by monopolistically competitive firms, each of which produces a single differentiated good. Each intermediate goods producer faces a demand function for its output good that varies inversely with its output price $P_t\left(f\right)$, and directly with aggregate demand Y_t :

$$Y_t\left(f\right) = \left[\frac{P_t\left(f\right)}{P_t} \right]^{\frac{-(1+\theta_p)}{\theta_p}} Y_t \tag{24}$$

Each intermediate goods producer utilizes capital services $K_t\left(f\right)$ and a labor index $L_t\left(f\right)$ (defined below) to produce its respective output good. The form of the production function is Cobb-Douglas:

$$Y_t\left(f\right) = K_t(f)^{\alpha} L_t(f)^{1-\alpha} \tag{25}$$

Firms face perfectly competitive factor markets for hiring capital and the labor index. Thus, each firm chooses $K_t(f)$ and $L_t(f)$, taking as given both the rental price of capital R_{Kt} and the aggregate wage index W_t (defined below). Firms can costlessly adjust either factor of production. Thus, the standard static first-order conditions for cost minimization imply that all firms have identical marginal cost per unit of output.

The prices of the intermediate goods are determined by Calvo-Yun style staggered nominal contracts. In each period, each firm f faces a constant probability, $1-\xi_p$, of being able to reoptimize its price $P_t(f)$. The probability that any firm receives a signal to reset its price is assumed to be independent of the time that it last reset its price. If a firm is not allowed to optimize its price in a given period, we follow Christiano, Eichenbaum and Evans (2005) by assuming that it adjusts its price by a weighted combination of the lagged and steady state rate of inflation, i.e., $P_t(f) = \pi_{t-1}^{\iota_p}\pi^{1-\iota_p}P_{t-1}(f)$ where $0 \leq \iota_p \leq 1$. A positive value of ι_p introduces structural inertia into the inflation process.

3.1.2. Households and Wage Setting

We assume a continuum of monopolistically competitive households (indexed on the unit interval), each of which supplies a differentiated labor service to the production sector; that is, goods-producing firms regard each household's labor services $N_t(h)$, $h \in [0,1]$, as an imperfect substitute for the labor services of other households. It is convenient to assume that a representative labor aggregator combines households' labor hours in the same proportions as firms would choose. Thus, the aggregator's demand for each household's labor is equal to the sum of firms' demands. The labor index L_t has the Dixit-Stiglitz form:

$$L_t = \left[\int_0^1 N_t(h)^{\frac{1}{1+\theta_w}}\, dh\right]^{1+\theta_w} \tag{26}$$

where $\theta_w > 0$. The aggregator minimizes the cost of producing a given amount of the aggregate labor index, taking each household's wage rate $W_t(h)$ as given, and then sells units of the labor index to the production sector at their unit cost W_t:

$$W_t = \left[\int_0^1 W_t(h)^{\frac{-1}{\theta_w}}\, dh\right]^{-\theta_w} \tag{27}$$

It is natural to interpret W_t as the aggregate wage index. The aggregator's demand for the labor hours of household h – or equivalently, the total demand for this household's labor by all goods-producing firms – is given by

$$N_t(h) = \left[\frac{W_t(h)}{W_t}\right]^{-\frac{1+\theta_w}{\theta_w}} L_t \tag{28}$$

The utility functional of a typical member of household h is

$$\mathrm{E}_t \sum_{j=0}^{\infty} \beta^j \{ \frac{1}{1-\sigma} C_{t+j}(h) - \varkappa C_{t+j-1} - \nu_c \nu_t \}^{1-\sigma} - \frac{\chi_0}{1+\chi} N_{t+j}(h)^{1+\chi} \} \tag{29}$$

where the discount factor β satisfies $0 < \beta < 1$. The period utility function depends on household h's current consumption $C_t(h)$, as well as lagged aggregate per capita consumption to allow for the possibility of external habit persistence (Smets and Wouters 2003). As in the simple model considered in the previous section, a positive taste shock ν_t raises the marginal utility of consumption associated with any given consumption level. The period utility function also depends inversely on hours worked $N_t(h)$.

Household h's budget constraint in period t states that its expenditure on goods and net purchases of financial assets must equal its disposable income:

$$\begin{aligned} & P_t C_t(h) + P_t I_t(h) + \frac{1}{2}\psi_I P_t \frac{(I_t(h) - I_{t-1}(h))^2}{I_{t-1}(h)} + \\ & P_{B,t} B_{G,t+1} - B_{G,t} + \int_s \xi_{t,t+1} B_{D,t+1}(h) - B_{D,t}(h) \\ = \; & (1-\tau_{N,t}) W_t(h) N_t(h) + (1-\tau_K) R_{K,t} K_t(h) + \delta \tau_K P_t K_t(h) + \Gamma_t(h) - T_t(h) \end{aligned} \tag{30}$$

Thus, the household purchases the final output good (at a price of P_t), which it chooses either to consume $C_t(h)$ or invest $I_t(h)$ in physical capital. The total cost of investment to each household h is assumed to depend on how rapidly the household changes its rate of investment (as well as on the purchase price). Our specification of investment adjustment costs as depending on the square of the change in the household's gross investment rate follows Christiano, Eichenbaum, and Evans (2005). Investment in physical capital augments the household's (end-of-period) capital stock $K_{t+1}(h)$ according to a linear transition law of the form:

$$K_{t+1}(h) = (1-\delta) K_t(h) + I_t(h) \tag{31}$$

In addition to accumulating physical capital, households may augment their financial assets through increasing their government bond holdings ($P_{B,t} B_{G,t+1} - B_{G,t}$), and through the net acquisition of state-contingent bonds. We assume that agents can engage in frictionless trading of a complete set of contingent claims. The term $\int_s \xi_{t,t+1} B_{D,t+1}(h) - B_{D,t}(h)$ represents net purchases of state-contingent domestic bonds, with $\xi_{t,t+1}$ denoting the state price, and $B_{D,t+1}(h)$ the quantity of such claims purchased at time t. Each member of household h earns after-tax labor income $(1-\tau_{N,t})\, W_t(h)\, N_t(h)$, after-tax capital rental income of $(1-\tau_K)R_{K,t}K_t(h)$, and a depreciation allowance of $\delta\tau_K P_t K_t(h)$. Each member also receives an aliquot share $\Gamma_t(h)$ of the profits of all firms, and pays a lump-sum tax of $T_t(h)$ (this may be regarded as taxes net of any transfers).

In every period t, each member of household h maximizes the utility functional (29) with respect to its consumption, investment, (end-of-period) capital stock, bond holdings, and holdings of contingent claims, subject to its labor demand function (28), budget constraint (30), and transition equation for capital (31). Households also set nominal wages in Calvo-style staggered contracts that are generally similar to the price contracts described above. Thus, the probability that a household receives a signal to reoptimize its wage contract in a given period is denoted by $1-\xi_w$. In addition, we specify a dynamic indexation scheme for the adjustment of the wages of those households that do not get a signal to reoptimize, i.e., $W_t(h) = \omega_{t-1}^{\iota_w}\pi^{1-\iota_w}W_{t-1}(h)$, where ω_{t-1} is gross nominal wage inflation in period $t-1$. Dynamic indexation of this form introduces some structural persistence into the wage-setting process.

3.1.3. Fiscal and Monetary Policy and the Aggregate Resource Constraint

Government purchases G_t are assumed to follow an exogenous stochastic process given by eq. (21). Government purchases have no effect on the marginal utility of private consumption, nor do they serve as an input into goods production. Government expenditures are financed by a combination of labor, capital, and lump-sum taxes. The government does not need to balance its budget each period, and issues nominal debt to finance budget deficits according to

$$P_{B,t}B_{G,t+1} - B_{G,t} = P_tG_t - T_t - \tau_{N,t}W_tL_t - \tau_K\left(R_{K,t} - \delta P_t\right)K_t. \tag{32}$$

In eq. (32), all quantity variables are aggregated across households, so that $B_{G,t}$ is the aggregate stock of government bonds, K_t is the aggregate capital stock, and $T_t = \left(\int_0^1 T_t(h)\,dh\right)$ aggregate lump-sum taxes. In our benchmark specification, the labor (and capital) tax rate is held fixed to abstract from the effects of time-varying tax distortions on equilibrium allocations. Accordingly, lump-sum taxes adjust endogenously according to a tax rate reaction function that allows taxes to respond both to government debt (as in Section 2) and the the gross budget deficit. In log-linearized form:

$$\tau_t - \tau = \varphi_\tau\left(\tau_t - \tau\right) + \varphi_b\left(b_{G,t} - b_G\right) + \varphi_d\left(b_{G,t} - b_{G,t-1}\right), \tag{33}$$

where $b_{G,t} = \frac{B_{G,t}}{4P_t\bar{Y}}$. Although the form of the tax rate reaction function has no effect on equilibrium allocations given that all agents are Ricardian (as assumed in this section), we also perform sensitivity analysis in which the distortionary tax rate on labor income adjusts according to equation (33), in which case $\tau_{N,t}$ replaces τ_t.

Monetary policy is assumed to be given by a Taylor-style interest rate reaction function similar to equation (3) except allowing for a smoothing coefficient γ_i:

$$i_t = \max\{-i, (1-\gamma_i)(\gamma_\pi \pi_t + \gamma_x x_t) + \gamma_i i_{t-1}\} \tag{34}$$

Finally, total output of the service sector is subject to the resource constraint:

$$Y_t = C_t + I_t + G_t + \psi_{I,t} \tag{35}$$

where $\psi_{I,t}$ is the adjustment cost on investment aggregated across all households (from eq. 30, $\psi_{I,t} \equiv \frac{1}{2}\psi_I \frac{(I_t(h)-I_{t-1}(h))^2}{I_{t-1}(h)}$).

3.1.4. Solution and Calibration

To analyze the behavior of the model, we log-linearize the model's equations around the non-stochastic steady state. Nominal variables, such as the contract price and wage, are rendered stationary by suitable transformations. To solve the unconstrained version of the model, we compute the reduced-form solution of the model for a given set of parameters using the numerical algorithm of Anderson and Moore (1985), which provides an efficient implementation of the solution method proposed by Blanchard and Kahn (1980).

When we solve the model subject to the non-linear monetary policy rule (34), we use the techniques described in Hebden, Lindé and Svensson (2009). An important feature of the Hebden, Lindé and Svensson algorithm is that the duration of the liquidity trap is endogenous, and is affected by the size of the fiscal impetus. Their algorithm consists of adding a sequence of current and future innvoations to the linear component of the policy rule to guarantee that the zero bound constraint is satisfied given the economy's state vector. The innovations are assumed to be correctly anticipated by private agents at each date, and in the case where they are all positive HLS shows that the solution satisfies (34). This solution method is easy to use, and well-suited to examine the implications of the zero bound constraint in models with large dimensional state spaces; moreover, it yields identical results to the method of Jung, Terinishi, and Watanabe (2005) under the assumption of perfect foresight.

As in Section 2, we set the discount factor $\beta = 0.995$, and steady state (net) inflation $\pi = .005$, implying a steady state nominal interest rate of $i = .01$ at a quarterly rate. The subutility function over consumption is logarithmic, so that $\sigma = 1$, and the parameter determining the degree of habit persistence in consumption $\varkappa$ is set at 0.6 (similar to the empirical estimate of Smets and Wouters 2003). The Frisch elasticity of labor supply $\frac{1}{\chi}$ of 0.4 is well within the range of most estimates from the empirical labor supply literature (see e.g. Domeij and Flodén, 2006).

The capital share parameter α is set to 0.35. The quarterly depreciation rate of the capital stock $\delta = 0.025$, implying an annual depreciation rate of 10 percent. We set the cost of adjusting investment parameter $\psi_I = 3$, which is somewhat smaller than the value estimated by Christiano, Eichenbaum, and Evans (2005) using a limited information approach; however, the analysis of Erceg, Guerrieri, and Gust (2006) suggests that a lower value may be better able to capture the unconditional volatility of investment.

We maintain the assumption of a relatively flat Phillips curve by setting the price contract duration parameter $\xi_p = 0.9$. As in Christiano, Eichenbaum and Evans (2005), we also allow for a fair amount of intrinsic persistence by setting the price indexation parameter $\iota_p = 0.9$. It bears emphasizing that our choice of ξ_p does not necessarily imply an average price contract duration of 10 quarters. Altig et al. (2010) show in a model very similar to ours that a low slope of the Phillips curve can be consistent with frequent price reoptimization if capital is firm-specific, at least provided that the markup is not too high, and it is costly to vary capital utilization; both of these conditions are satisfied in our model, as the steady state markup is 10 percent ($\theta_p = .10$), and capital utilization is fixed. Specifically, our choice of ξ_p implies a Phillips curve slope of about 0.007. For reasons discussed in further detail in Section 3.1.5, this slope coefficient is a bit lower than the median estimates of recent empirical studies. For example, the median estimates of Adolfson et al (2005), Altig et al. (2010), Galí and Gertler (1999), Galí, Gertler, and López-Salido, Lindé (2005), and Smets and Wouters (2003, 2007) cluster in the range of 0.009-.014; even so, our slope coefficient is well within the confidence intervals provided by these studies.

Given strategic complementarities in wage-setting across households, the wage markup influences the slope of the wage Phillips curve. Our choices of a wage markup of $\theta_W = 1/3$ and a wage contract duration parameter of $\xi_w = 0.85-$ along with a wage indexation parameter of $\iota_w = 0.9$ - imply that wage inflation is about as responsive to the wage markup as price inflation is to the price markup.

The share of government spending of total expenditure is set equal to 20 percent. The government debt to GDP ratio is 0.5, close to the total estimated U.S. federal government debt to output ratio at end-2009. The steady state capital income tax rate, τ_K, is set to 0.2, while the lump-sum tax revenue to GDP ratio is set to 0.02. The government's intertemporal budget constraint implies that labor income tax rate τ_N equals 0.27 in steady state. The parameters in the fiscal policy rule in equation (33) are set to $\varphi_\tau = 1$, $\varphi_b = 0.05$ and $\varphi_d = 0.10$, implying that the tax rule is not very aggressive. Importantly, given the low share of government revenue accounted for by lump-sum taxes and low sensitivity of lump-sum taxes to government debt/deficits, most of the variation in the primary government budget deficit reflects fluctuations in revenue from the capital and labor income tax (due to variations in the tax base).

Finally, the parameters of the monetary policy rule are set as $\gamma_i = 0.7$, $\gamma_\pi = 3$ and $\gamma_x = 0.25$. These parameter choices are supported by simple regression analysis using instrumental variables over the 1993:Q1-2008:Q4 period. This analysis suggests that the response of the policy rate to inflation and the output gap has increased in recent years, which helps account for somewhat higher response coefficients than typically estimated when using sample periods which include the 1970s and 1980s.

3.1.5. Initial Economic Conditions

As emphasized in Section 2, the effects of fiscal policy depend on the perceived depth and duration of the underlying liquidity trap. Accordingly, we begin by using our workhorse model to generate initial macroeconomic conditions that capture some key features of the recent U.S. recession, including a sharp and persistent fall in output, some decline in inflation, and a protracted period of near-zero policy rates.

The solid lines in Figure 5a depict this "severe U.S. recession" scenario under the benchmark calibration of our model. The scenario is generated by a sequence of three unanticipated negative taste shocks ν_t that begin in 2008:Q3 and continue through 2009:Q1. Each shock ν_t follows an AR(2) to allow for some persistence in the growth rate.[15] The shock innovations are scaled to induce a maximum output contraction of about 8 percent relative to steady state. Although the initial shock is too small to push the economy into a liquidity trap, the subsequent shocks deepen the recession and generate a liquidity trap. Upon the arrival of the final shock in 2009:Q1, agents expect that the short-term nominal interest rate will remain at its lower bound of zero for 8 quarters

[15] Paralleling the case in which government spending requires implementation lags, we assume that ν_t follows $\nu_t - \nu_{t-1}=\rho_{\nu_1}(\nu_{t-1} - \nu_{t-2})$-$\rho_{\nu_2}\nu_{t-1}+\varepsilon_{\nu,t}$, where $\rho_{\nu_1} = 0.2$ and $\rho_{\nu_2} = 0.05$.

through 2010:Q4 (to highlight the zero bound constraint, the short-term nominal interest rate and inflation rates are shown in levels). Inflation falls from its steady state level of 2 percent to a trough of slightly below zero, and remains close to zero for about a year.

The peak output contraction in this scenario comes close to matching the maximum decline in U.S. output relative to trend that occurred following the intensification of the financial crisis in 2008:Q3 (the detrended U.S. output series is depicted by cross-hatches). The implication of a prolonged liquidity trap seems consistent with historical experience thus far, as actual policy rates have remained near zero since late 2008 (the cross-hatches show realized values of the federal funds rate). Moreover, given that the *perceived* duration of the liquidity trap plays a crucial role in determining the effects of fiscal stimulus, we also compare the expected duration of the liquidity trap based on our model simulation with an empirical proxy for the expected path of the policy rate based on overnight index swap rates. These projections are available 1-24 months ahead, and 36 months ahead. As seen in the lower panel, the "projected" path of the federal funds rate in the first quarter of 2009 – shortly after the current federal funds rate target was reduced to nearly zero – is below 1 percent for a horizon extending out eight quarters.[16] Although there are difficulties with interpreting this path as measuring the expected policy rate due to e.g., time-varying risk premia, this evidence suggests that the implications of our benchmark calibration are not unreasonable; in addition, we investigate the sensitivity of our results to the duration of the liquidity trap.

As seen in Figure 5a, the decline in price inflation implied by our benchmark calibration is somewhat larger than in the corresponding data (the price inflation measure is the core CPI inflation rate, and is depicted by cross-hatches). In fact, a striking feature of the recession is that both actual inflation and inflation forecasts have responded very little to large and persistent output declines. The right panel of Figure 5b plots the median forecast *path* of expected inflation over the next six quarters from the Survey of Professional Forecasters, beginning in 2008-Q3 (the solid line) and continuing through 2009-Q2. Clearly even short-term inflation expectations remained quite stable as the recession deepened.

Although our benchmark calibration implies a larger decline in inflation than occurred during the financial crisis episode, it bears emphasizing that it implies *much less* movement in inflation (and expected inflation) than other commonly-adopted calibrations. As noted previously, our chosen values of both the contract duration parameters and the coefficient on inflation in the monetary rule are towards the higher side of empirical estimates. To highlight this, Figure 5a also reports

[16] The data are from Bloomberg. Monthly OIS rates are averaged to obtain the quarterly values shown in the figure, with the values in quarters 9-11 derived from interpolation using a cubic spline.

results for two alternative calibrations. In the case labelled "more flexible p and w," the mean duration of price and wage contracts is reduced to four quarters, while another alternative labelled "loose rule" adopts the standard Taylor rule coefficients in the monetary policy rule (i.e., the parameters γ_π and γ_x in equation (34) are set to 1.5 and 0.125, respectively, compared with 3 and 0.25 under our benchmark). Under each of these alternatives, the taste shock is rescaled (reduced modestly) to account for roughly the same-sized output contraction as in the benchmark, and to imply an eight quarter liquidity trap. Inflation declines by considerably more under either of these alternative calibrations. Overall, we take these results as providing support for our benchmark calibration relative to these alternatives, while acknowledging the possibility suggested by fitting the recent recession that even our benchmark may perhaps overstate the response of inflation to highly persistent economic shocks.

3.2. Dynamic Effects of Government Spending

The solid lines in Figure 6 (labeled "ZLB benchmark") show the effects of a front-loaded increase in government expenditures equal to 1 percent of steady state output against the backdrop of the negative taste shocks described above above. The government spending shock follows an AR(1) with a persistence of 0.9. The impulse response functions shown are computed as the difference between this scenario which includes both the consumption taste shocks and government spending shock, and the previous scenario (i.e., the benchmark in Figure 5a) with only the taste shocks. While the government spending shock occurs in 2009:Q1 – at which point agents expect the liquidity trap would last 8 quarters in the absence of stimulus– this corresponds to "period 0" in the figure.[17] The fiscal expansion is assumed to be financed by lump-sum taxes as specified by equation (33)

As in the stylized model analyzed in Section 2, the fiscal policy expansion implies larger and more persistent effects on output relative to a normal situation in which policy is unconstrained (the dotted line). The outsized effects on output reflect that higher government spending boosts the potential real interest rate, while the (ex ante) real interest rate falls as nominal interest rates do not respond and expected inflation rises. The lower right panel shows the government spending multiplier. The impact multiplier is simply defined as $\frac{1}{g_y}\frac{\Delta y_t}{\Delta g_t}$, i.e. the increase in output per unit increase in government spending (both relative to steady state). More generally, the multiplier at horizon K is defined as $\frac{1}{g_y}\frac{\sum_0^K \Delta y_{t+K}}{\sum_0^K \Delta g_{t+K}}$. The implied government spending multiplier equals unity in

[17] The American Reinvestment and Recovery Act was passed in 2009:Q1, the first quarter in which the federal funds rate averaged less than 1/4 percentage point.

the four quarters following the spending shock before declining gradually.[18] Given the amplified multiplier, the government debt/GDP ratio rises by less than in a normal situation, though it still increases above zero after only a couple of quarters.[19]

The same factors identified as key determinants of the multiplier in Section 2 – including the duration of the liquidity trap and structural parameters determining the response of expected inflation – also have a major influence on the multiplier in this more empirically-realistic model. The dashed lines in Figure 6 show impulse responses to the same government spending shock against the backdrop of a longer-lived liquidity trap of 11 quarters. The multiplier is well over 2, reflecting the more persistent rise in inflation and larger implied fall in the real interest rate. The bigger multiplier contributes to a highly persistent fall in the government debt/GDP ratio, which is tantamount to a "fiscal free lunch." Similarly, the multiplier is larger under a more accomodative monetary policy rule (not shown), again due to a larger expected inflation response.

Conversely, the multiplier shows considerably less variation with the duration of the liquidity trap under conditions that give rise to a small inflation response. As in Section 2, the multiplier depends on the gap that the higher fiscal spending induces between the potential real interest rate and actual real interest rate over the period in which the economy remains in a liquidity trap. In the limiting case in which inflation is constant, the effect of increasing the liquidity trap duration on the multiplier hinges on how government spending affects the potential real interest rate at relatively distant horizons; and since these effects are small, the peak value of the multiplier shows relatively little increase with the duration of the trap. To illustrate this, Figure 6 shows the multiplier under a calibration labelled 'ZLB long duration, very sticky p and w' which imposes extremely long-lived price and wage contracts with a mean duration of 40 quarters, and against the backdrop of a Great Depression-sized output decline of almost 30 percent (the initial conditions are depicted in Figure 5a with the same label). Although the liquidity trap lasts 16 quarters, the multiplier peaks at a value of unity before declining slowly, and the higher spending boosts government debt significantly.

[18] Using our terminology in Section 2, this definition corresponds to an "average" multiplier, since it indicates how output on average responds to a fiscal stimulus package of a given size. We will examine the behavior of the "marginal" multiplier in the next subsection.

[19] Note that while the tax-rule (33) responds to government debt as a ratio of annualized trend nominal output $b_{G,t} = \frac{B_{Gt}}{4P_tY}$, the figure reports government debt relative to actual output $\tilde{b}_{G,t} \equiv \frac{B_{Gt}}{4P_tY_t}$.

3.2.1. Marginal vs. Average Responses

In Section 2, we used the simple New Keynesian model to illustrate that the marginal effects of government spending shocks can diverge substantially from the average effects captured by impulse response functions. We next provide a parallel analysis in our workhorse model. The solid line in the upper left panel of Figure 7 shows the multiplier as a function of government spending under our benchmark calibration. The output and government spending responses used in computing the multiplier are an average over the first four quarters following the spending hike. Because it is useful to first show how the multiplier behaves in a long-lived liquidity trap, the negative taste shocks which generate the initial conditions are scaled so that the liquidity trap would last 11 quarters in the absence of fiscal stimulus (as in the "ZLB longer duration" case in Figure 6).

As in the stylized model in Section 2, the multiplier follows a step function. The multiplier is constant until government spending reaches a threshold value that is large enough to shorten the duration of the liquidity trap by one period, and then drops discretely as spending surpasses this threshold. Quantitatively, the multiplier is 3.3 for small additions to spending less than 0.5 percent of GDP. However, as seen from the tick marks on the upper axis, the multiplier drops abruptly to around unity as the duration of the liquidity trap shortens by a couple of quarters. Using the terminology of Section 2, the multiplier is labeled the "marginal multiplier" to differentiate it from the average response of output per unit increase in spending, which is labeled the "average multiplier" (though when the meaning is reasonably clear from context, we simply refer to the multiplier). For the one percent of GDP rise in spending considered in Figure 6, the average multiplier of nearly 2.5 reflects a (marginal) multiplier of 3.3 for small additions to spending less than 0.5 percent of GDP, and a multiplier of 1.7 on the additional spending. As in Section 2, the rapid falloff in the multiplier reflects that fiscal stimulus is extremely effective in mitigating the effects of the recession in an environment in which monetary policy is constrained for a prolonged period; but with a shallower recession, the benefits of additional stimulus drop substantially.

The solid line in the right upper panel plots the marginal response of the government deficit/GDP ratio, inclusive of interest payments, to government spending (the deficit and spending levels are averaged over the four quarters following the spending hike). Given the long duration of the underlying liquidity trap, the marginal response is negative for spending increments less than 3 percent of GDP. As additional spending shrinks the duration of the liquidity trap below 9 quarters (as seen by the upper tick marks), the marginal effect on the government deficit turns positive, even though the average response – shown by the dotted line – remains negative. This underscores how marginal increments to fiscal spending can put upward pressure on budget deficits even when the average effects appear small.

The lower panel of Figure 7 shows the multiplier under our benchmark calibration (solid lines) and several alternatives; in each case, the initial conditions are chosen to imply a shorter-lived liquidity trap of 8 quarters. The multiplier under our benchmark calibration is about or below unity at all spending levels (as can be surmised from the upper panel as well). The multiplier can be much higher under calibrations that imply a larger response of expected inflation. The amplification of the multiplier is particularly dramatic under the case of "more flexible prices and wages," which specifies that both price and wage contracts last only four quarters. In this case, the multiplier exceeds 10 for low spending levels below 0.2 percent of GDP. The high potency of fiscal policy reflects that it offsets the strong deflationary pressure that would otherwise occur in response to the contractionary (taste) shocks. However, the multiplier drops precipitously as spending increases and the liquidity trap duration shortens (the duration is indicated by the upper tick markets *for this particular calibration*[20]). For a spending increment a little over 1 percent of GDP, the liquidity trap shortens to only 6 quarters, and the multiplier is only about unity. A second alternative examines a case in which only prices are less sticky than under our baseline (i.e. ξ_p is lowered from 0.90 to 0.75), but wage-setting remains unaltered. The multiplier function in this case (the dotted line) is only slightly higher than under our benchmark, reflecting that the sluggish behavior of wages keeps price inflation from moving as much as under the previous alternative. This calibration underscores that the dramatically higher multiplier under the "more flexible p and w" calibration hinges on *both prices and wages* being considerably more flexible than under our benchmark. The third alternative ("looser policy rule") assumes that monetary policy is less aggressive in responding to inflation and the output gap than under our benchmark (i.e,. γ_π = 1.5 and γ_x = 0.125). The multiplier is about 1.5 for a low level of spending, and the multiplier exceeds that under our benchmark calibration by a noticeable margin at all spending levels.

Overall, these results provide a strong rationale for expanding fiscal spending in a very deep liquidity trap lasting roughly 3 years, with the caveats that it is important to take account of how the

[20] In general, the exit period from the liquidity trap depends on model's structural parameters. Hence, the spending levels at which the multiplier jumps vary across calibrations. Given the four alternative calibrations in the lower panel of Figure 7 (and also 10), the upper tick marks simply highlight where these jumps occur for the calibration in which wages and prices are highly flexible. By contrast, the jump points in the corresponding panel in Figure 3 are uniform across the alternative calibrations examined, reflecting that the exit from the liquidity trap depends only on the potential real interest rate under our simplifying assumption that monetary policy completely stabilizes the economy.

multiplier declines with higher spending, and that the high multipliers assume no implementation lags. For a shorter-lived liquidity trap of two years or less, our benchmark calibration implies a multiplier of only about unity for small increases in spending. Although we have shown that the workhorse model can generate very high multipliers if prices and wages adjust more quickly, we are skeptical of this implication it implies that the adverse taste shocks that generate the underlying liquidity trap would induce an extremely large decline in inflation. Recalling Figure 5a, inflation falls 7 percentage points below baseline under the "more flexible price and wage" calibration in response to the shocks that depress output about 8 percent. This behavior seems in contrast with the resilence of both inflation and expected inflation during the past recession that was documented in Figure 5a and 5b.

3.2.2. Effects of Implementation Lags

Figure 8 examines the sensitivity of the results to alternative assumptions about how the fiscal stimulus is financed and how quickly it can be implemented. First, we consider the implications of gradual rise in government spending of 1 percent of GDP that peaks after eight quarters, rather than immediately, while continuing to assume lump-sum tax adjustment. The (average) multiplier – the dotted line, labelled "8 quarter implementation lag lump-sum"- is only about half as large as under our benchmark with immediate implementation (the solid line), and government debt rises immediately. The smaller output response reflects that the higher spending reduces the potential real interest rate in the short-run, as the promise to increase future public spending encourages households to save. Second, we replace the benchmark assumption of lump-sum tax adjustment with the alternative in which labor taxes adjust dynamically in response to higher government debt, while retaining the assumption that government spending is front-loaded as in the benchmark.[21] The multiplier (dashed line) is reduced modestly relative to the benchmark, reflecting that the higher labor taxes dampen labor supply. But it is worth emphasizing that the parameters in the tax rate reaction function are small, and that output would be affected more if the parameters of the tax rate reaction function implied a larger tax rate response. Finally, we also consider a third variant that merges these alternatives by embedding both implementation lags, and the assumption of labor tax adjustment. The multiplier (dash-dotted line) under this alternative is only slightly positive. Clearly, implementation lags and the requirement that government purchases be financed

[21] Uhlig (2009) emphasizes that highly persistent (or permanent) increases in the level of government spending tend depress output significantly at horizons beyond a couple of years if the higher spending must be financed by a hike in the labor income tax rate.

by distortionary taxes can significantly reduce the multiplier.[22]

4. Robustness analysis: The Empirical Model Augmented with Financial Frictions and Keynesian Households

The workhorse model in Section 3 has been criticized by Galí, López-Salido and Vallés (2007), among others, for its inability to account for VAR-based empirical evidence indicating that private consumption rises in response to higher government spending. As shown by GLV, the inclusion of non-Ricardian households helps account for this empirical evidence, and allows their model to generate a somewhat higher government spending multiplier even in normal times when monetary policy raises interest rates. In addition, the workhorse model of Section 3 omits potentially important financial channels, such as movements in private credit spreads due to balance sheet effects. Given these potential shortcomings of the workhorse model, we investigate the sensitivity of our results to the inclusion of non-Ricardian households and financial frictions.

4.1. Key Model Equations and Calibration

To incorporate these features, we modify the model in Section 3 along two dimensions. First, we assume that a fraction s_{kh} of the population consists of "Keynesian" households whose members consume their current after-tax income each period, and set their wage equal to the average wage of the optimizing households. Because all households face the same labor demand schedule, each Keynesian household works the same number of hours as the average optimizing household. Thus, the consumption of Keynesian households $C_t^K(h)$ is simply determined as

$$P_t C_t^K(h) = (1 - \tau_{Nt}) W_t(h) N_t(h) - T_t.$$

Second, we incorporate a financial accelerator following the basic approach of Bernanke, Gertler and Gilchrist (1999). Thus, entrepreneurs acquire capital to supply to homogeneous factor markets, but must pay an external finance premium on the funds they borrow from households due to an agency problem. We follow Christiano, Motto and Rostagno (2007) by assuming that the debt contract between the entrepreneurs and lenders (households) is written in nominal terms (rather than real terms as in BGG 1999). At an aggregate level, the corporate finance premium varies with the degree of leverage of the economy.

[22] We have also examined a case in which the capital income tax rate adjusts dynamically to government debt. This alternative implies an even lower output response than in Figure 8 (actually negative if capital taxes adjust quickly enough).

We set the population share of the Keynesian households to optimizing households, s_{kh}, to 1/2, which implies that the Keynesian households' share of total consumption is about 1/3. This calibration perhaps probably overstates the role of non-Ricardian households in affecting consumption behavior, but seems useful to help put plausible bounds on how the multiplier may vary with the degree of non-Ricardian behavior in consumption (recognizing that the workhorse model is a special case in which $s_{kh} = 0$ and there are no financial frictions). Our calibration of the parameters affecting the financial accelerator follow BGG (1999).[23]

4.2. Dynamic Effects of Fiscal Policy Expansions

We analyze the effects of a government spending shock against the backdrop of initial economic conditions that are generated in the same manner as described in Section 3.1.5. In particular, we calibrate the same AR(2) process for the consumption taste shock, and again feed in a sequence of three innovations beginning in 2008:Q3 to roughly match the peak decline in U.S. output relative to trend that occurred during the financial crisis. The short-term interest rate falls to zero in 2009:Q1, and remains at this level for 8 quarters, while inflation falls about 2.5 percent below its steady state value of 2 percent. Given the stronger propagation mechanisms in this model, the size of the consumption innovations are considerably smaller than those considered in the previous section.

Figure 9 shows the effects of a front-loaded increase in government expenditures of 1 percent to trend GDP (as in Figure 6, the impulse responses show the "partial effect" of the rise in government spending). We report results both for the benchmark parameterization of the model where monetary policy is constrained by the zero lower bound (labeled 'ZLB Full Model'), and for the case in which policy is unconstrained (labeled 'Normal Full Model'). The (average) spending multiplier is about 1.6 in the four quarters following the shock, considerably larger than in a normal situation in which policy raises real interest rates.

The figure also shows the spending multiplier for the benchmark calibration of the model analyzed in the previous section for both the constrained ('ZLB Workhorse') and unconstrained ('Normal Workhorse) settings of the policy rule. The government spending multiplier in the liquidity trap case is substantially enhanced by the inclusion of Keynesian households and financial frictions. This is mainly due to a much larger rise in the potential real interest rate relative to the workhorse

[23] The monitoring cost, μ, expressed as a proportion of entrepreneurs' total gross revenue, is 0.12. The default rate of entrepeneurs is 3 percent per year, and the variance of the idiosyncratic productivity to entrepreneurs is 0.28.

model: because government spending boosts wage income and hence the consumption of Keynesian households, real interest rates must rise more to keep output at potential.[24] In the normal case when monetary policy is unconstrained, the larger increase in the potential real rate in the full model is largely offset by a larger monetary policy response, so that the disparity in the multiplier across the two models is more modest; but in a liquidity trap, the disparity in the potential real rate translates into a much larger difference in the output responses.

The upper left panel of Figure 10 reports the government spending multiplier as a function of the size of the increment to government spending for the full model under our benchmark calibration. The initial conditions imply a liquidity trap lasting 10 quarters to emphasize how the multiplier behaves in a very long-lived trap. The multiplier is about 4 for increments to spending below 1/2 percent of GDP, and remains above unity even for relatively large spending increases above 4 percent of GDP. Moreover, even the marginal impact on the government budget balance is negative for spending increments in this range. These results provide an even stronger rationale for increasing fiscal spending in a very deep liquidity trap than the workhorse model.

The lower left panel shows the multiplier when initial conditions imply a liquidity trap lasting only 8 quarters. Under our benchmark calibration, the multiplier is about 1.6 for a small increase in spending, and declines to 1.2 for spending increases exceeding 2.5 percent of GDP. As seen in the right panel, the government deficit falls in the first year following the shock for spending increments in this range. As with the workhorse model, while the multiplier can be much higher under conditions that induce a sizeable inflation response, the multiplier tends to drop extremely sharply. Thus, the multiplier is dramatically higher under the calibration with four quarter price and wage contracts ("more flexible prices and wages"), exceeding 10 for spending below 0.2 percent of GDP, but falls to about unity as spending rises a bit above 2 percent of GDP.

5. Conclusions

Our results suggest that for an economy facing a deep recession that appears likely to keep monetary policy constrained by the zero bound for well over two years, there is a strong argument for increasing government spending on a temporary basis. But even under such conditions, it is important to recognize that the marginal benefits of fiscal stimulus may drop substantially as spending rises, so that there is some risk that larger spending programs may have a low marginal

[24] Most of the difference between the workhorse model and the full model is driven by the inclusion of Keynesian households (and the assumption that they account for half of all households).

payoff. Against the backdrop of a shallower recession and expected liquidity trap duration of under two years, there may still be substantial benefits of increasing some forms of spending with short implementation lags; but "outsized" multipliers are only likely to apply to relatively small spending programs.

Governments and central banks clearly have an array of options in addition to stimulative fiscal policy for mitigating the effects of a liquidity trap. For example, many central banks have used the asset side of their balance sheet to support credit markets by providing liquidity and purchasing long-term securities. Although the models we have examined are not designed to assess the effectiveness of such actions, our analysis highlights the importance of analyzing the effects of such actions *jointly* with the fiscal stimulus packages in order to properly assess their marginal impact.

Several extensions of our framework would be useful to explore in future research. First, it would be desirable to build on the work of Christiano, Eichenbaum and Rebelo (2009), Nakata (2009), and Woodford (2010) by investigating the normative question of the optimal level of government spending in a liquidity trap in our framework with endogenous exit. Second, while our paper has focused on government consumption spending exclusively as the tool of fiscal policy, it would be interesting to consider alternative fiscal measures such as tax cuts and targeted transfers.[25] Finally, as the multiplier may be affected through cross-country linkages, it would desirable to extend our analysis to an open economy setting.

[25] Eggertsson (2009) argues that tax cuts aimed at stimulating aggregate demand rather than aggregate supply are preferable in a liquidity trap (e.g. sales taxes and implementing an investment tax credit).

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Figure 1a: Negative Taste Shock and Fiscal Response

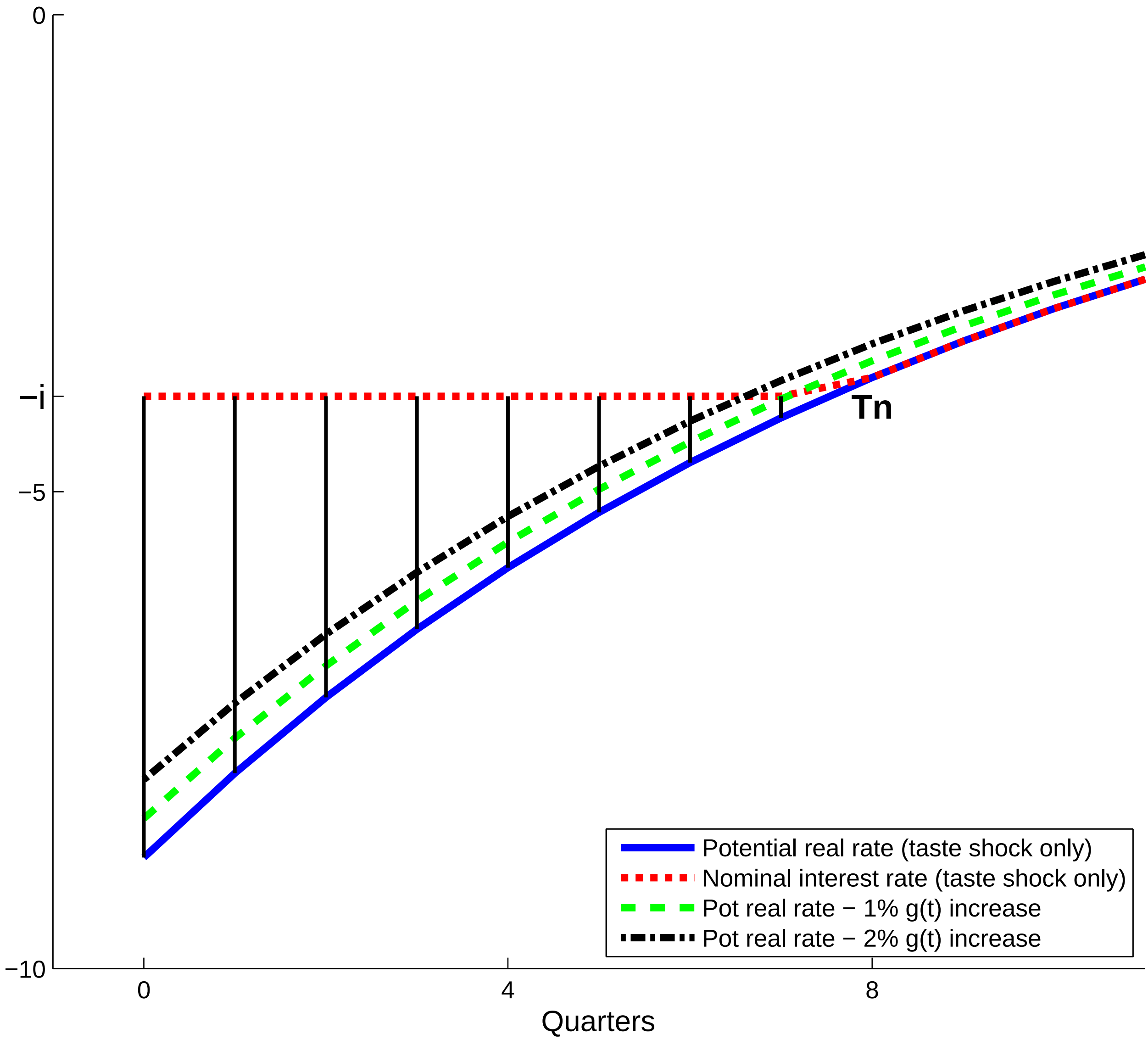

Figure 1b: Liquidity Trap Duration and Potential Real Rate

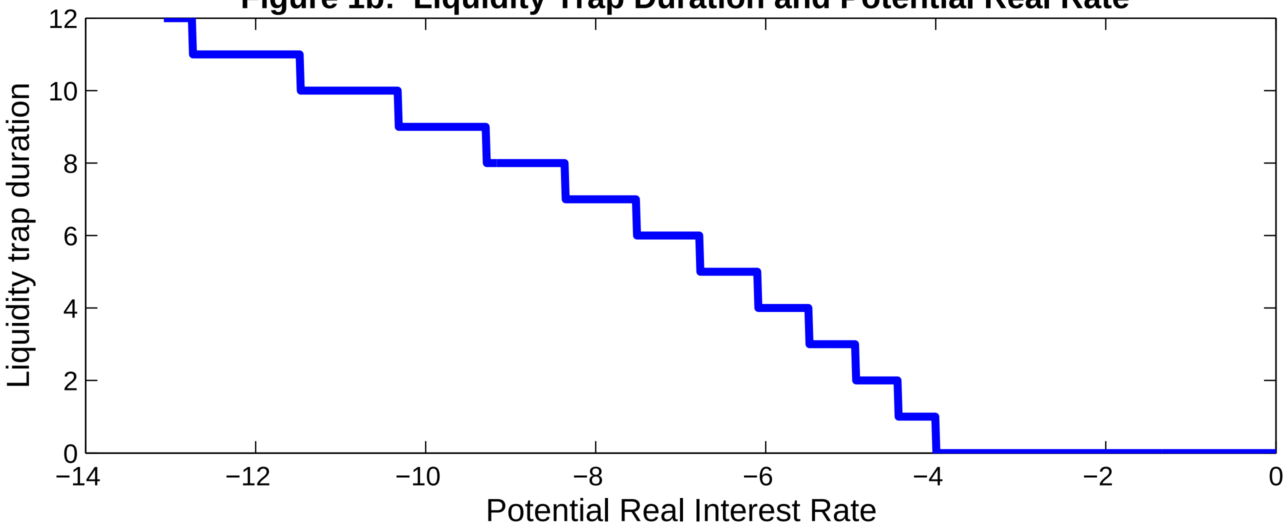

Figure 2: Immediate Rise in Government Spending

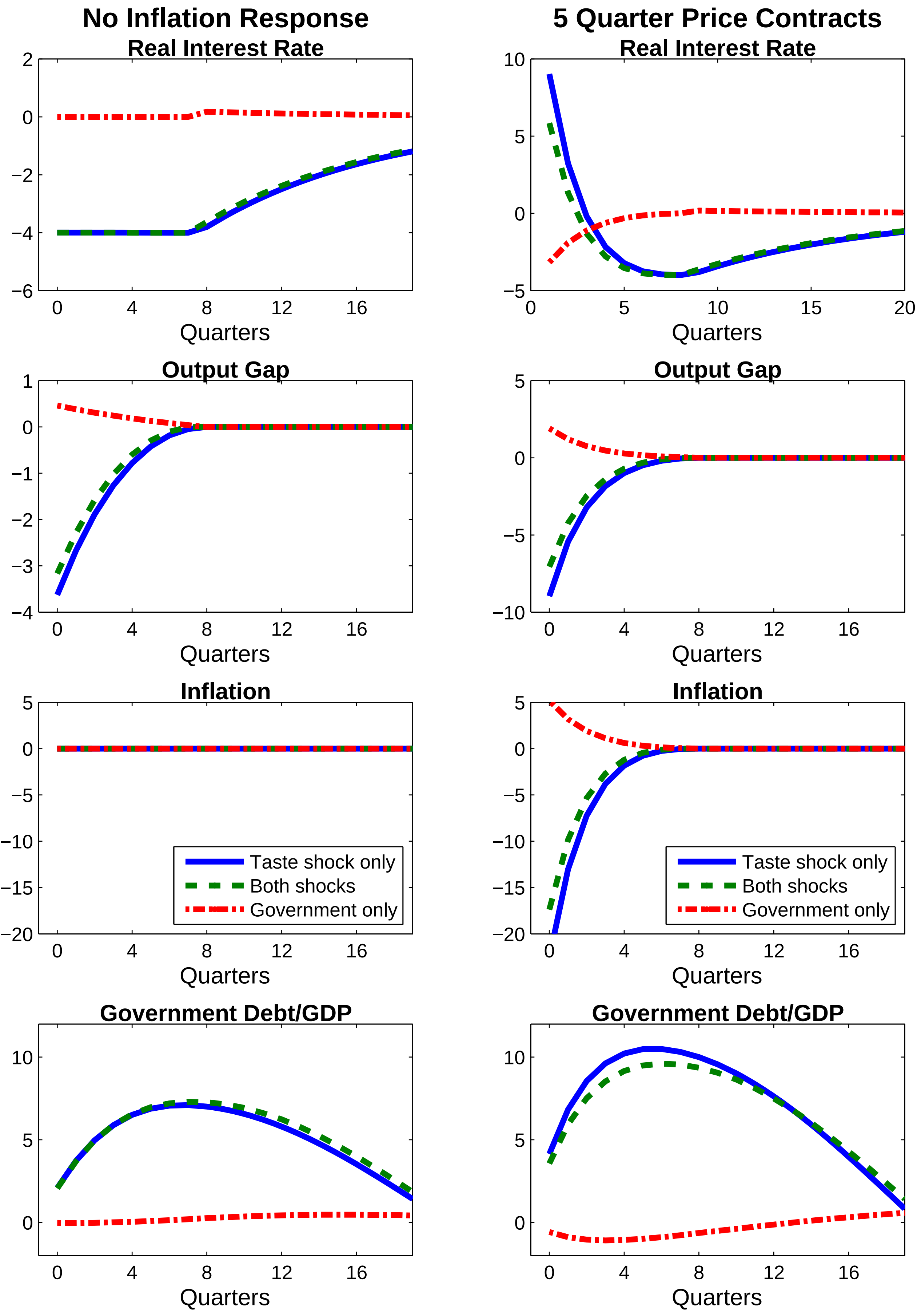

Figure 3: Spending Multipliers and Government Debt Responses in Simple New−Keynesian Model

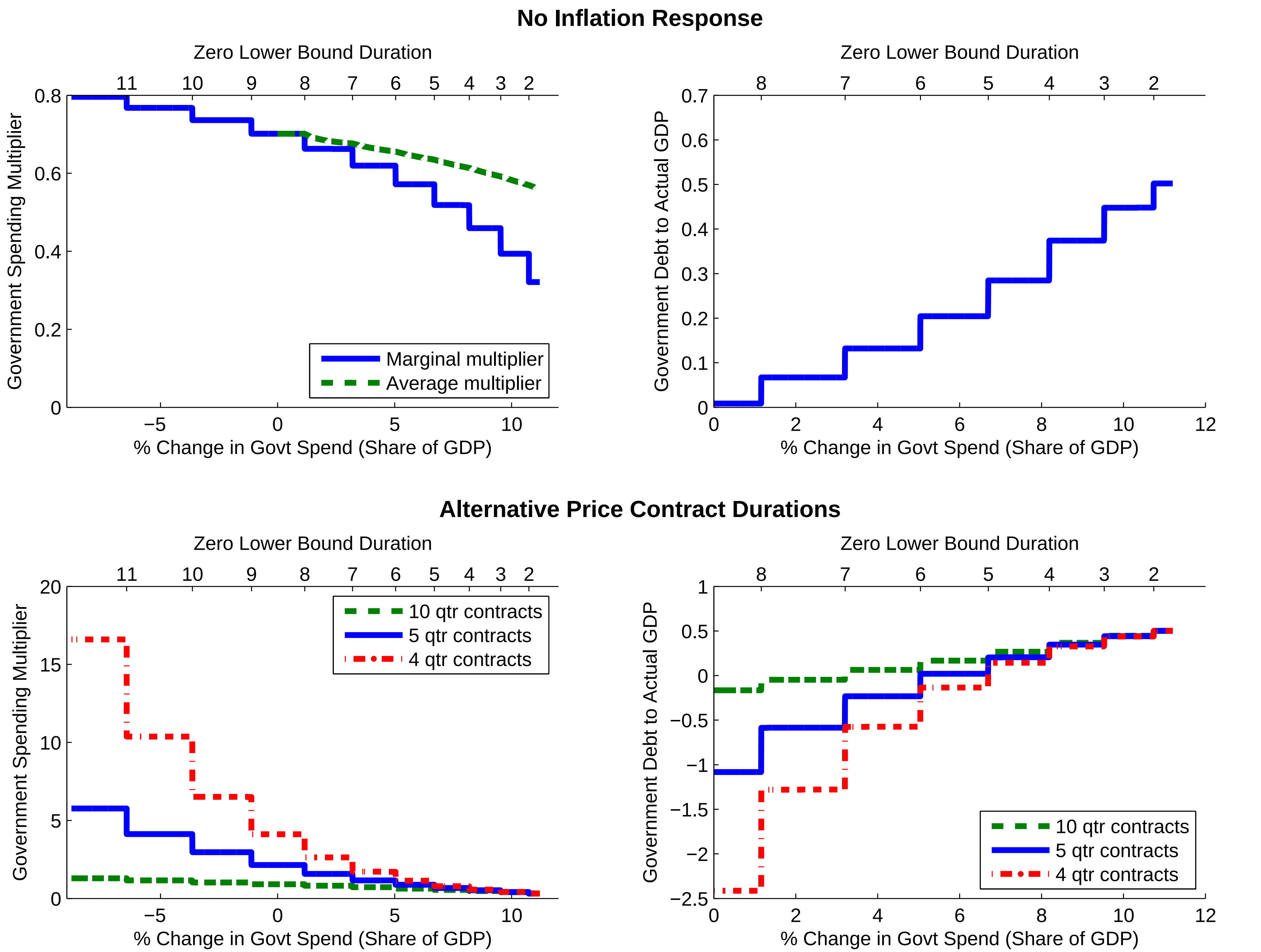

Figure 4a: Weights on Leads of the Interest Rate Gap in Inflation Equation

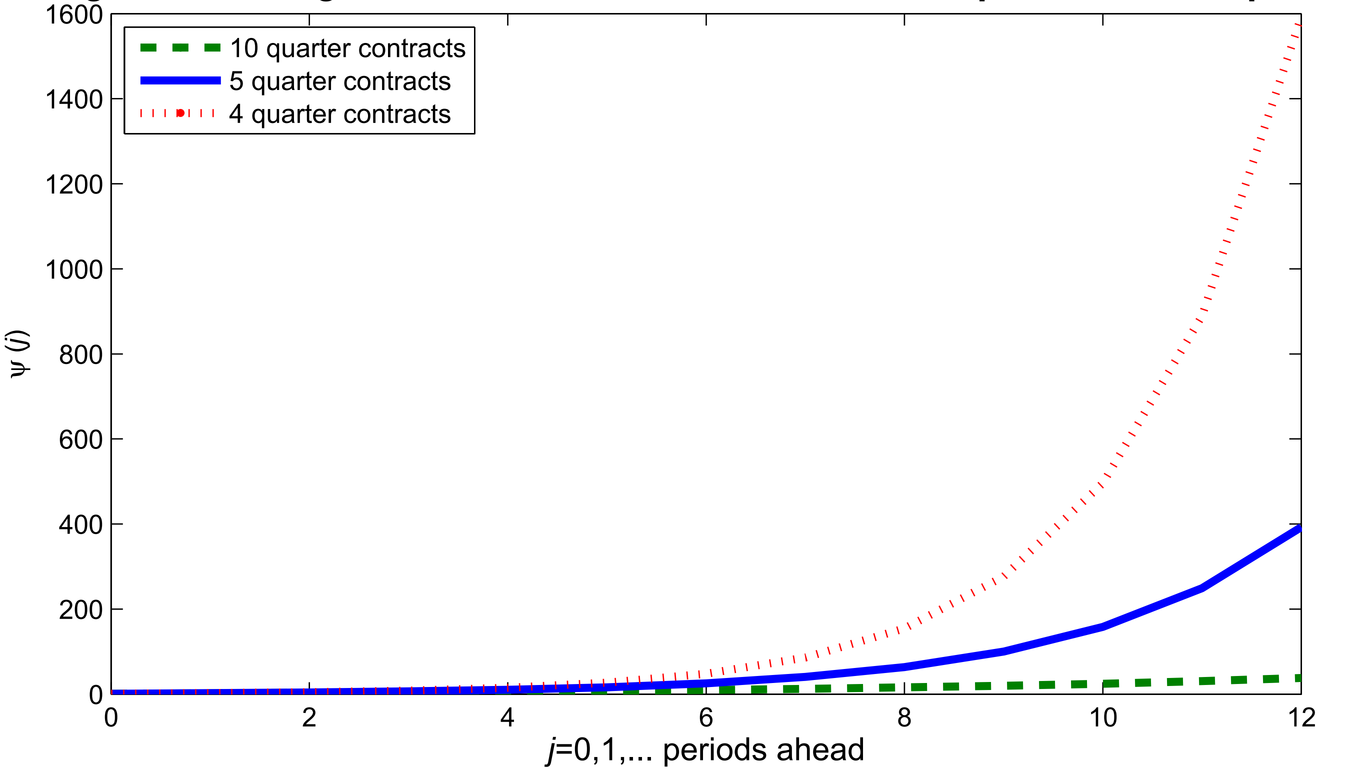

Figure 4b: Government Spending Peaks after Eight Quarters

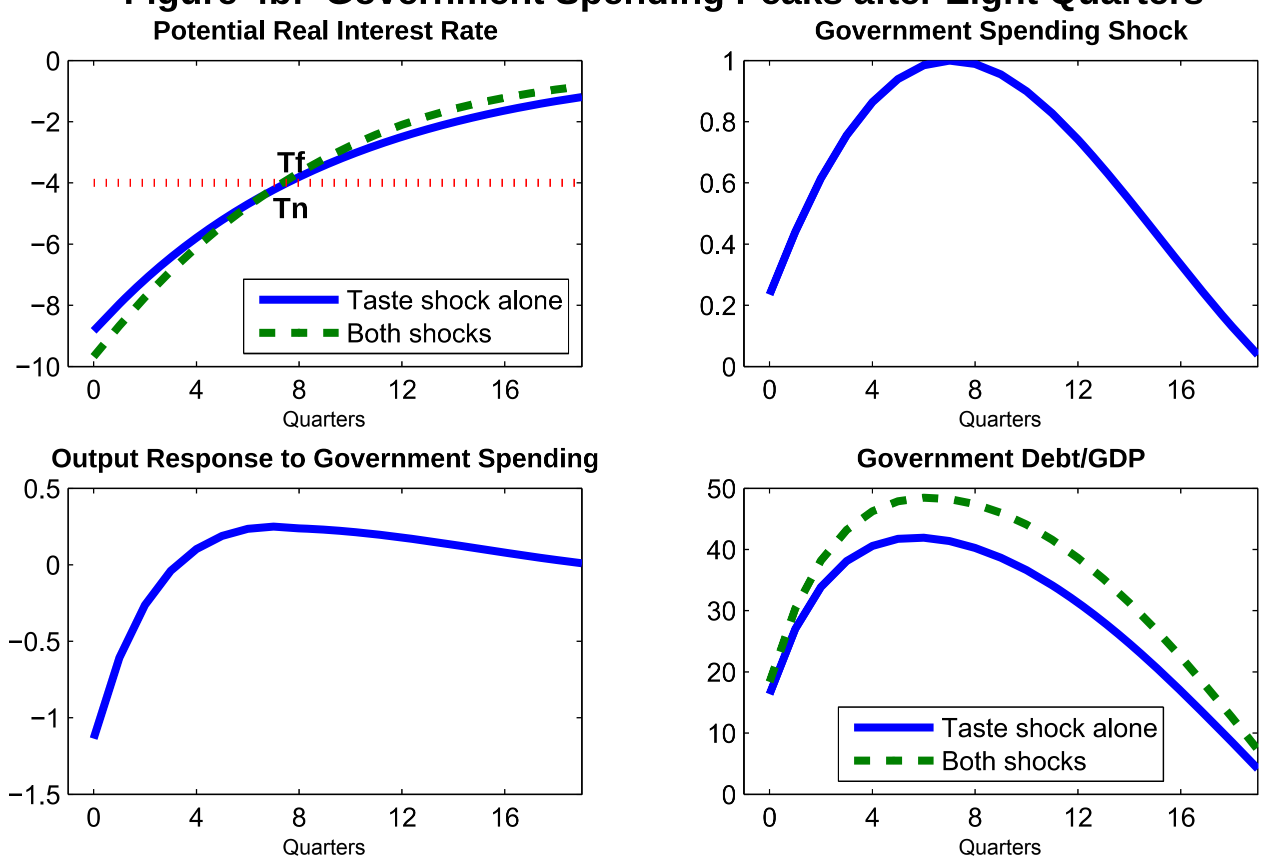

Figure 5a: Simulated and Actual Paths for Key Macroeconomic Variables in Workhorse Model

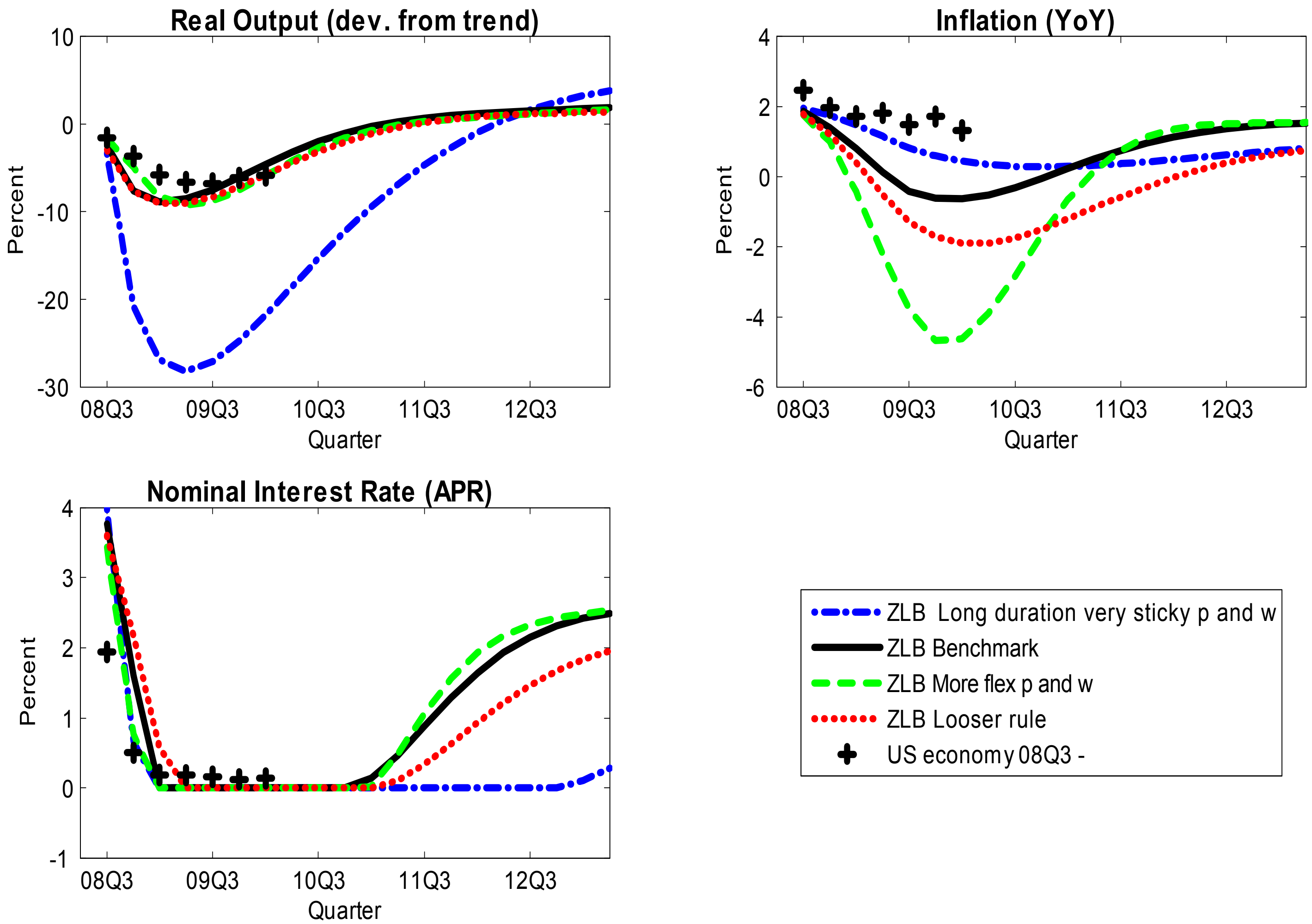

Figure 5b: Actual and Expected FFR and Core Inflation Rates

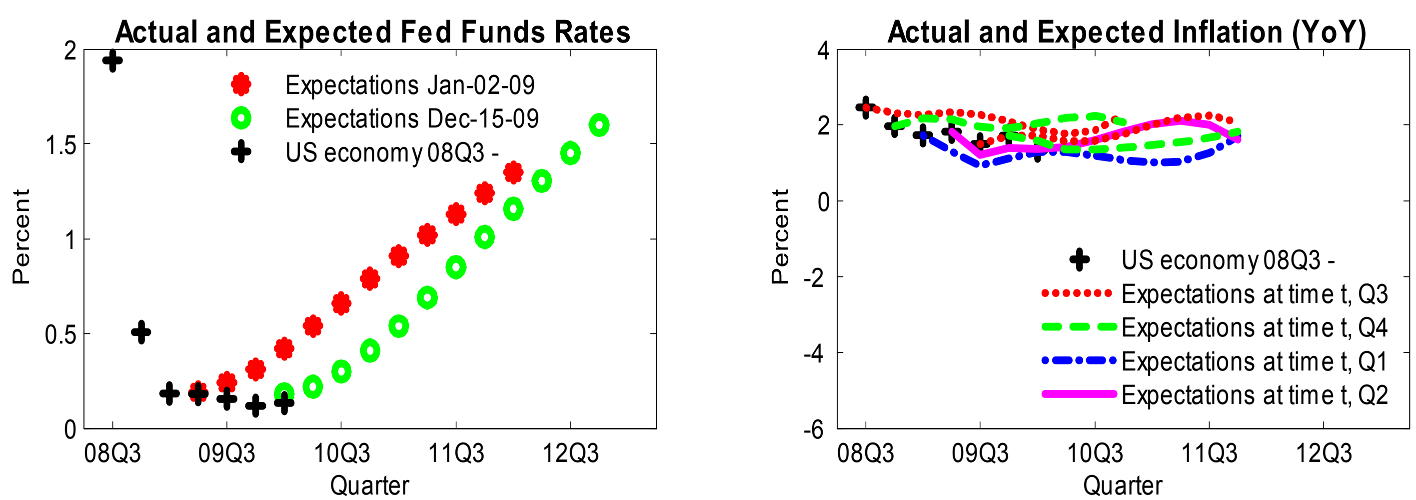

Figure 6: Responses to a Front−Loaded Increase in Government Spending in a Liquidity Trap and in Normal Times in the Workhorse Model

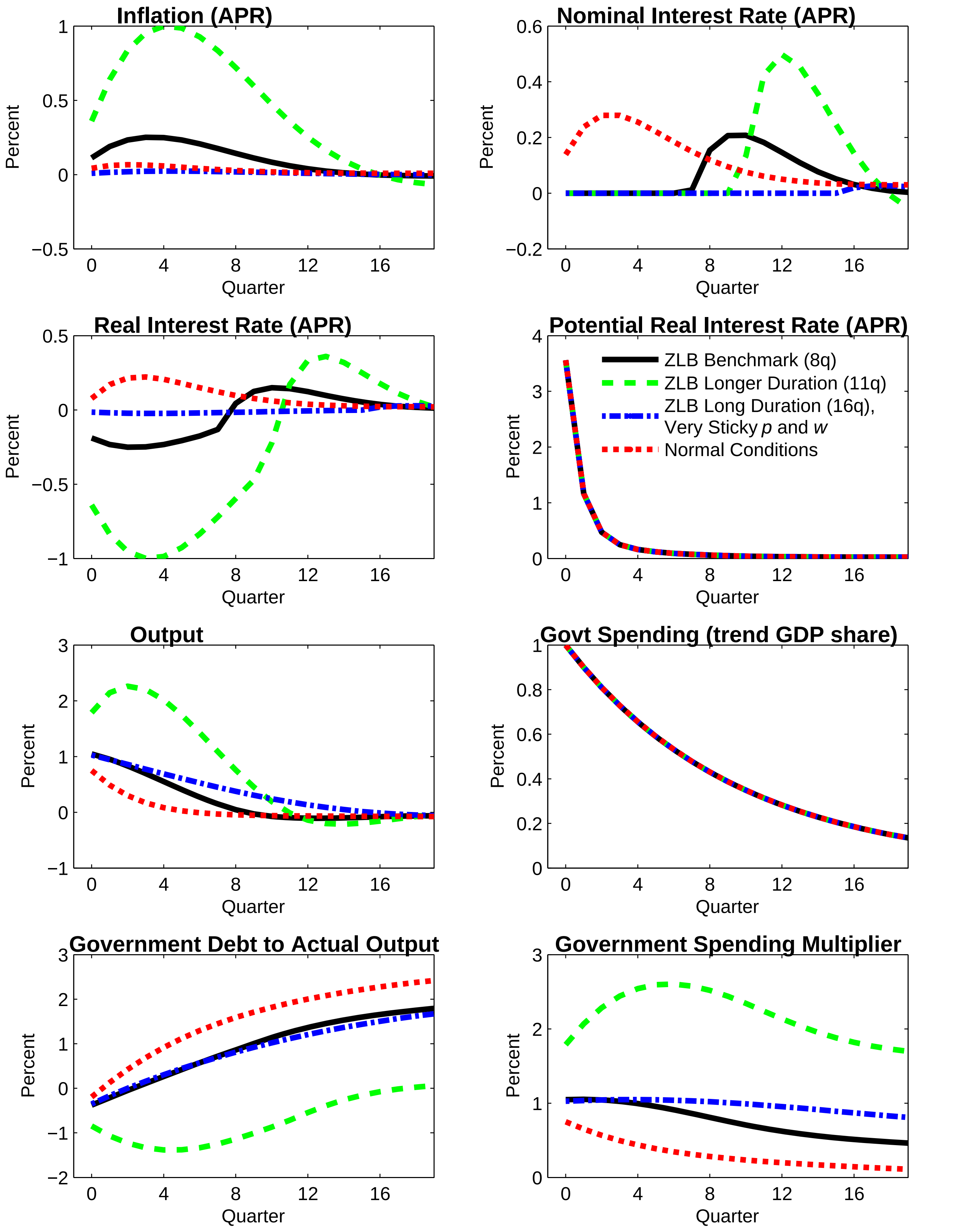

Figure 7: Spending Multipliers and Government Deficit Responses in Workhorse Model

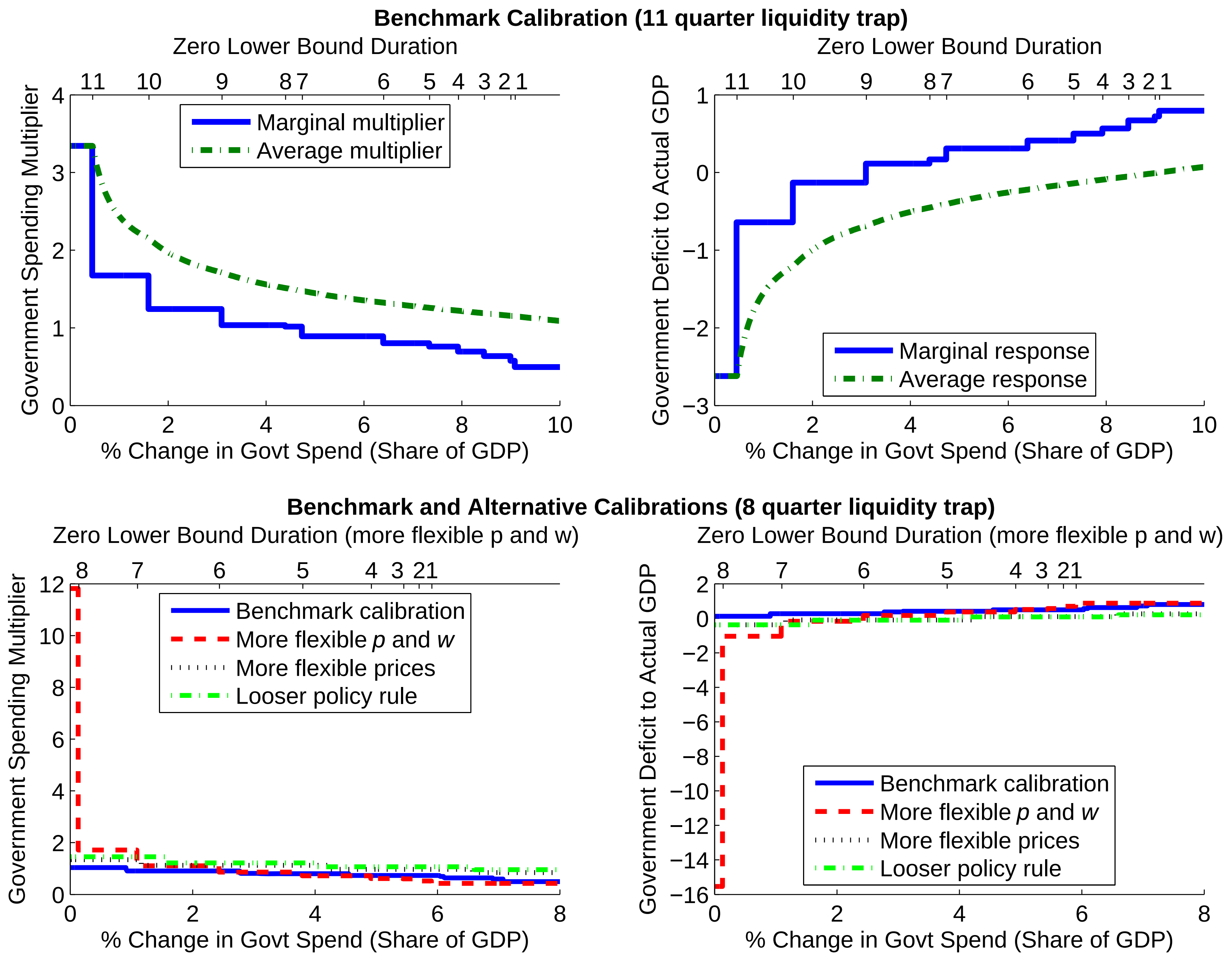

Figure 8: Responses to Alternative Implementation and Financing of the Government Spending Increase in the Workhorse Model

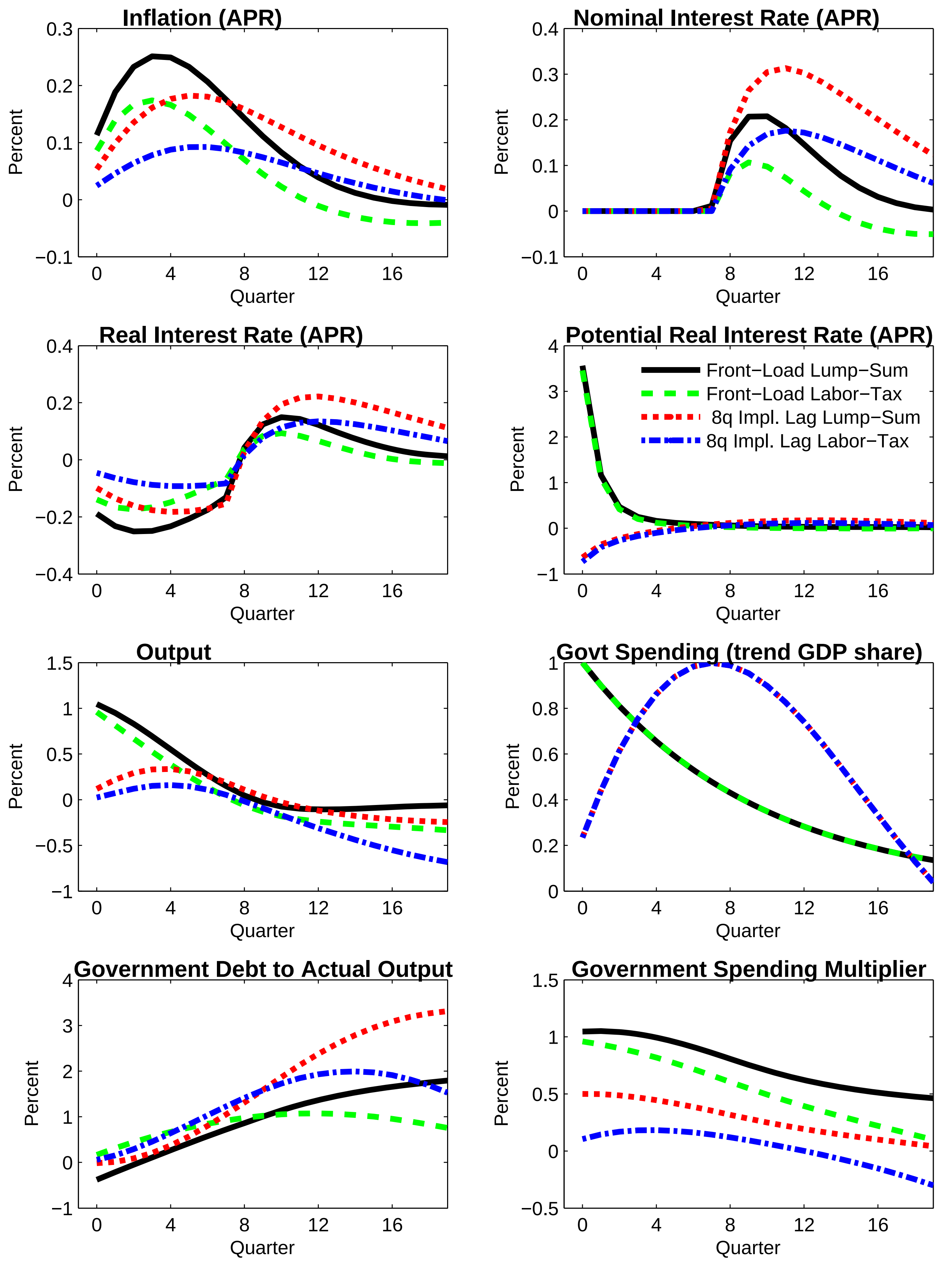

Figure 9: Responses to a Front−Loaded Government Spending Hike in Model With Keynesian Agents and Financial Frictions and Workhorse Model

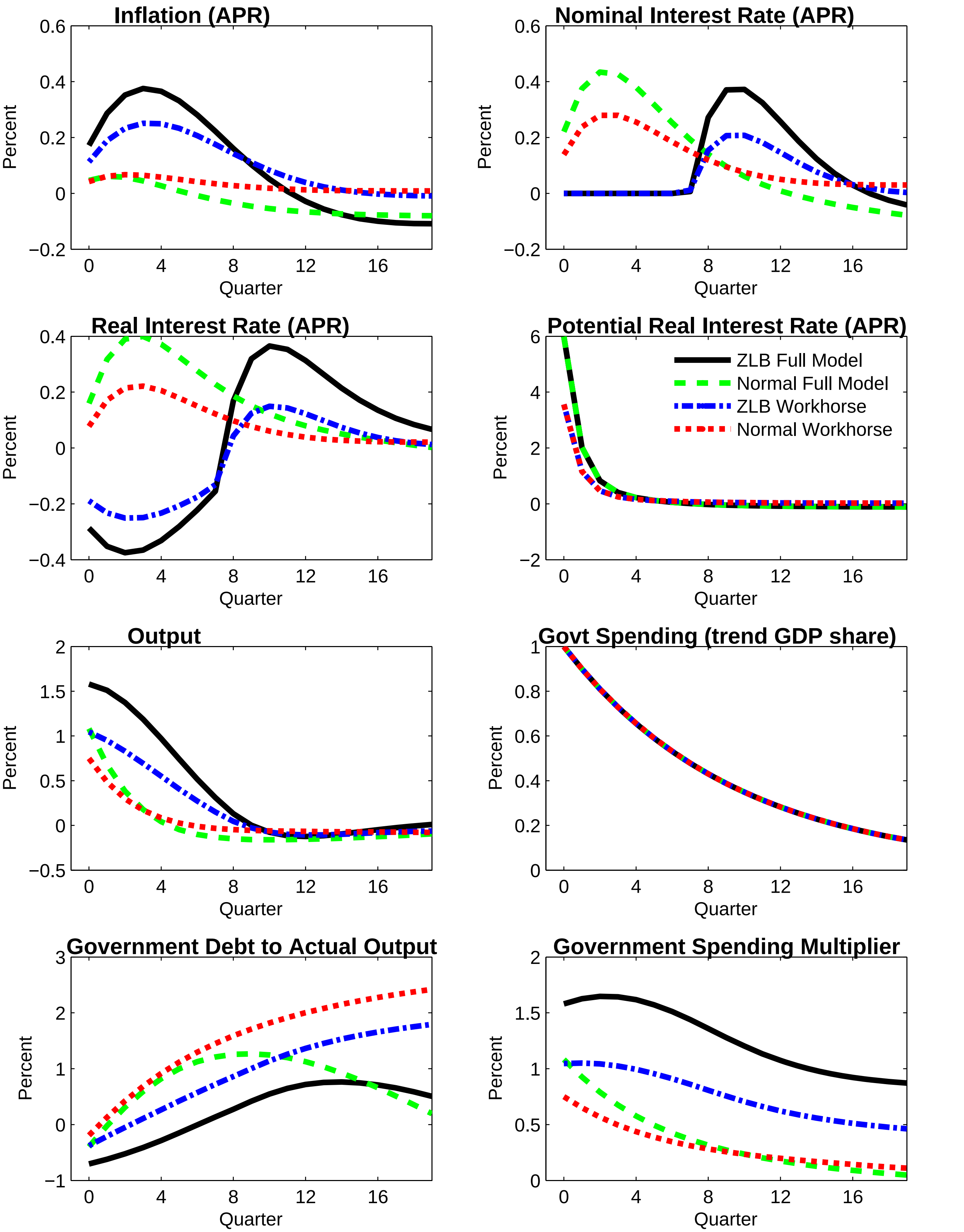

Figure 10: Spending Multipliers and Government Deficit Responses in Full Model with Keynesian Agents and Financial Frictions