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Abstract

This paper uses a DSGE model to examine the effects of an expansion in government spending in a liquidity trap. If the liquidity trap is very prolonged, the spending multiplier can be much larger than in normal circumstances, and the budgetary costs minimal. But given this "fiscal free lunch," it is unclear why policymakers would want to limit the size of fiscal expansion. Our paper addresses this question in a model environment in which the duration of the liquidity trap is determined endogenously, and depends on the size of the fiscal stimulus. We show that even if the multiplier is high for small increases in government spending, it may decrease substantially at higher spending levels; thus, it is crucial to distinguish between the marginal and average responses of output and government debt.

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1. Introduction

Keynes argued in favor of aggressive fiscal expansion during the Great Depression on the grounds that the fiscal multiplier was likely to be much larger in a liquidity trap than in normal times, and the financing burden correspondingly smaller.[1] Recent analysis using New Keynesian DSGE models –including by Eggertsson (2008), Davig and Leeper (2011), and Christiano, Eichenbaum, and Rebelo (2011) and Woodford (2011) – has corroborated Keynes' view by showing that increases in government spending can indeed have outsized effects on output when monetary policy allows real interest rates to fall.[2] However, these results raise the important question of why policymakers would want to limit the magnitude of fiscal expansion.

Our paper addresses this question by showing that the spending multiplier in a liquidity trap decreases with the level of government spending. The novel feature of our approach is to allow the economy's exit from a liquidity trap – and return to conventional monetary policy – to be determined *endogenously,* with the consequence that the multiplier depends on the size of the fiscal response. Quite intuitively, a large fiscal response pushes the economy out of a liquidity trap more quickly. Because the multiplier is smaller upon exiting the liquidity trap – reflecting that monetary policy reacts by raising real interest rates – the marginal impact of a given-sized increase in government spending on output decreases with the magnitude of the spending hike. Accordingly, it is crucial to understand the marginal effect of higher government spending on output to make informed choices about the appropriate scale of fiscal intervention in a liquidity trap.

Toward this end, we use a simple New Keynesian model in which policy rates are constrained by the zero bound to derive a government spending multiplier schedule. This schedule shows how the spending multiplier varies with the level of government spending conditional on the state of the economy (which determines how long the liquidity trap would last in the absence of fiscal stimulus). A key result is that the spending multiplier – measured as the contemporaneous impact on output of a small increment in government spending – is a step function in the level of government spending. If the increment to spending is sufficiently small, it does not affect the economy's exit date from the liquidity trap, and the multiplier is constant at a value that is higher than in a normal situation in which monetary policy would raise real interest rates. However, as spending rises to higher levels, the economy emerges from the liquidity trap more quickly and the multiplier drops (eventually

[1] A large empirical literature has examined the effects of government spending shocks, mainly focusing on the post-WWII period in which monetary policy had latitude to adjust interest rates. The bulk of this research suggests a government spending multiplier in the range of 0.5 to slightly above unity. See e.g. Hall (2009) and Ramey (2011) and the references therein.

[2] Some model-based analysis has questioned whether the multiplier is larger in a liquidity trap than in normal times. Cogan et al. (2009) and Drautzburg and Uhlig (2011) conclude that the multiplier is only slightly amplified even in liquidity traps lasting 2-3 years. Mertens and Ravn (2010) develop a stylized model which rationalizes a low and possibly negative spending multiplier in a liquidity trap in an environment with multiple equilibria driven by expectational shocks. In their model, an increase in fiscal spending confirms and reinforces the pessimistic expectations of the private sector.

leveling off at a value equal to that under normal conditions).

Structural factors that affect inflation expectations, including the slope of the Phillips Curve, play a key role in determining the contour of the government spending multiplier schedule. If prices are fairly responsive to marginal cost – as implied by relatively short-lived price contracts – the multiplier is extremely high for small increments to government spending, but drops quickly at higher spending levels. By contrast, the multiplier function is much flatter if the slope of the Phillips Curve is lower.

A second major focus of our analysis is on how the budgetary impact of higher government spending in a liquidity trap differs from normal times, an important policy question that has received relatively little attention in the recent literature. We show – consistent with the conjecture of Keynes (1933, 1936) – that because the multiplier is higher in a liquidity trap, a given-sized government spending hike can stimulate a much larger response of tax revenues than in normal times, making the fiscal expansion less costly. Moreover, the multiplier may even be high enough in a deep liquidity trap that the government spending hike becomes self-financing, reflecting that tax revenue increases enough to pay for the higher spending even at unchanged tax rates. However, while the prospect of such a "fiscal free lunch" is clearly attractive, our analysis also underscores the importance of distinguishing average from marginal effects. Because the multiplier may drop sharply with the level of spending, the marginal impact on government debt may increase rapidly.[3] The flipside, which is very relevant in an environment of fiscal austerity, is that spending cuts – at least if perceived as temporary – may prolong a recession and boost government debt, with the marginal effects rising in the size of the cut.

While the budgetary implications of a rise in government spending clearly are more favorable the larger the spending multiplier, our analysis shows that the composition of the tax base also plays an important role in determining how a government spending hike affects the government budget. In particular, the government deficit (and debt stock) rises by less in response to a spending increase if the tax base is more cyclically-sensitive. For example, given that labor income is considerably more procylical than consumption expenditure in the New Keynesian model, the effects of higher government spending on tax revenue are more favorable to the extent that labor taxes – rather than sales taxes – comprise a larger fraction of the steady state tax base.

In our benchmark model, dynamic tax adjustment occurs exclusively through lump-sum taxes (with distortionary tax rates fixed), and government spending is purely exogenous. But we also

[3] There is an important difference between our framework – in which higher government spending depresses the real interest rate because of the zero lower bound – and that of Davig and Leeper (2011). Davig and Leeper (2011) show that the government spending multiplier may be well above unity even after many years under a passive monetary policy regime. Although their model does not impose the zero lower bound, real interest rates remain low because the passive monetary policy stance fails to satisfy the Taylor Principle. As a consequence, tax rates would never have to adjust much in such a regime. By contrast, the outsized multiplier and potential fiscal free lunch in our model is state contingent, depending on the depth and duration of the liquidity trap; once the liquidity trap ends, the multiplier returns to normal, and there is no fiscal free lunch.

consider the implications of an alternative fiscal rule whereby the labor income tax rate adjusts to stabilize government debt. If tax rates adjust very gradually to the level of government debt, the multiplier (schedule) associated with a liquidity trap of a given duration is only reduced slightly relative to the lump-sum case, and the effects of higher spending on the government budget are nearly the same.[4] However, we show that a more aggressive tax rule – in which the labor tax rate is quite responsive to government debt even in the near-term – tends to reduce the multiplier substantially *relative to* the lump-sum case, especially in a prolonged liquidity trap. At first glance, this result seemingly contrasts with that of Eggertson (2010) and Christiano, Eichenbaum, and Rebelo (2011), who find that a purely *exogenous* rise in a distortionary tax amplifies the multiplier. The apparent disparity reflects that the inflation response is damped under an aggressive *endogenous* rule, since lower tax rates than in normal times (associated with the higher fiscal multiplier in a liquidity trap) reduce upward pressure on marginal cost; this effect is not present under exogenous tax adjustment. We also consider the implications of an endogenous component of government spending that responds to the output gap to proxy for automatic stabilizers. In this environment, the multiplier associated with a given-sized increase in discretionary spending (i.e., the exogenous component) is reduced relative to our benchmark in which all spending is exogenous.

Our paper concludes by examining the effects of government spending shocks on output and the government budget in a more empirically-realistic model. In particular, we utilize a model that is similar to the estimated models of Christiano, Eichenbaum and Evans (2005) and Smets and Wouters (2007), but also incorporates "Keynesian" hand-to-mouth agents and financial frictions. As argued by Galí, López-Salido, and Vallés (2007), the inclusion of Keynesian households can help account for the positive response of private consumption to a government spending shock documented in structural VAR studies by e.g., Blanchard and Perotti (2002) and Perotti (2007); more generally, "Keynesian" hand-to-mouth agents and financial frictions can increase the multiplier by amplifying the response of the potential real interest rate.

We find that the government spending multiplier exceeds 4 against the backdrop of a deep recession that would generate a 10 quarter liquidity trap in the absence of a fiscal response, and that small increments to spending generate enough tax revenue to be self-financing (in fact, the impact on government debt remains persistently negative even at a horizon of five years). Even so, the government spending multiplier declines fairly abruptly as spending increases. With a spending increase of over 3 percent of GDP, the marginal multiplier declines to 1.3, and the marginal impact on government debt is solidly positive. Moreover, while the multiplier is even higher for small increments to spending under calibrations that allow for *both prices and wages* to adjust more rapidly, the multiplier declines with the level of spending more precipitously. Thus, the multiplier tends to drop sharply in the level of government spending under exactly the conditions (i.e., relatively high inflation responsiveness) that are favorable to a large multiplier.

[4] Consistent with Uhlig (2010), output is lower in the longer-term under distortionary tax rather than lump-sum tax financing.

Overall, our results corroborate previous analysis suggesting a strong argument in favor of increasing government spending on a temporary basis for an economy facing a deep recession and long-lived liquidity trap. Consistent with the view originally espoused by Keynes, a temporary spending boost can have much larger effects on output than under usual conditions, and comes at a low cost to the Treasury. But our analysis highlights the importance of taking account of how the multiplier varies with the level of spending at the margin. Insofar as the multiplier can drop quickly with the level of fiscal spending, larger spending programs may suffer from sharply diminishing returns, and hence increase government debt at the margin. Conversely, large-scale fiscal consolidations under some conditions may run the risk of deepening a recession and boosting government debt.

The remainder of this paper is organized as follows. Section 2 derives the multiplier schedule in a stylized New Keynesian model in which all dynamic tax adjustment occurs through lump-sum taxes, and government spending is exogenous; we then consider modifications that allow for distortionary taxes, and for some component of government spending to be endogenous. Section 3 examines the more empirically-realistic model, and Section 4 concludes.

2. A stylized New Keynesian model

As in Eggertsson and Woodford (2003), we use a standard log-linearized version of the New Keynesian model that imposes a zero bound constraint on interest rates. Our framework allows exit from the liquidity trap to be determined endogenously, rather than fixed arbitrarily, an innovation that is crucial in showing how the multiplier varies with the level of fiscal spending.

2.1. The Model

The key equations of the model are:

$$x_t = x_{t+1|t} - \hat{\sigma}(i_t - \pi_{t+1|t} - r_t^{pot}), \tag{1}$$

$$\pi_t = \beta\pi_{t+1|t} + \kappa_p x_t, \tag{2}$$

$$i_t = \max\left(-i, \gamma_\pi \pi_t + \gamma_x x_t\right), \tag{3}$$

$$y_t^{pot} = \frac{1}{\phi_{mc}\hat{\sigma}}[g_y g_t + (1-g_y)\nu_c \nu_t], \tag{4}$$

$$r_t^{pot} = \frac{1}{\hat{\sigma}}\left(1 - \frac{1}{\phi_{mc}\hat{\sigma}}\right)\left[g_y(g_t - g_{t+1|t}) + (1-g_y)\nu_c(\nu_t - \nu_{t+1|t})\right], \tag{5}$$

where $\hat{\sigma}$, κ_p, and ϕ_{mc} are composite parameters defined as:

$$\hat{\sigma} = \sigma(1 - g_y)(1 - \nu_c), \tag{6}$$

$$\kappa_p = \frac{(1-\xi_p)(1-\beta\xi_p)}{\xi_p}\phi_{mc}, \tag{7}$$

$$\phi_{mc} = \frac{\chi}{1-\alpha} + \frac{1}{\hat{\sigma}} + \frac{\alpha}{1-\alpha}. \tag{8}$$

All variables are measured as percent or percentage point deviations from their steady state level.[5]

Equation (1) expresses the "New Keynesian" IS curve in terms of the output and real interest rate gaps. Thus, the output gap x_t depends inversely on the deviation of the real interest rate ($i_t - \pi_{t+1|t}$) from its potential rate r_t^{pot}, as well as on the expected output gap in the following period. The parameter $\hat{\sigma}$ determines the sensitivity of the output gap to the real interest rate; as indicated by (6), it depends on the household's intertemporal elasticity of substitution in consumption σ, the steady state government spending share of output g_y, and a (small) adjustment factor ν_c which scales the consumption taste shock ν_t. The price-setting equation (2) specifies current inflation π_t to depend on expected inflation and the output gap, where the sensitivity to the latter is determined by the composite parameter κ_p. Given the Calvo-Yun contract structure, equation (7) implies that κ_p varies directly with the sensitivity of marginal cost to the output gap ϕ_{mc}, and inversely with the mean contract duration ($\frac{1}{1-\xi_p}$). The marginal cost sensitivity equals the sum of the absolute value of the slopes of the labor supply and labor demand schedules that would prevail under flexible prices: accordingly, as seen in (8), ϕ_{mc} varies inversely with the Frisch elasticity of labor supply $\frac{1}{\chi}$, the interest-sensitivity of aggregate demand $\hat{\sigma}$, and the labor share in production $(1-\alpha)$. The policy rate i_t follows a Taylor rule subject to the zero lower bound (equation 3).

Equation (4) indicates that potential output y_t^{pot} varies directly with two exogenous shocks, including a consumption taste shock ν_t and government spending shock g_t. Both shocks are assumed to follow an AR(1) process with the same persistence parameter $(1-\rho_\nu)$, e.g., the taste shock follows:

$$\nu_t = (1 - \rho_\nu)\nu_{t-1} + \varepsilon_{\nu,t}, \tag{9}$$

where $0 < \rho_\nu < 1$. Given the front-loaded nature of the shocks, equation (5) indicates that positive realizations of these shocks boosts the potential real interest rate (noting $\phi_{mc}\hat{\sigma} > 1$); this reflects that each shock – if positive – raises the marginal utility of consumption associated with any given output level.

The government does not need to balance its budget each period, and issues nominal debt as needed to finance budget deficits. Under the simplifying assumption that government debt is zero in steady state, the log-linearized government budget constraint is given by:

$$b_{G,t} = (1 + r)b_{G,t-1} + g_y g_t - \tau_N s_N (y_t + \phi_{mc} x_t) - \tau_t, \tag{10}$$

[5] We use the notation $y_{t+j|t}$ to denote the conditional expectation of a variable y at period $t+j$ based on information available at t, i.e., $y_{t+j|t} = \mathrm{E}_t y_{t+j}$. The superscript 'pot' denotes the level of a variable that would prevail under completely flexible prices, e.g., y_t^{pot} is potential output. See Appendix A (available online) for the model derivation.

where $b_{G,t}$ is end-of-period real government debt, $(y_t + \phi_{mc} x_t)$ equals real labor income, τ_t is a lump-sum tax, and s_N is the steady state labor share.[6] The government derives tax revenue from a fixed tax on labor income τ_N, and from the time-varying lump-sum tax τ_t. The tax rate τ_N is set so that government spending is financed exclusively by the distortionary labor tax in the steady state (so $\tau_N s_N = g_y$). Lump-sum taxes adjust according to the reaction function:

$$\tau_t = \varphi_b b_{G,t-1}. \tag{11}$$

Given that agents are Ricardian and that only lump-sum taxes adjust, the fiscal rule only affects the evolution of the stock of debt and lump-sum taxes, with no effect on other macro variables.

Our benchmark calibration is fairly standard at a quarterly frequency. We set the discount factor $\beta = 0.995$, and the steady state net inflation rate $\pi = .005$; this implies a steady state interest rate of $i = .01$ (i.e., four percent at an annualized rate). We set the intertemporal substitution elasticity $\sigma = 1$ (log utility), the capital share parameter $\alpha = 0.3$, the Frisch elasticity of labor supply $\frac{1}{\chi} = 0.4$, the government share of steady state output $g_y = 0.2$, and the scale parameter on the consumption taste shock $\nu_c = 0.01$. We examine a range of values of the price contract duration parameter ξ_p to highlight the sensitivity of the fiscal multiplier to the Phillips Curve slope κ_p. We assume that monetary policy would completely stabilize inflation and the output gap in the absence of a zero bound constraint, which can be regarded as a limiting case in which the coefficients on inflation, γ_π, and the output gap, γ_x, in the interest rate reaction function become arbitrarily large. The tax rule parameter φ_b is set equal to .01, which implies that the contribution of lump-sum taxes to the response of government debt is extremely small in the first couple of years following a shock (so that almost all variation in tax revenue comes from fluctuations in labor tax revenues). Finally, the preference and government spending shocks are assumed to follow an AR(1) process with persistence of 0.9, so that $\rho_\nu = 0.1$ in equation (9).

2.2. Impulse Responses to a Rise in Government Spending

The effects of fiscal policy in a liquidity trap depend crucially on agents' perceptions about the likely duration of the liquidity trap. The liquidity trap duration in turn generally depends on a number of factors, including the parameters of the monetary policy rule, the type of shocks causing the liquidity trap, and the fiscal response. For simplicity, we follow the recent literature – including Eggertson and Woodford (2003), and Eggertson (2009), and Adam and (2008) – by assuming that the liquidity trap is caused by an adverse taste shock ν_t that sharply depresses

[6] In (10), real government debt $b_{G,t}$ and real transfers τ_t are defined as a share of steady state GDP and expressed as percentage point deviations from their steady state values. That is, $b_{G,t} = \left(\frac{B_{G,t}}{P_t Y}\right) - b_G$, where $B_{G,t}$ is nominal government debt, P_t is the price level, and Y is real steady state output; and similarly, $\tau_t = \left(\frac{T_t}{P_t Y}\right) - \tau$. Because of our simplifying assumption that the steady state government debt $b_G = 0$, a time-varying real interest rate does not enter in eq. (10). In the full model analyzed in Section 3, we allow for positive steady state government debt, and hence a role for time-varying debt service costs.

the potential real interest rate r_t^{pot}. Because this type of shock does not imply a tradeoff between stabilizing inflation and the output gap, our assumed highly aggressive monetary policy reaction function would keep both inflation at target and output at potential if unconstrained by the zero lower bound. Thus, if the zero bound is not binding, equation (1) implies that the nominal interest rate i_t simply tracks r_t^{pot} (i.e., $i_t = r_t^{pot}$, recalling that both variables are measured as percentage point deviations from baseline).[7] A key implication is that the duration of the liquidity trap depends solely on how long r_t^{pot} remains below $-i$. Given that r_t^{pot} is a simple function of both the taste and government spending shocks by equation (5), this setting makes it very tractable to show how different government spending choices affect the duration of the liquidity trap and hence the spending multiplier.

Figure 1.a illustrates how the liquidity trap duration and the path of the policy rate is determined. The solid line shows the impact of an adverse taste shock ν_t on r_t^{pot}. The liquidity trap lasts as long as $r_t^{pot} < -i$, over which period $i_t = -i = -1$ percent (the figure shows the annualized interest rate, so -4 percent). Beginning in period T_n, which is the first period in which r_t^{pot} exceeds $-i$, the policy rate i_t simply rises with r_t^{pot}(the taste shock is scaled so that the liquidity trap lasts for $T_n = 8$ quarters). A government spending shock equal to one percentage point of steady state GDP – shown by the dashed line – simply offsets some of the decline in r_t^{pot} induced by the negative taste shock, shifting up the path of r_t^{pot} in a proportional manner (recalling the shocks are equally persistent). Because this government spending hike turns out to be too small to affect the duration of the liquidity trap, monetary policy continues to hold the nominal interest rate unchanged for $T_n = 8$ quarters.

The solid lines in Figure 2 show the dynamic effects of the taste shock ν_t on the real interest rate, output gap, inflation, and government debt as a share of GDP. To highlight the role of expected inflation in amplifying the effects of the shock, it is useful to begin with a limiting case in which inflation is (essentially) constant, which is achieved by setting κ_p in equation (2) arbitrarily close to zero. The left column of Figure 2 shows this limiting case. Because the taste shock causes the potential interest rate r_t^{pot} to decline persistently below the policy rate $-i$, output falls below potential (noting equation (1), and that the real rate tracks the policy rate). The dash-dotted lines show the partial effect of the one percentage point government spending rise (i.e., the difference between the response to both shocks and the taste shock alone). Because the increase in government spending boosts r_t^{pot} while leaving the real interest rate unaffected, the higher spending has a positive effect on the output gap while the economy remains in a liquidity

[7] The result that the monetary policy rule in equation (3), if unconstrained, can stabilize inflation and the output gap at target (implying the the policy rate tracks r_t^{pot}) is shown in Woodford (2003), and reproduced in Appendix A. Moreover, given the absence of a policy tradeoff, the relative weight on inflation stabilization and output gap stabilization in the rule is immaterial so long as one policy rule coefficient is sufficiently large. Other shocks that induced the same path of r_t^{pot} as the taste shock – including to tax rates or to productivity – would have the same implications for the spending multiplier in this model (provided that the shocks did not induce a policy tradeoff, as discussed below).

trap. The government spending multiplier,

$$m_t = \frac{1}{g_y}\frac{dy_t}{dg_t}, \tag{12}$$

which is equal to the sum of the output gap response of 0.5 shown in the figure and the potential output response of 0.2, (not shown), is considerably larger than in a normal situation in which the output gap would be unaffected.

The multiplier is amplified substantially more when expected inflation responds to shocks, as illustrated in the right column of Figure 2 for a calibration implying a mean duration of price contracts of 5 quarters (i.e. $\xi_p = 0.8$). In this case, the negative taste shock causes expected inflation to fall persistently, which raises the real interest rate and augments the output decline relative to the no-inflation response case. Higher government spending partially reverses these effects by boosting r_t^{pot} and expected inflation. The impact spending multiplier, m_0 is about 2.

Because the fiscal multiplier is relatively high in a liquidity trap, the effects of a rise in government spending on government debt are smaller than in normal times in which policy rates adjust. For the case of 5 quarter contracts, government debt falls for several quarters (lower right panel), a sharp contrast from the progressive rise in debt that occurs in normal times. These debt dynamics largely reflect that labor income tax revenue, and hence the primary balance p_{Gt}, varies sharply with the spending multiplier:

$$\frac{1}{g_y}\frac{dp_{Gt}}{dg_t} = g_y\{m_t + \phi_{mc}(m_t - m_t^{pot})\} - 1. \tag{13}$$

where the bracketed term premultiplied by g_y reflects the response of labor income to higher government spending. Thus, holding lump-sum taxes unchanged (as assumed in equation 13), the primary balance p_{Gt} (expressed relative to GDP) actually *improves* in response to higher spending if $m_t > \frac{1}{1+\phi_{mc}}(\frac{1}{g_y} + \phi_{mc}m_t^{pot})$, which corresponds to a multiplier greater than unity under our benchmark calibration. By contrast, in normal times the multiplier of m_t^{pot} implies that labor tax revenue rises by $g_y m_t^{pot}$, or only about 0.05 under our calibration, so that a 1 percentage point rise in spending causes the primary deficit to rise by 0.95 percent.

Accordingly, conditions that imply a larger and more persistent boost in the multiplier than under the 5 quarter contracts calibration cause debt to fall even more than shown in the figure. These include a higher Phillips Curve slope (as suggested by comparing the lower two panels), or a longer-lived liquidity trap. For example, in the case of four quarter price contracts – which imply an impact multiplier of 3 – debt falls progressively. Debt eventually converges to baseline through a *reduction* in lump-sum taxes, which may be regarded as tantamount to a "fiscal free lunch."

2.3. The Multiplier and the Size of Fiscal Spending

In the log-linearized model that ignores the zero bound constraint, the government spending multiplier is invariant to the size of the change in spending. By contrast – as we next proceed to show

– the multiplier in a liquidity trap *declines* in the level of government spending. Intuitively, this behavior reflects that the multiplier varies positively with the duration of the liquidity trap, and that the duration shortens as the level of spending rises.

Because government spending and taste shocks have the same linear dynamic effects on r_t^{pot} under our assumption that the shocks are equally persistent, and only the path of r_t^{pot} matters for the output gap and inflation response, we can simply focus on how r_t^{pot} affects the output gap and inflation in a liquidity trap. Solving the IS curve forward yields:

$$x_t = -\hat{\sigma} \sum_{j=0}^{T_n-1} (-i - r_{t+j|t}^{pot}) + \hat{\sigma} \sum_{j=1}^{T_n} \pi_{t+j|t} + x_{t+T_n|t}, \tag{14}$$

where T_n is the duration of the liquidity trap. Hence, the output gap x_t in a liquidity trap depends on four terms. First, it depends on the *cumulative* gap between the nominal interest rate $-i$ and the potential real interest rate over the interval in which the economy remains in a liquidity trap. This cumulative interest rate gap $\sum_{j=0}^{T_n-1}(-i - r_{t+j|t}^{pot})$ can be interpreted as indicating how shocks to the potential real interest rate would affect the output gap *if expected inflation remained constant*. Second, the output gap depends on cumulative expected inflation over the liquidity trap (or equivalently, the log change in the price level $log(P_{t+Tn}) - log(P_t)$); as indicated above, the effects of shocks to the potential real rate on the output gap are amplified through changes in expected inflation. Third, the current output gap also depends on the expected output gap $x_{t+T_n|t}$ when the economy exits the liquidity trap, though both the terminal output gap and inflation terms drop under the assumption that monetary policy completely stabilizes the economy ($x_{t+T_n|t} = \pi_{t+T_n|t} = 0$). Finally, $t + T_n$ can be interpreted as the exit date of the liquidity trap and is determined endogenously as the first period in which the expected potential real interest rate exceeds $-i$. Thus:

$$T_n = \min_j (r_{t+j|t}^{pot} > -i), \tag{15}$$

where $j = 0, 1, 2, ...$

T_n depends both on the size and persistence of the shocks to r_t^{pot}. The relation between T_n and r_t^{pot} under our baseline calibration is shown in Figure 1.b. Because T_n is only affected as r_t^{pot} exceeds certain threshold values, it is a step function in the level of r_t^{pot} (rising as r_t^{pot} assumes more negative values). Thus, a slightly larger adverse taste shock that caused r_t^{pot} to drop more than shown in Figure 1.a would leave the duration of the liquidity trap unchanged at 8 quarters; but a large enough adverse shock would extend the duration of the trap, and a sufficiently smaller shock would shorten it.

In the limiting case in which expected inflation remains constant, we can derive a simple closed form solution for the multiplier. Because r_t^{pot} follows an AR(1) with persistence parameter 1-ρ_v, equation (14) implies that the output gap x_t equals:

$$x_t = -\hat{\sigma} \sum_{j=0}^{T_n-1} (-i - (1-\rho_v)^j r_t^{pot}) = -\hat{\sigma} i T_n + \hat{\sigma} r_t^{pot} \frac{1 - (1-\rho_v)^{T_n}}{\rho_v} < 0. \tag{16}$$

For changes in government spending that are small enough to keep the liquidity trap duration unchanged at T_n periods, the multiplier $\frac{1}{g_y}\frac{dy_t}{dg_t}$ is derived by differentiating equation (16) with respect to g_t, and adding the effect on potential output $(\frac{dy_t^{pot}}{dg_t})$:

$$\frac{1}{g_y}\frac{dy_t}{dg_t} = \frac{1}{g_y}(\frac{dx_t}{dg_t} + \frac{dy_t^{pot}}{dg_t}) = \hat{\sigma}\frac{1-(1-\rho_v)^{T_n}}{\rho_v}\frac{1}{g_y}\frac{dr_t^{pot}}{dg_t} + \frac{1}{g_y}\frac{dy_t^{pot}}{dg_t}. \tag{17}$$

The first term – the output gap component – is positive. It varies directly with the duration of the underlying liquidity trap T_n, reflecting that fiscal policy can only affect the output gap over the period in which the economy remains in the trap. The second term $\frac{1}{g_y}\frac{dy_t^{pot}}{dg_t}$ is equal to the spending multiplier in the flexible price equilibrium, as well as during normal times given our assumption that monetary policy, if unconstrained, keeps output at potential. From equations (4) , (6) , and (8), the latter may be expressed as $\frac{1}{g_y}\frac{dy_t^{pot}}{dg_t} = \frac{1}{\phi_{mc}\hat{\sigma}}$ which is less than unity since $\phi_{mc}\hat{\sigma} = 1 + \frac{(\alpha+\chi)\hat{\sigma}}{1-\alpha} > 1$.

Substituting $\frac{1}{g_y}\frac{dr_t^{pot}}{dg_t} = \frac{1}{\hat{\sigma}}(1 - \frac{1}{\phi_{mc}\hat{\sigma}})\rho_v$ into equation (17), the multiplier can be expressed in the simple form:

$$\frac{1}{g_y}\frac{dy_t}{dg_t} = 1 - (1 - \frac{1}{\phi_{mc}\hat{\sigma}})(1-\rho_v)^{T_n}. \tag{18}$$

The solid lines in the upper left panel of Figure 3 show how the marginal multiplier varies with the duration of the liquidity trap, where the latter is indicated by the tick marks along the upper axis. The multiplier associated with a tiny increment to government spending in an 8 quarter liquidity trap is about 0.7, but rises to about 0.8 against the backdrop of an 12 quarter liquidity trap (caused by a larger contractionary taste shock than in Figure 1.a). The multiplier increases monotonically with the duration of the trap, but in a concave manner; and importantly, the multiplier remains less than or equal to unity provided that the liquidity trap is of finite duration, however long.

These results provide a key stepping stone for understanding how the multiplier varies with the size of the increase in government spending. While the foregoing analysis examined the effects of tiny increments to government spending against the backdrop of different initial conditions (i.e., associated with liquidity traps of varying length), we now take "initial conditions" – summarized by a given-sized taste shock – as fixed, and assess how progressively larger increases in government spending affect the multiplier by reducing the duration of the liquidity trap.

In this vein, the upper left panel of Figure 3 can be reinterpeted as showing how the multiplier varies with alternative levels of government spending – that is, a *government spending multiplier schedule*. For concreteness, we assume that the liquidity trap is generated by the same adverse taste shock shown in Figure 1.a, so that the "0" government spending level on the lower horizontal axis implies an 8 quarter liquidity trap (shown by the tick mark on the upper horizontal axis). For a government spending hike of less than a threshold value of 1.2 percent of GDP, the duration of the liquidity trap remains unchanged at $T_n = 8$ quarters, and the multiplier equals about 0.7.[8] As government spending exceeds this threshold, the potential real rate is boosted enough that the

[8] As seen in the figure, cuts in government spending exert a progressively more negative marginal impact as they become large enough to extend the duration of the liquidity trap.

liquidity trap is shortened by one period (as determined by equation 15), and the multiplier falls discontinuously (to the value implied by equation (18) with $T_n = 7$). The multiplier continues to decline in a step-wise fashion – with equation (15) determining the threshold levels of spending at which the multiplier drops – until leveling off at a constant value of $\frac{1}{g_y}\frac{dy_t^{pot}}{dg_t}$ corresponding to a spending level high enough to keep the economy from entering a liquidity trap. Given that the multiplier declines with spending, the average change in output per unit increase in government spending, $\frac{1}{g_y}\frac{\Delta y_t}{\Delta g_t}$, lies well above the marginal response $\frac{1}{g_y}\frac{dy_t}{dg_t}$; to differentiate between these concepts in the figure, the former is labeled the "average multiplier," and the latter the "marginal multiplier" (in a slight abuse of terminology, since the multiplier is inherently a marginal concept).

The relationship between the multiplier and the size of government spending can be given an alternative graphical interpretation using Figure 1.a. Recall that the effect of the adverse taste shock alone on the potential real interest rate is shown by the solid line. This shock's effect on the output gap is proportional to $\sum_{j=0}^{T_n-1}(-i - r_{t+j|t}^{pot})$, the sum of the bold vertical line segments between $-i$ and the path of r_{t+j}^{pot} (the "interest rate gaps") implied by the taste shock through period $T_n - 1$. The 1 percent of GDP rise in government spending shown by the dashed line leaves the liquidity trap duration unchanged, implying that the higher government spending narrows the gap between between $-i$ and r_{t+j}^{pot} over a full $T_n = 8$ periods. But for a spending program equal to 2 percent of GDP (the dash-dotted line), $r_{t+T_{n-1}}^{pot}$ exceeds $-i$. Thus, further increments to spending have no effect on the interest rate gap at $T_n - 1$, as the effect on the potential real rate in this period is completely offset by monetary policy. Because additional spending only shrinks the interest rate gap for 7 quarters, the multiplier is lower.

The variation in the multiplier with spending becomes more pronounced with an upward-sloping Phillips Curve. When expected inflation responds, movements in the potential real interest rate r_t^{pot} have larger effects on the output gap than implied by equation (16), so that the same taste shock has a larger contractionary effect, and higher government spending has a more stimulative effect. To see how the effects of variation in r_t^{pot} are magnified, equations (1) and (2) can be solved forward (imposing the zero bound constraint that $i_t = -i$) to express inflation in terms of current and future interest rate gaps:

$$\pi_t = -\hat{\sigma}\kappa_p \sum_{j=0}^{T_n-1} \psi(j)(-i - r_{t+j|t}^{pot}), \tag{19}$$

where the weighting function $\psi(j)$ is given by

$$\psi(j) = \lambda_1\psi(j-1) + \lambda_2^j, \tag{20}$$

with the initial condition $\psi(0) = 1$, and where λ_1 and λ_2 are determined by:

$$\lambda_1 + \lambda_2 = 1 + \beta + \hat{\sigma}\kappa_p, \qquad \lambda_1\lambda_2 = \beta. \tag{21}$$

Given that $\kappa_p > 0$, the coefficients $\psi(j)$ premultiplying the interest rate gap grow exponentially with the duration of the liquidity trap T_n. Moreover, the contour is extremely sensitive to κ_p, as illustrated in Figure 1.c for several values of κ_p associated with price contract durations ranging from four to ten quarters.[9] The convex pattern of weights reflects that deflationary pressure associated with any given-sized interest rate gap ($-i - r^{pot}_{t+j|t}$) is compounded as the liquidity trap lengthens by the interaction between the response of the output gap and expected inflation.

Consistent with the analysis of Eggertson (2009 and 2010), Christiano, Eichenbaum, and Rebelo (2011), and Woodford (2011), the multiplier can be amplified substantially relative to normal circumstances in a long-lived liquidity trap, especially if the Phillips Curve slope is relatively high. This is illustrated in the lower left panel of Figure 3, which plots the impact government spending multiplier under alternative specifications of the parameter ξ_p that imply price contracts with a mean duration between 4 and 10 quarters. With short enough price contracts, the multiplier increases in a sharply convex manner with the duration of the liquidity trap, in contrast to the concave relation that obtains when expected inflation is less responsive. For example, in a liquidity trap lasting 12 quarters (see the tick marks on the upper axis), the multiplier is about 7 with five quarter contracts, and 25 with four quarter contracts. Clearly, fiscal policy can be very effective in providing economic stimulus.

Even so, under exactly the same conditions in which the government spending multiplier is very large – a long-lived trap, and shorter-lived price contracts – the multiplier drops substantially as government spending increases. Intuitively, the multiplier is large under these conditions because fiscal stimulus helps reverse the strong deflationary pressure arising from the adverse taste shock. But insofar as the higher spending is very efficacious and shortens the liquidity trap, the deflationary pressure abates and the benefits of additional stimulus diminish substantially. In this vein, the lower left panel of Figure 3 can be interpreted as showing how the impact multiplier varies with the level of government spending assuming that the taste shock induces an eight quarter liquidity trap.[10] The multiplier associated with four quarter price contracts drops from about 4 for a spending level of 1 percent of GDP to about 1.5 for spending increments above 3.5 percent of GDP. The marginal multiplier schedules for the cases of 5 and 10 quarter price contracts are considerably flatter, though the multipliers clearly decrease in the level of spending.[11]

Our assumptions that the timing of fiscal stimulus coincides exactly with the arrival of the shock causing the liquidity trap, and that the spending shock is equally persistent, are useful for expositional clarity in showing how the multiplier varies with spending. But while the downward-

[9] As is clear from equations (19) and (21) ,the aggregate demand elasticity $\hat{\sigma}$ also influences the inflation response, reflecting that the contour is determined by *the product* of $\hat{\sigma}$ and κ_p.

[10] As in the upper left panel, the "0" spending level on the lower horizontal axis implies an 8 quarter liquidity trap (denoted by the tick marks on the upper horizontal axis).

[11] The two-state Markov framework adopted by Eggertson (2009 and 2010), Christiano, Eichenbaum, and Rebelo (2011), and Woodford (2011) provides a great deal of clarity in identifying factors that can potentially account for a high multiplier, which is the focus of their analysis. But given that the depth of the recession – and associated fall in the potential real interest rate – is assumed to be constant in the liquidity trap state, the multiplier also turns out to be constant in a liquidity trap irrespective of the level of spending (i.e., until spending rises enough to snap the economy out of the liquidity trap entirely).

sloping multiplier schedule is a robust implication, the exact contour of the government spending multiplier schedule depends on a range of factors, including both the timing and persistence of government spending shocks, as well as on the characteristics of the adverse shocks causing the liquidity trap. Christiano, Eichenbaum, and Rebelo (2011) show that the spending multiplier tends to be larger if the spending is timed to coincide with the period in which monetary policy is constrained by the zero lower bound. Erceg and Linde (2010) emphasize that even the profile of fiscal stimulus over the period in which monetary policy is constrained can markedly affect the multiplier: in particular, significant lags in the implementation of the spending hike – through damping the impact on the potential real rate – can reduce the multiplier substantially.[12]

The multiplier schedule also depends on the conduct of monetary policy. In general, the coefficients of the monetary rule influence the size of the government spending multiplier both through affecting the duration of the liquidity trap, and through their effect on the "normal times" spending multiplier that applies when monetary policy is no longer constrained. The relative weight attached to inflation compared with output gap stabilization in the monetary rule can have particularly important implications for the fiscal multiplier in the event that the underlying shocks induce a policy tradeoff between stabilizing inflation and the output gap. As an illustration, suppose that the adverse demand (i.e., taste) shock in our baseline were accompanied by a markup shock that boosted inflation. Under some conditions, a monetary policy rule that put a large weight on inflation stabilization would imply a shorter-lived liquidity trap than a policy rule more tilted towards stabilizing the output gap, implying a smaller fiscal multiplier in the former case.[13]

Our benchmark monetary policy rule assumes that policy rates, if constrained, respond only to contemporaneous values of inflation and the output gap. The government spending multiplier would tend to be lower under an inertial, or "history dependent" policy rule that more closely approximated the optimal policy under commitment (Woodford 2003). In this case, monetary policy is more effective in cushioning the economy from the effects of adverse demand (or financial) shocks, which reduces the benefits of discretionary fiscal actions.[14]

2.4. Marginal Impact on Government Budget

We next consider the budgetary impact of government spending in our benchmark model. The right panels of Figure 3 show how the response of the government debt/GDP ratio after four quarters

[12] Interestingly, committing to a constant increase in government spending through date T_{N-1} yields a somewhat larger marginal multiplier than shown in Figure 3. This policy boosts the potential interest rate sharply at T_{N-1}, which is very useful in limiting some of the downward pressure on inflation associated with a long-lived liquidity trap (noting that equation 19 implies that the response of expected inflation is highly sensitive to the real interest rate gap at more distant horizons). Even so, the multiplier drops even more precipitously with the level of spending than under our benchmark calibration (reflecting that spending remains at its initial level for "too long" as the liquidity trap duration shortens).

[13] This case is illustrated in Figure A.1 of Appendix A.

[14] This case is illustrated in Figure A.2 of Appendix A.

varies with the level of government spending assuming that the baseline taste shock generates an eight quarter liquidity trap. Consistent with the impulse responses discussed earlier, a 1 percent of GDP rise in government spending *reduces* government debt by 1/2 percentage point under our benchmark calibration with five quarter price contracts (lower right panel), and over 1.5 percentage points under four quarter price contracts; as noted, the government debt response in the latter case remains well below baseline even for several years. With longer price contracts of 10 quarters, the lower spending multiplier implies a rise in government debt; hence, taxes must increase, though by much less than in normal times. Under all of these calibrations, the implications for the debt/GDP ratio would be even more favorable if we allowed for a positive steady state debt/GDP ratio, reflecting that higher inflation – and a consequently higher price level – would reduce the real value of the outstanding stock government debt.

While the possibility of a fiscal free lunch – or even a cheap lunch – is intriguing, our analysis highlights that it is vital for policymakers to distinguish between the marginal and average effects on government debt (and hence taxes) of different-sized spending programs. Although government debt may fall in response to small spending increments under conditions that imply a high spending multiplier – i.e., a substantial responsiveness of expected inflation – larger increases in spending may put sizeable upward pressure on government debt and tax rates, as is particularly apparent in the case of four quarter contracts in the lower right panel.

Our analysis also has implications for how fiscal *consolidation* may affect the economy when monetary policy is constrained by the zero bound. Figure 3 indicates that a persistent reduction in the level of government spending – if large enough – lengthens the duration of the liquidity trap, and can have very contractionary effects if the multiplier schedule is sufficiently convex. In this case, a reduction in government spending may boost government debt persistently. For example, a 5 percent of GDP spending cut under our benchmark causes the debt/GDP ratio to rise by almost 2 percentage points after 4 quarters, and the debt/GDP ratio remains about 1.3 percentage point above baseline even after 3 years.[15] This analysis of a temporary spending cut may well overstate the negative impact of fiscal consolidation to the extent that the latter is perceived as more enduring (in which case crowding in effects could be substantially larger). Even so, it provides a strong caution that fiscal consolidation that is perceived as temporary – perhaps due to low credibility – can generate a deep output contraction and cause government debt to rise for a prolonged period. Insofar as larger spending cuts may prolong a liquidity trap significantly and increase the marginal impact on output and government debt, it is clearly important for policymakers to understand the full contour of the multiplier schedule.

In addition to the output multiplier, the government debt response also depends crucially on the composition of the tax base, and the cyclical responsiveness of its key components. If the tax base is less (more) cyclically-sensitive, the consequences of higher government spending for government

[15] For visual clarity, Figure 3 does not show the effects of cuts in spending on the debt/GDP ratio (since a cut implies a rise in debt).

debt and taxes may appear less (more) benign than in Figure 3, even if the spending multiplier is unchanged. To illustrate this, it is helpful to amend the benchmark model slightly by assuming that government spending in the steady state is financed by both a labor tax τ_N *and* a sales tax τ_C that is levied on private consumption. Under the assumption that government debt is zero in steady state, the government budget constraint implies that tax rates satisfy the relation that $g_y = \tau_N s_N + \tau_C(1-g_y)$, where s_N is the steady state labor income share. Retaining the assumption that only lump-sum taxes adjust dynamically, so that the dynamics of the government spending multiplier are unaffected by the form of tax financing, government debt evolves according to:

$$b_{G,t} = (1+r)b_{G,t-1} + g_y g_t - \tau_N s_N(y_t + \phi_{mc}x_t) - \tau_C(1-g_y)c_t - \tau_t. \tag{22}$$

Hence, the impact of a one percent of GDP rise in government spending on the government debt/GDP ratio depends on the response of labor income $y_t + \phi_{mc}x_t$, of private consumption c_t (the base for the sales tax, which equals $\frac{1}{1-g_y}(y_t - g_y g_t)$), and on the share of government spending financed by each type of tax. The response of labor income to a one percent of GDP rise in government spending may be expressed as $\frac{1}{g_y}\frac{dy_t}{dg_t} + \phi_{mc}\frac{1}{g_y}\frac{dx_t}{dg_t} = m_t + \phi_{mc}(m_t - m_t^{pot})$, where m_t is the spending multiplier. The term involving ϕ_{mc} is nonnegative, and increases sharply with the multiplier if marginal cost is relatively sensitive to the output gap (i.e., if ϕ_{mc} is relatively high). The response of consumption to the same spending hike is given by $\frac{1}{g_y}\frac{dc_t}{dg_t} = \frac{1}{1-g_y}(m_t - 1)$, which implies much less variation with the multiplier. For example, with a multiplier of unity, labor tax revenue would rise 5 percent in response to the spending increase, while no revenue would accrue from a sales tax (since private consumption would remain unchanged). As noted earlier, the primary balance would improve – and hence government debt decline – under a pure labor income tax if the multiplier exceeded unity, while the multiplier would have to exceed 5 for government debt to decline under a pure sales tax.

Thus, the cyclical responsiveness of the tax base can play a major role in determining the budgetary implications of higher government spending. From a policy perspective, the budgetary implications of fiscal stimulus may diverge markedly across countries with different tax bases, such as between the United States (where labor taxes, inclusive of payroll taxes, comprise a large share of revenue at the federal level) and European countries in which sales taxes are more important.

From the perspective of the literature, our model's implication that a fiscal expansion may induce government debt to contract has an important parallel in the analysis of Davig and Leeper (2011). Within the context of a New Keynesian sticky price model, these authors show that a spending expansion both has large effects on output and can cause government debt to decline if monetary policy behaves passively (allowing real interest rates to fall), and if fiscal policy is "active" (so that the tax rule implies a small response of taxes to debt). In this environment, because tax policy is not expected to be aggressive enough to generate eventual primary surpluses, the price level must rise, reducing government debt enough to satisfy the government's intertemporal budget constraint. Importantly, the implications for the fiscal multiplier and debt reflect the imprint of the "passive monetary, active fiscal" regime, so that taxes or tax rates would never have to adjust much in such a regime (and the spending multiplier would always be high). By contrast, our model assumes that monetary policy is active once the zero bound constraint is no longer binding, and that fiscal policy is passive, so that the rule is committed to adjusting taxes aggressively enough to stabilize debt. Thus, both the multiplier and budgetary implications of alternative spending choices are highly state-dependent, rather than a characteristic of the regime.

2.5. Distortionary vs. Lump-Sum Taxes

We next compare our benchmark specification with lump-sum taxes to an alternative in which the labor income tax rate $\tau_{N,t}$ adjusts according to the rule:

$$\tau_{N,t} = \psi_1 b_{G,t-1} + \psi_2 \{g_y g_t - \tau_N s_N (y_t + \phi_{mc} x_t)\}. \tag{23}$$

According to (23), the tax rate on labor income $\tau_{N,t}$ reacts endogenously to the lagged stock of government debt $b_{G,t-1}$, and to the primary budget deficit that would accrue if the labor income tax rate remained fixed at its steady state value of τ_N (the term in parentheses that postmultiplies ψ_2). The latter term is convenient for analyzing tax rules designed to stabilize debt aggressively, with a balanced budget rule a special case in which $\psi_1 = 0$ and $\psi_2 = \frac{1}{s_N}$.

Allowing the labor tax rate to be determined endogenously has potentially important implications for the government spending multiplier that arise through effects on inflation, potential output, and the potential real rate. With regard to inflation, the price-setting equation (2) becomes:

$$\pi_t = \beta \pi_{t+1|t} + \kappa_p \{\phi_{mc} x_t + \frac{1}{(1-\tau_N)}(\tau_{N,t} - \tau_{N,t}^{pot})\}. \tag{24}$$

The salient change relative to the standard price-setting equation is that real marginal cost – the expression in parentheses – depends on the "tax gap" $\tau_{N,t} - \tau_{N,t}^{pot}$ in addition to the standard output gap term. The tax gap enters because the endogenous labor tax rule (23) implies that the tax rate $\tau_{N,t}$ may differ from its level under flexible prices of $\tau_{N,t}^{pot}$. As suggested by equation (23), if a shock to government spending boosts labor income by more than would occur under flexible prices (because the multiplier is large), the tax reaction function implies a lower labor income tax rate ($\tau_{N,t}$) than would occur under flexible prices ($\tau_{N,t}^{pot}$). This puts downward pressure on marginal cost and inflation, which reduces the multiplier.[16]

[16] The tax gap is inversely related to the current output gap x_t and directly related to the lag of the debt gap $(b_{G,t-1} - b_{G,t-1}^{pot})$:

$$(\tau_{N,t} - \tau_{N,t}^{pot}) = -\psi_2 \tau_N s_N (1+\phi_{mc}) x_t + \psi_1 \frac{1}{(1+\psi_2 \frac{s_N}{(1-\tau_N)})}(b_{G,t-1} - b_{G,t-1}^{pot}). \tag{25}$$

This equation follows from equation (23) and the government budget constraint (equation 11, after making the appropriate substitution for the distortionary tax rate in place of the the lump-sum tax). Because the debt gap varies inversely with past output gaps (i.e., positive output gaps reduce debt relative to the level implied under flexible prices), the tax gap varies inversely with current and past output gaps.

The second key channel through which distortionary taxes influences the multiplier is through potential output, which may be expressed:

$$y_t^{pot} = \frac{g_y}{\phi_{mc}\hat{\sigma}\delta_1} g_t - \frac{\psi_2}{\phi_{mc}\hat{\sigma}\delta_1} \frac{\hat{\sigma} g_y}{(1-\tau_N)\delta_1} g_t - \frac{\psi_1}{\phi_{mc}(1-\tau_L)\delta_1} b_{G,t-1}^{pot} + \frac{(1-g_y)}{\phi_{mc}\hat{\sigma}\delta_1} \nu_c \nu_t, \qquad (26)$$

where $0 < \delta_1 \leqq 1$.[17] The standard wealth effect of a government spending shock is captured by the first term: as in the benchmark model, higher government spending reduces consumption, expands potential labor supply, and boosts y_t^{pot}. The second term is negative provided that $\psi_2 > 0$, and reflects that y_t^{pot} is reduced to the extent that the labor tax rate reacts contemporaneously to higher government spending. The third term reflects how past increments to government spending – by raising potential government debt $b_{G,t-1}^{pot}$ and hence the tax labor rate by equation (23) – reduce potential output y_t^{pot}. Thus, labor tax financing tends to reduce the spending multiplier by reducing the response of potential output relative to lump-sum financing.

The third channel is through the potential real interest rate r_t^{pot}. Both the form of the IS curve given by equation (1) and the equation determining r_t^{pot} are the same as under lump-sum taxes; the latter can be written:

$$r_t^{pot} = y_{t+1|t}^{pot} - y_t^{pot} + g_y(g_t - g_{t+1|t}) + v_c(\nu_t - v_{t+1|t}). \qquad (27)$$

The effect of higher spending on r_t^{pot}under the alternative modes of financing – and thus the stimulus to aggregate demand – depends on the response of potential output growth. Although distortionary taxes depress the response of the *level* of potential output to a spending hike relative to lump-sum taxes, the *growth rate* of potential output may be higher or lower depending on the parameters of the tax rule (with a larger response of potential output growth possible under a rule that responds aggressively to the deficit, as we will illustrate).

In the case in which the tax rate reaction function reaction reacts only to the stock of debt ($\psi_1 > 0$ and $\psi_2 = 0$), the three channels discussed above operate in the same direction to reduce the government spending multiplier. This is illustrated in Figure 4, which compares the effects of a 1 percent of GDP rise in government spending under our benchmark with lump-sum financing to an alternative ("simple debt labor tax rule") which sets $\psi_1 = 0.01$ in the tax reaction function (23) (this calibration implies that a 10 percentage point increase in the annualized debt/GDP ratio boosts the labor tax rate by 0.4 percentage points). The peak multiplier in the latter case is only about 3/4 as large as under lump-sum taxes. The aggregate demand stimulus is smaller under the labor tax because r_t^{pot} rises by less (potential output growth is *relatively* lower because $\tau_{N,t}^{pot}$ rises gradually as potential government debt increases), and because inflation rises less as the negative tax gap restrains pressure on marginal cost. On the supply side, the response of potential output is also smaller. The smaller multiplier translates into a more substantial increase in government debt. Given that inflation is less responsive under labor tax financing, the disparity between the

[17] The parameter $\delta_1 = 1 - \frac{\psi_2}{\phi_{mc}} \frac{s_N}{(1-\tau_N)}$.

spending multiplier under these alternative taxation schemes becomes considerably larger as the duration of the liquidity trap lengthens beyond the eight quarters shown in the figure. For example, the multiplier exceeds 7 for a 12 quarter liquidity trap under lump-sum taxes, but is only slightly above 2 under labor tax financing.[18]

Conversely, the differences across taxation schemes become smaller to the extent that the tax rule responds more gradually to government debt. For example, under an inertial form of the same distortionary tax rule considered in Figure 4 – of the form $\tau_{N,t} = 0.98\tau_{N,t-1} + (1 - 0.98)\,\psi_1 b_{G,t-1}$ – the spending multiplier and government debt responses are virtually identical to the lump-sum tax case (and hence are omitted for visual clarity). Under this more inertial tax rule, the tax gap in equation (24) is smaller, so inflation determination isn't much affected relative to lump-sum tax case.[19] As the liquidity trap duration extends beyond the eight quarters shown in the figure, the multiplier rises only a bit faster with duration under lump-sum taxes than labor tax financing.[20]

Given that previous research, including by Eggertson (2010) and Christiano, Eichenbaum, and Rebelo (2011), has shown that the government spending multiplier is *amplified* in a liquidity trap if accompanied by an exogenous rise in the labor tax rate, our result that the multiplier is reduced might appear to hinge on the gradual adjustment of tax rates to debt implied by setting $\psi_2 = 0$ in the tax reaction function (23). In particular, a key channel highlighted by this previous literature in accounting for greater stimulus is that a higher distortionary tax *raises* the potential real rate r_t^{pot} relative to the lump-sum case, which boosts expected inflation and hence the multiplier (given that monetary policy does not react).[21] However, the different implications turn out to mainly reflect that we specify an endogenous tax reaction function, rather than assume that the labor tax adjusts exogenously. In our framework, the effects of endogenous tax changes on marginal cost and inflation implied by equation (24), which lower the responsiveness of inflation, can dominate the behavior of the multiplier. Consequently, even a tax reaction function that reacts aggressively to the current deficit in order to stabilize government debt and that generates a much larger rise in r_t^{pot} can imply a smaller multiplier than in the lump-sum case.

To show how the multiplier may be reduced (relative to the lump-sum case) even under an aggressive tax rule, it is useful to consider the special case of a balanced budget rule (which sets $\psi_1 = 0$ and $\psi_2 = \frac{1}{s_N}$ in equation (23). Figure 4 shows the effects of the 1 percent of GDP increase in government spending under the balanced budget rule. Because y_t^{pot} declines in a front-loaded

[18] The upper panels of Figure A.1 in Appendix A shows how the (marginal) government spending multiplier and government debt response vary with the liquidity trap duration under both lump-sum taxes and the distortionary tax rate rule; the figure also includes an inertial version of the latter, and a balanced budget rule.

[19] Moreover, because $\tau_{N,t}^{pot}$ adjust even more slowly to potential debt, the evolution of y_t^{pot} and r_t^{pot} is also very similar to the case of lump-sum taxes.

[20] See Figure A.3 in Appendix A.

[21] Because the tax hike is assumed to be front-loaded, potential output falls immediately, but potential output growth (and hence r_t^{pot}) increases. Thus, the tax hike stimulates aggregate demand if interest rates are left unchanged.

manner under the balanced budget rule, r_t^{pot} rises more sharply than in the lump-sum tax case.[22] While the partial effect of an increase in r_t^{pot} is to boost the multiplier relative to the lump-sum case, the overall impact hinges on the extent to which inflation behavior is influenced by the endogenous reaction of the labor tax. In the case of a balanced budget rule, the tax gap $\tau_{N,t}-\ \tau_{N,t}^{pot}$ is simply proportional to the current output gap, so that endogenous tax adjustment in effect reduces the Phillips Curve slope by a factor that depends on the steady state level of the distortionary tax, i.e., equation (24) becomes:

$$\pi_t = \beta\pi_{t+1|t} + (\kappa_p - \psi_2 \frac{\tau_N}{(1-\tau_N)} s_N)x_t. \tag{28}$$

Under our benchmark calibration with a government spending share of 20 percent ($\tau_N = 0.27$), Figure 4 shows that the inflation response is noticeably smaller under the balanced budget rule than under lump-sum taxes, which explains the smaller output response. The lower Phillips Curve slope also means that the gap between the spending multipliers rises as the liquidity trap duration becomes more prolonged. But interestingly, because the effect of the tax gap on the Phillips Curve slope declines as the tax rate becomes very low, our model can also yield results more in line with the exogenous tax specifications considered in the literature. For example, a calibration with a steady state government spending share of 5 percent generates a larger spending multiplier under the balanced budget rule with distortionary taxes than under lump-sum taxes if the liquidity trap extends beyond six quarters (since the effect of higher r_t^{pot} dominates the effect of a lower Phillips Curve slope under this calibration).[23]

Overall, our analysis suggests that the response of output and government debt to government spending under distortionary taxes is probably only modestly lower than under lump-sum taxes provided that tax rates adjust fairly inertially to government debt. This seems important from a practical perspective, as tax rates typically exhibit considerable inertia. Even so, our analysis underscores that the nature of tax adjustment can potentially have substantial consequences for the multiplier if taxes adjust rapidly to debt or current deficits, an environment that may become more relevant going forward in the wake of mounting concerns about fiscal sustainability. In terms of the recent literature on fiscal spending multipliers, our results help explain why Drautzburg and Uhlig (2011) - who uses an aggressive labor income tax rule to stabilize debt - obtain only a moderately higher spending multiplier in a liquidity trap than in normal times, while other authors – including Christiano, Eichenbaum and Rebelo (2011), Eggertsson (2010) and Woodford (2011) – find much higher multipliers under the assumption of lump-sum financing.

[22] Referring to equation (26), the front-loaded decline in potential output reflects that the wealth effect of higher government spending – the first term – is more than offset by the effect of immediately higher labor tax rates – the second term.

[23] Figure A.3 in Appendix A shows marginal multipliers under the balanced budget rule for alternative government spending shares.

2.6. Endogenous Government Spending

The fiscal multiplier may also be affected by the presence of automatic stabilizers.[24] In particular, while some component of government spending g_t^{exo} may be exogenous (and follow the same autoregression as in our benchmark), another component g_t^{endo} may be endogenous and vary with cylical conditions. To illustrate some key channels through which endogenous spending operates, it is instructive to consider the simple specification $g_t^{endo} = -\mu x_t$, where $\mu > 0$, so that a 1 percent rise in the output gap induces a contraction of μ percent in the endogenous component of spending (with g_t^{endo} equal to zero in the steady state). In this case, the New Keynesian IS curve given by equation (1) is unchanged, except that the interest-sensitivity of demand is reduced from $\hat{\sigma}$ to $\hat{\sigma}^A = \frac{\hat{\sigma}}{1+\mu}$. Intuitively, because the endogenous spending "leans against the wind" by contracting spending when output rises above potential, the response of the output gap to a given-sized interest rate gap is reduced. The endogenous spending response – as in the case of the distortionary tax rule – also affects the slope of the Phillips Curve. Because government spending falls when the output gap is positive, labor supply shifts inward (due to a positive wealth effect), which translates into a heightened sensitivity of real marginal cost to the output gap. Again, the form of the Phillips Curve is unchanged, but the marginal cost sensitivity to output increases from ϕ_{mc} to $\phi_{mc}^A = \frac{\alpha+\chi}{1-\alpha} + \frac{1+\mu}{\hat{\sigma}}$.

Given that the New Keynesian model equations (1)-(5) are unchanged aside from these parameter adjustments – and importantly, the duration of the liquidity trap is unaffected – we can easily analyze the implications of alternative response coefficients μ. Thus, in the special case of a flat Phillips Curve, the spending multiplier associated with a given-sized increase in the exogenous component of spending g_t^{exo} is smaller if $\mu > 0$, since automatic stabilizers reduce the interest-sensitivity of demand; the quantitative effect on the multiplier is given by equation (18). While it is natural to conjecture that the higher Phillips Curve slope under automatic stabilizers might raise the multiplier under certain conditions, the inflation responsiveness depends on the product of ϕ_{mc}^A and $\hat{\sigma}$ (from equations 19 and and 7), which declines in μ. Thus, automatic stabilizers unambiguously reduce the multiplier. Moreover, the multiplier schedule relating the multiplier to the level of spending becomes less convex as μ increases.[25]

The responsiveness of automatic stabilizers to cyclical conditions may have important implications for the expected benefits of discretionary fiscal stimulus in the presence of uncertainty. In particular, decisions about the size of a discretionary fiscal spending program must often be made against the backdrop of considerable uncertainty about how long the liquidity trap would be likely to last in the absence of fiscal stimulus. To the extent that the multiplier is convex in the liquidity trap duration – and the duration is uncertain – the expected spending multiplier will exceed the

[24] We thank Eric Leeper for suggesting an analysis of automatic stabilizers and uncertainty along the lines of this subsection.

[25] Figure A.4 in Appendix A compares the effects of a one percent of baseline GDP increase in the exogenous component of government spending under our benchmark with $\mu = 0$ to an alternative specification in which μ is set to unity.

multiplier under perfect foresight (i.e., the multiplier associated with the mean expected duration of the liquidity trap). Quite intuitively, this wedge between the multiplier under uncertainty and perfect foresight reflects that the payoff to fiscal expansion in bad states is especially high. Because automatic stabilizers reduce the convexity of the multiplier schedule, the difference between the expected multiplier and multiplier under perfect foresight is comparatively smaller. Accordingly, the desirability of committing to a large fiscal expansion to hedge against adverse tail risks would appear stronger to the extent that automatic stabilizers are relatively weak.[26]

3. Fiscal Stimulus in a New-Keynesian Model with Keynesian Households and Financial Frictions

In this section, we examine how the spending multiplier and government debt responses depend on the level of spending in a more empirically realistic framework with endogenous capital accumulation. The core of our model is a close variant of the models developed and estimated by Christiano, Eichenbaum and Evans (2005), CEE henceforth, and Smets and Wouters (2003, 2007), SW henceforth. CEE show that their model can account well for the dynamic effects of a monetary policy innovation during the post-war period. SW consider a much broader set of shocks, and argue that their model – which is estimated by Bayesian methods – is able to fit many key features of U.S. and euro area-business cycles.

However, we depart from the CEE/SW environment in two substantive ways. First, we assume that a fraction of the households are "Keynesian", and simply consume their current after-tax income. Galí, López-Salido and Vallés (2007) show that the inclusion of non-Ricardian households helps account for structural VAR evidence indicating that private consumption rises in response to higher government spending, and also allows their model to generate a higher spending multiplier. Second, to capture financial channels omitted from the CEE/SW framework, we incorporate a financial accelerator following the basic approach of Bernanke, Gertler and Gilchrist (1999). In this framework, the corporate finance premium varies with the degree of leverage of the economy due to an agency problem in private lending markets.[27]

We set the share of Keynesian households to optimizing households to 0.5, implying that the former comprise about 1/3 of aggregate consumption in the steady state, and calibrate the parameters affecting the financial accelerator as in BGG (1999). However, we also report some results from a CEE/SW-type specification to help gauge the sensitivity to these factors.

Given space limitations, we relegate most of the remaining details about the model, solution

[26] While a complete treatment of the interaction between automatic stabilizers and uncertainty is well beyond the scope of this paper, Figure A.4 of Appendix A illustrates how automatic stabilizers affect the expected spending multiplier in a simple framework in which the government spending choice must be made before the size of the adverse shock(s) is revealed (see panels B and C and the associated discussion).

[27] Following Christiano, Motto and Rostagno (2008), we assume that the debt contract between the entrepreneurs and lenders (households) is written in nominal terms (rather than real terms as in BGG 1999).

method, and calibration to Appendix B.[28] Even so, it is important to highlight two features. First, in the model's fiscal block, government revenue is assumed to be derived from taxes on labor and capital.[29] While the tax rate on capital income is fixed, the distortionary tax on labor income reacts to annualized government debt according to the calibrated rule:

$$\tau_{N,t} = 0.92\tau_{N,t-1} + (1 - .92)\, 0.1\tilde{b}_{G,t}. \tag{29}$$

Because this tax rule has substantial inertia – and is not very aggressive even in the long-run – the consequences for the multiplier are very similar to lump-sum taxes.[30]

Second, our calibration of both the parameters of the monetary policy rule and the Calvo price and wage contract duration parameters – while within the range of empirical estimates – tilt in the direction of reducing the sensitivity of inflation to shocks. In particular, the monetary rule that is followed when policy is unconstrained is a Taylor rule with a fairly aggressive long-run coefficient of 3 on inflation, of unity on the output gap, and 0.7 on the lagged interest rate. Our choice of a price contract duration parameter of $\xi_P = .90$ implies a Phillips Curve slope of about .007, which is on the low side of the median estimates reported in the empirical literature, even if well within reported confidence intervals; and wages exhibit a commensurate degree of stickiness.[31] These parameter choices are aimed at capturing the resilience of core inflation, and measures of expected inflation, during the global recession.

3.1. Dynamic Effects of Government Spending

Figure 5 shows the effects of a front-loaded increase in government expenditures equal to 1 percent of steady state output under our benchmark calibration. The government spending shock follows an AR(1) with a persistence of 0.9. The spending hike occurs against the backdrop of initial conditions consistent with a deep recession and liquidity trap expected to last eight quarters (these initial conditions, which are generated by a sequence of adverse taste shocks, are described in Appendix B). The figure shows results both for the benchmark model (labeled ZLB Full Model), and for

[28] In the models used in this paper, we have worked with log-linearized equations, aside from imposing the zero lower bound on policy rates. Given that we examine model dynamics well away from the steady state, a useful extension of our work would be to solve all model equations using nonlinear methods.

[29] Given a steady state government spending share of 20 percent and debt/GDP ratio of 50 percent, the steady state tax rate on labor income is 27 percent, and capital income 20 percent.

[30] The coefficients in the rule are taken from Traum and Yang (2011), who estimate a DSGE model using Bayesian methods; but even simple regression analysis suggests a high degree of tax smoothing. The multiplier would be reduced substantially with a more aggressive response to debt (both for reasons noted in Section 2.5, and because it reduces the disposable income of Keynesian households).

[31] The median estimates of the Phillips Curve slope in recent empirical studies by e.g. Adolfson et al (2005), Altig et al. (2011), Galí and Gertler (1999), Galí, Gertler, and López-Salido (2001), Lindé (2005), and Smets and Wouters (2003, 2007) are in the range of $0.009 - .014$. Given our specification of the steady-state wage markup and a wage contract duration parameter of $\xi_w = 0.85-$ along with a wage indexation parameter of $\iota_w = 0.9$ – wage inflation is about as responsive to the wage markup as price inflation is to the price markup.

a variant (labeled ZLB CEE/SW) which excludes financial frictions and Keynesian households. Results for each model variant are also presented for normal times in which monetary policy is unconstrained by the zero lower bound.

Under either model variant, the fiscal policy expansion implies larger effects on output in a liquidity trap than in a normal situation in which policy is unconstrained. As in the stylized model in Section 2, this reflects that higher government spending in a liquidity trap boosts the potential real interest rate while causing real interest rates to fall (as seen in the figure, the nominal interest rate does not respond for some time and expected inflation rises). Because the rise in the potential interest rate is amplified by Keynesian households and financial frictions, the output response is considerably larger in the full model than in the CEE/SW alternative when the economy is constrained by the zero bound; by contrast, the disparity in the output responses across models is much smaller in normal conditions, reflecting that monetary policy would raise interest rates more to offset a bigger rise in the potential real rate. The lower right panel shows the present value government spending multiplier as in Uhlig (2010), which at horizon K is defined as

$$m_K = \frac{1}{g_y} \frac{\sum_0^K \beta^K \Delta y_{t+K}}{\sum_0^K \beta^K \Delta g_{t+K}}. \tag{30}$$

Thus, the impact multiplier m_0 is simply given by $\frac{1}{g_y}\frac{\Delta y_t}{\Delta g_t}$.The implied government spending multiplier exceeds 1.5 in the full model for well over a year after the spending shock, but is only around unity in the CEE/SW model.

Against the backdrop of the eight quarter liquidity trap, the response of the government debt/GDP ratio is considerably smaller than under normal conditions under either model variant. In the full model, the government debt/GDP ratio rises only about half as much at a medium-run horizon of 3-5 years as under normal conditions, implying a much smaller rise in the labor tax rate than in normal times.[32] The smaller debt/GDP response is attributable to three factors. First, the higher multiplier in the liquidity trap boosts labor tax receipts substantially (i.e., holding the labor tax rate constant). Second, the fall in real interest rates lowers the cost of debt service relative to a normal situation in which real interest rates rise. Finally, capital income and hence revenue from the capital income tax rises by more.

3.2. Marginal vs. Average Responses

The solid line in the upper left panel of Figure 6 shows the multiplier as function of government spending for the full model with Keynesian households and financial frictions. Although benchmark parameters are unchanged, we consider a somewhat deeper liquidity trap that would last 10 quarters absent fiscal intervention (generated by larger taste shocks) to illustrate conditions sufficient to

[32] Note that while the tax-rule (29) responds to government debt as a ratio of annualized trend nominal output $\frac{B_{Gt}}{4P_tY}$, the figure reports government debt relative to actual output $\frac{B_{Gt}}{4P_tY_t}$. The difference between the responses becomes negligible after about 10 quarters as the effect of the spending shock on output dissipates.

produce a "fiscal free lunch." The average multiplier reported in the figure is for a four quarter horizon (i.e. m_3 in eq. 30); similarly, the marginal multiplier is derived from m_3, albeit for infinitesmal increments to spending that keep the liquidity trap duration unchanged.

As in the stylized model in Section 2, the marginal multiplier follows a step function. The marginal multiplier is nearly 5 for small additions to spending of less than 0.5 percent of GDP, but drops to 2.2 for a somewhat larger increments; thus, the average multiplier of 3.5 associated with a one percent of GDP spending hike lies well above the marginal.[33] The rapid falloff in the multiplier reflects that fiscal stimulus is very effective in mitigating the effects of recession when monetary policy is constrained for a prolonged period; but with a shallower recession, the benefits of additional stimulus decline substantially.[34]

The solid line in the right upper panel plots the marginal response of the government debt/GDP ratio for various levels of government spending. The debt responses shown are at a "medium-run" horizon of 12 quarters, and indicate the direction in which labor tax rates must eventually move. For a long-lived 10 quarter liquidity trap, the marginal multiplier is large enough that government debt falls substantially below baseline after several years, consistent with an enduring reduction in labor tax rates and a fiscal free lunch. A large rise in labor tax revenue (notwithstanding a slight decline in the labor tax rate) plays the biggest role in allowing government debt to fall at the margin, while lower debt servicing costs also make a major contribution (as higher nominal GDP reduces the ratio of existing nominal debt to GDP).[35] As additional spending shrinks the duration of the liquidity trap below 10 quarters (see the upper tick marks), the marginal effect on government debt turns positive, even though the average response – shown by the dotted line – remains negative. These results underscore how marginal increments to spending can put significant upward pressure on government debt and tax rates even when the average effects appear small.

The contour of the multiplier schedule can be highly sensitive to parameters determining the responsiveness of inflation. The lower left panel of Figure 6 shows the multiplier schedule under the benchmark calibration and several alternatives based on initial conditions that imply a liquidity trap of 8 quarters (as in Figure 5). While the multiplier is below 2 under our benchmark calibration at all spending levels, the multiplier can be much higher under calibrations that imply a larger response of expected inflation. The amplification of the multiplier is especially dramatic under a calibration in which both price and wage contracts are relatively short-lived ("more flexible prices and wages"), with the contract duration specified at four and five quarters, respectively. However, although the multiplier exceeds 9 for low spending levels below 0.3 percent of GDP, the multiplier

[33] The tick marks in along the upper horizontal axis indicate how the duration of the liquidity trap varies with the level of government spending.

[34] Under lump-sum tax adjustment, the multiplier is slightly higher for a deep liquidity trap of 10 quarters, but nearly identical for short-lived liquidity traps, reflecting that the tax rule responds gradually – and not very aggressively – to government debt.

[35] Figure B.2 in Appendix B, and the associated discussion, provides a more detailed analysis of the channels through which higher government spending generates a fiscal free lunch in a 10 quarter liquidity trap.

drops precipitously as spending increases and the liquidity trap duration shortens. This contour of the multiplier schedule reflects that even relatively small adverse shocks can cause a deep recession when inflation is fairly responsive; hence, only a small dose of fiscal expansion is called for to reverse most of these adverse effects. A second alternative examines a calibration in which prices are less sticky than under our baseline (i.e. the contract duration parameter ξ_p is lowered from 0.90 to 0.75), but wage-setting remains unaltered. The multiplier schedule in this case (the dotted line) is only slightly higher than under our benchmark, reflecting that the sluggish behavior of wages keeps price inflation from moving as much as under the previous alternative. This calibration underscores that the much higher multiplier under the "more flexible price and wage" calibration hinges on *both prices and wages* being much more flexible than under our benchmark. Finally, the marginal multiplier is also larger under the standard Taylor rule ("looser policy rule") than the benchmark for small spending increments, but the quantitative disparity appears small.

4. Conclusions

For an economy facing a deep recession and prolonged liquidity trap, there is a strong argument for increasing government spending on a temporary basis. But our analysis highlights the importance of recognizing that the marginal benefits of fiscal stimulus may drop substantially as spending rises, so that there is some risk that larger spending programs may have a low marginal payoff, and put substantial pressure on government budgets.

Governments and central banks have an array of options in addition to stimulative fiscal policy for mitigating the effects of a liquidity trap. For example, many central banks have used the asset side of their balance sheet to support credit markets by providing liquidity and purchasing long-term securities. Although the models we have examined are not designed to assess the effectiveness of such actions, our analysis highlights the importance of analyzing the effects of such actions *jointly* with the fiscal stimulus packages in order to properly assess their marginal impact.

As emphasized by Canova and Pappa (2011), a major issue for future research is to assess whether conditions that have been identified as likely to make fiscal policy highly effective hold empirically.[36] Many recent papers, including our own, have used calibrated models with a binding zero lower bound constraint to show that a sizeable response of inflation plays a crucial role in generating a large spending multiplier well above unity; this is also true in models in models in which the monetary policy regime is passive at least for some time.[37] However, the resilience of inflation in the aftermath of the global financial crisis gives reason to question whether inflation is

[36] These authors provide empirical evidence suggesting that the conditions for a high multiplier did not appear to be satisfied during the global recession.

[37] For example, Davig and Leeper (2011) show in a regime-switching model that the government spending multiplier under a passive monetary policy regime is around 1-1/2 after 10 quarters, roughly twice as high as under an active policy regime. The disparity mainly reflects a much larger and more persistent response of inflation under the passive regime.

as responsive to fiscal policy, and to macro shocks generally, as implied by existing models that are calibrated based on estimates derived from pre-crisis data. Even more directly, recent analysis by Canova and Pappa (2011) – using a structural VAR with sign restrictions – found that stimulative government spending shocks induce only a transient increase in inflation, rather than the persistent inflation rise required for a big spending multiplier. In future research, it will be important to draw on evidence from the global recession to further refine our empirical understanding of the role of different factors and policies in influencing the response of inflation to fiscal policy, including the characteristics of the monetary and fiscal policy regimes, the parameters of the price and wage Phillips Curve, and the nature of the shocks driving the economy into a liquidity trap.

There are also open questions about whether the traditional channels through which fiscal policy affects aggregate demand remain operative in a severe recession. The potency of the interest rate channel might be impaired to the extent that tight credit and heavy debt burdens reduce the interest-sensitivity of households and firms. As argued by Merten and Ravn (2010), the stimulative effects of government spending may also be muted if the source of recession is a self-fufilling loss in confidence, reflecting that the higher spending is perceived as a negative signal about the state of the economy. Conversely, various types of fiscal interventions could have a heightened impact through easing collateral constraints on borrowers, reducing precautionary savings, or by affecting financial market risk premia. From a modeling perspective, addressing some of these questions will require a non-linear stochastic framework to capture key channels through which fiscal interventions may operate in the presence of uncertainty such as in recent work by Bi, Leeper, and Leith (2011).[38]

[38] These authors examine how the effects of fiscal consolidation vary with the state of the economy, including the level of government debt.

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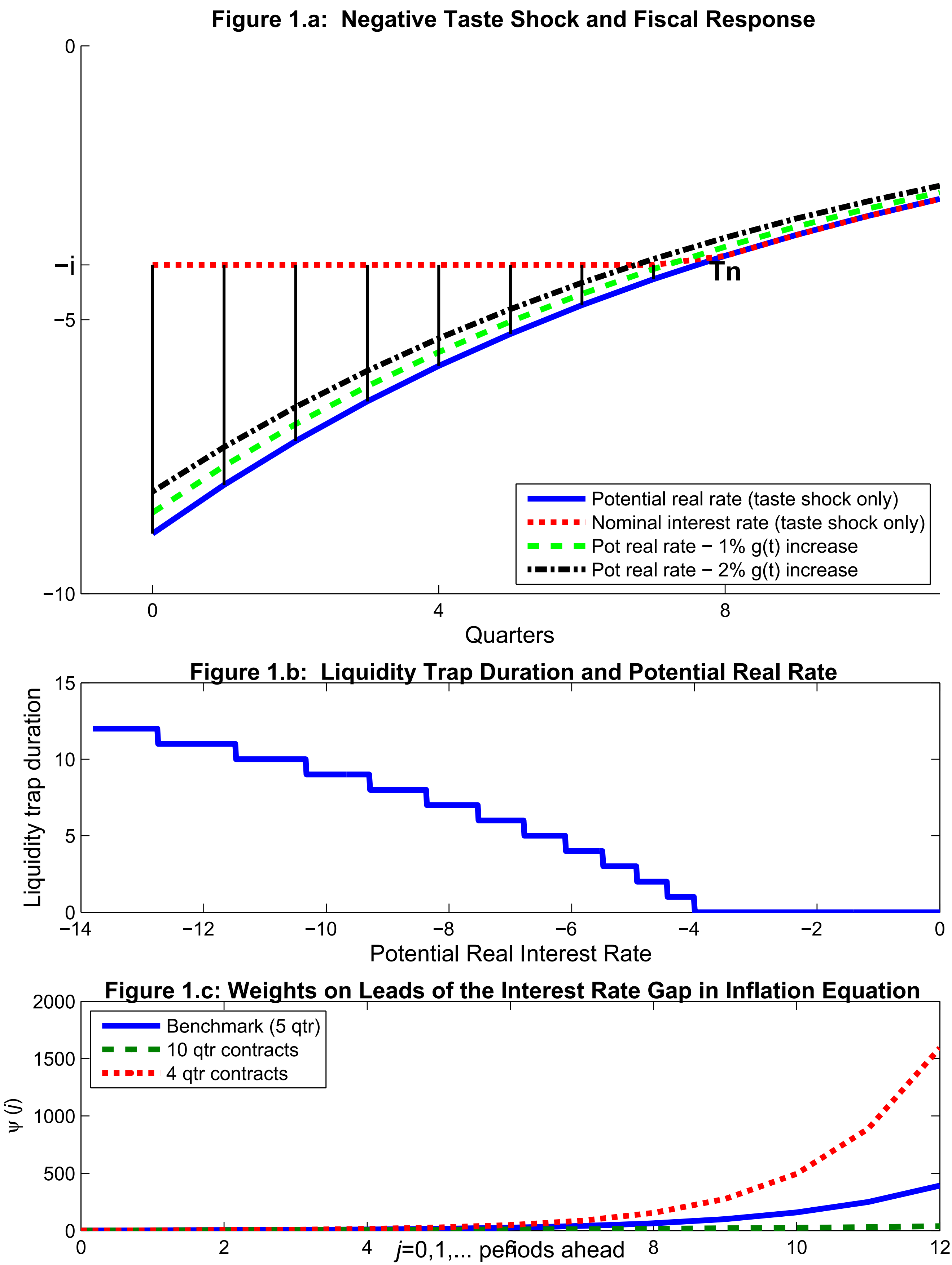


Figure 1.a: Negative Taste Shock and Fiscal Response
0
−i
−5
−10
Tn
Potential real rate (taste shock only)
Nominal interest rate (taste shock only)
Pot real rate − 1% g(t) increase
Pot real rate − 2% g(t) increase
0
4
8
Quarters
Figure 1.b: Liquidity Trap Duration and Potential Real Rate
Liquidity trap duration
15
10
5
0
−14
−12
−10
−8
−6
−4
−2
0
Potential Real Interest Rate
Figure 1.c: Weights on Leads of the Interest Rate Gap in Inflation Equation
ψ (j)
2000
1500
1000
500
0
Benchmark (5 qtr)
10 qtr contracts
4 qtr contracts
0
2
4
6
8
10
12
j=0,1,... periods ahead

Figure 2: Immediate Rise in Government Spending

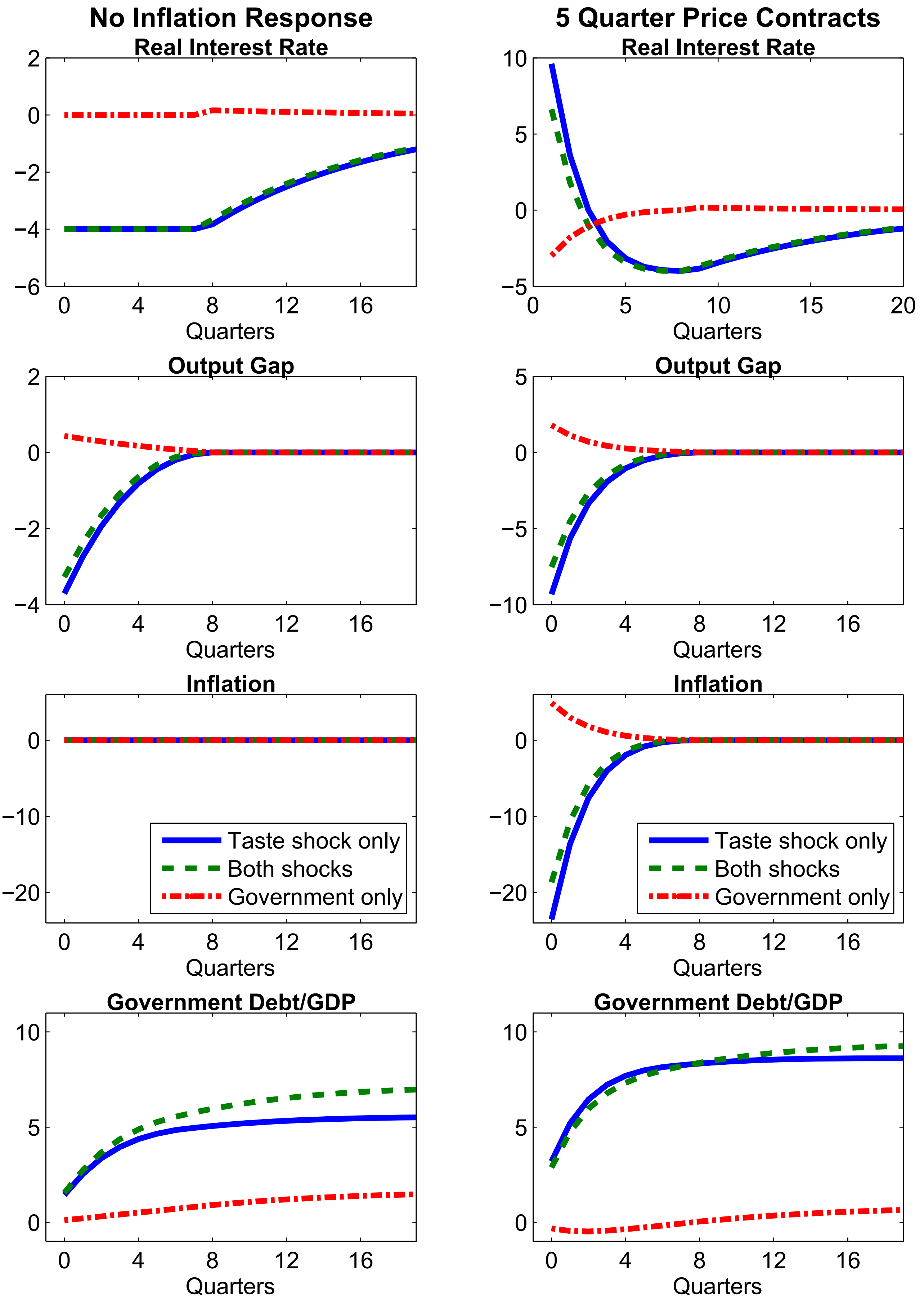


Figure 3: Spending Multipliers and Government Debt Responses in Simple New−Keynesian Model

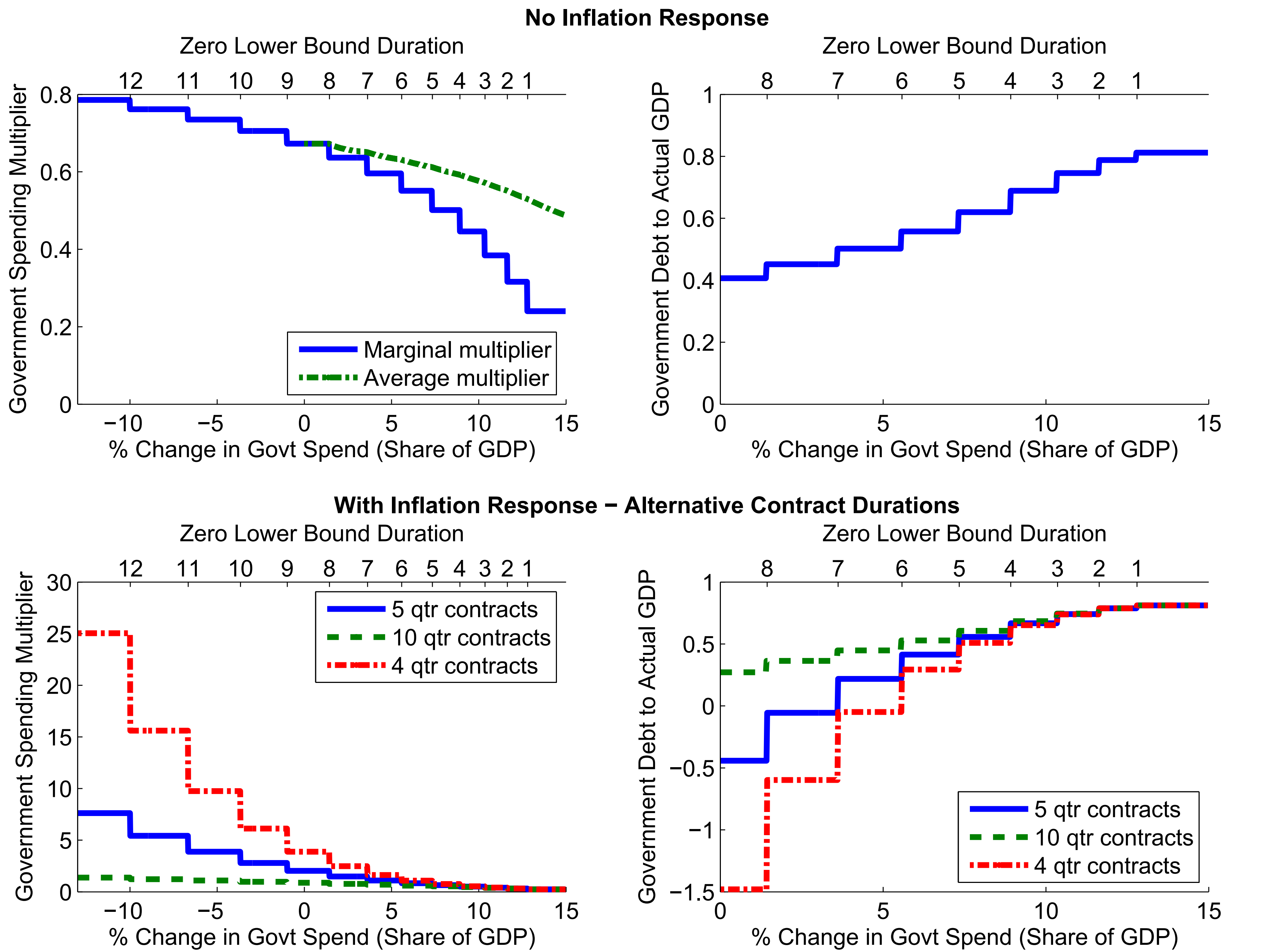


Figure 4: Immediate Government Spending Rise Under Alternative Financing Assumptions: Lump−Sum Vs. Labor−Income Taxes

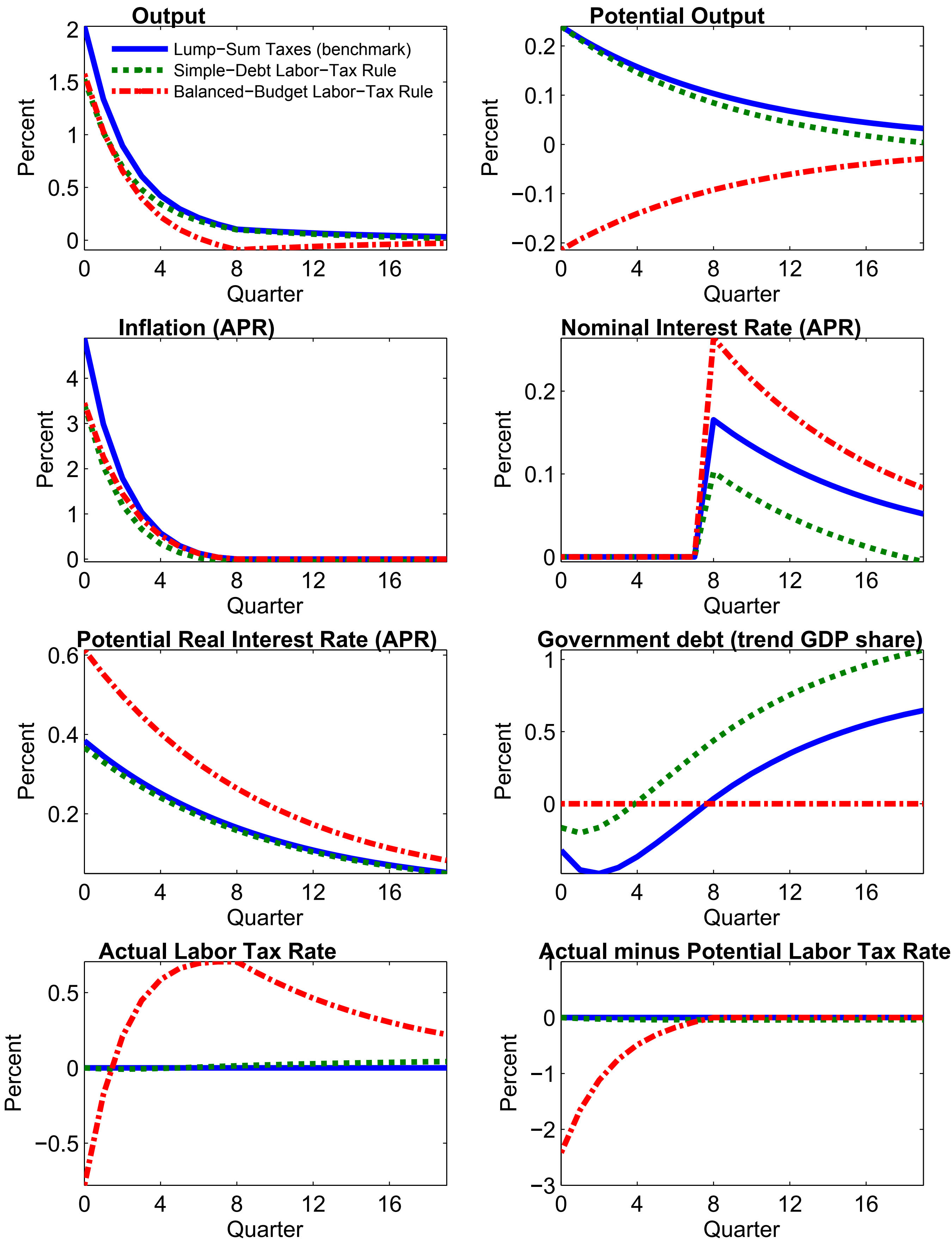


Figure 5: Spending Hike in Normal Times and a Liquidity Trap in Full Model With Keynesian Agents and Financial Frictions and in CEE−SW Model

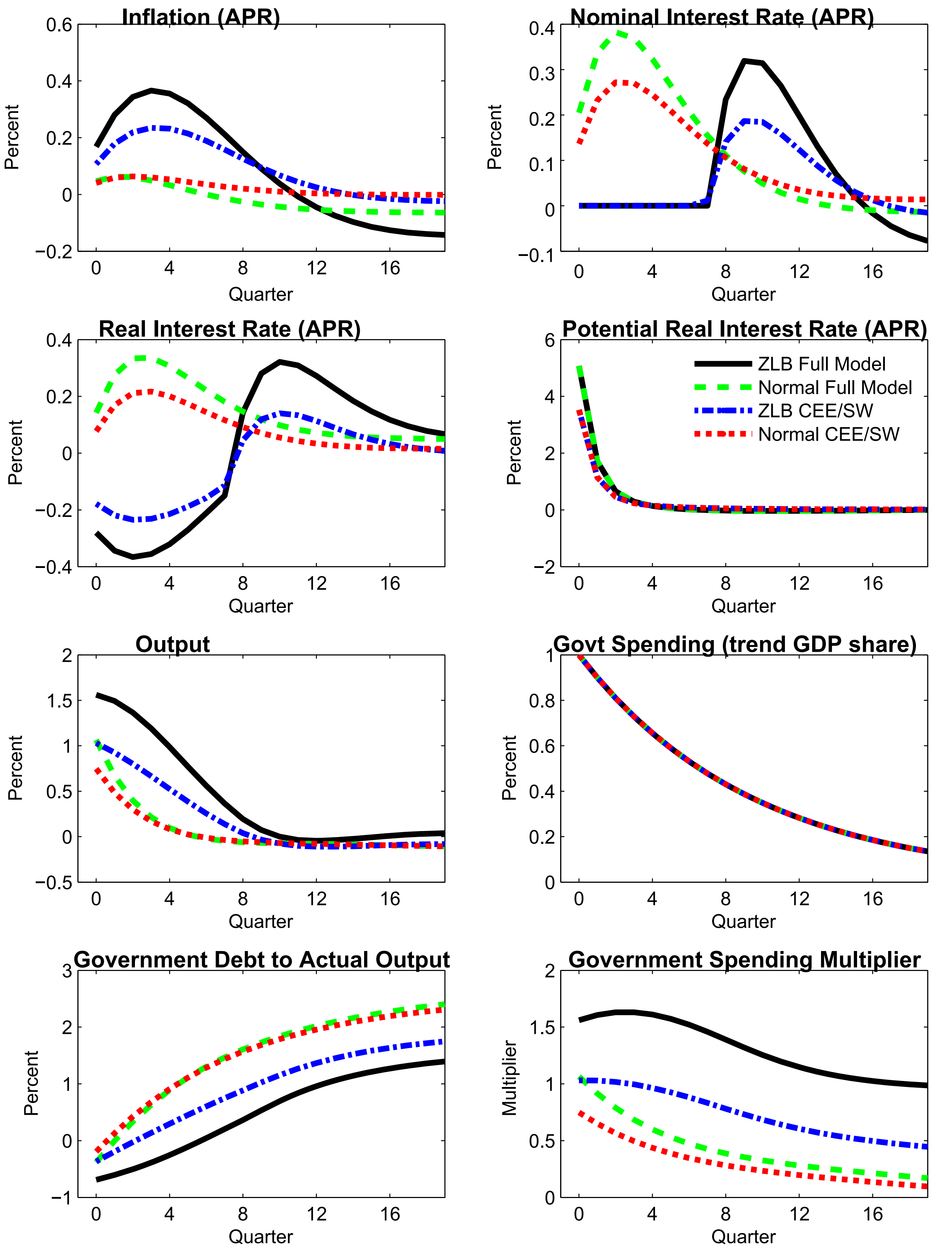
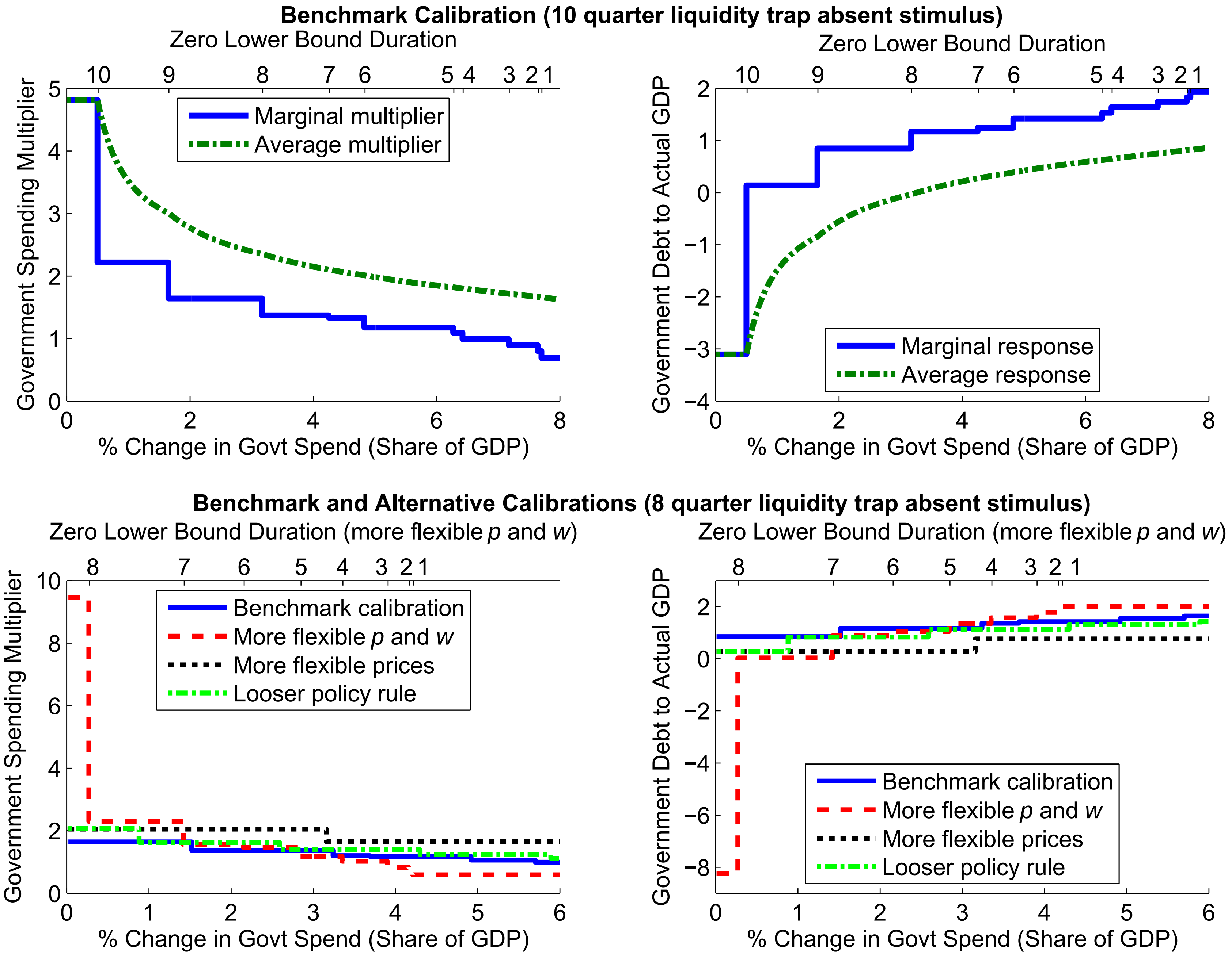


Figure 6: Marginal Output Multipliers and Government Debt Responses in Full Model with Keynesian Agents and Financial Frictions



Appendix A. The Simple New-Keynesian Model

Appendix A contains two parts. Section A.1 describes and derives the model used in Section 2, including both the benchmark model with lump-sum taxes and the variant with distortionary labor income taxes.[A.1] In Section A.2, we discuss some additional results referred to in Section 2 of the main text (including several supplementary figures).

A.1. The Model

A.1.1. Households

The utility functional for the representative household is

$$\mathbb{E}_t \sum_{j=0}^{\infty} \beta^j \left\{ \frac{1}{1-\frac{1}{\sigma}} \left(C_{t+j} - C\nu_{t+j}\right)^{1-\frac{1}{\sigma}} - \frac{N_{t+j}^{1+\chi}}{1+\chi} + \mu_0 F\left(\frac{MB_{t+j+1}(h)}{P_{t+j}}\right) \right\} \tag{A.1}$$

where the discount factor β satisfies $0 < \beta < 1$. The period utility function depends on the household's current consumption C_t as deviation from a "reference level" $C\nu_{t+j}$, where a positive taste shock ν_t raises this reference level and thus the marginal utility of consumption associated with any given consumption level. The period utility function also depends inversely on hours worked N_t. Following Eggertsson and Woodford (2003), the subutility function over real balances, $F\left(\frac{MB_{t+j+1}(h)}{P_{t+j}}\right)$, is assumed to have a satiation point for $\overline{MB}/P$. Hence, inclusion of money - which is a zero nominal interest asset - provides a rationale for the zero lower bound on nominal interest rates. However, we maintain the assumptions that money is additive and that μ_0 is arbitrarily small so that changes in real money balances have negligible implications for seignorage. Together, these assumptions imply that we can disregard the implications of money for government debt and output.

The household's budget constraint in period t states that its expenditure on goods and net purchases of (zero-coupon) government bonds $B_{G,t}$ must equal its disposable income:

$$P_t\left(1+\tau_{C,t}\right)C_t + B_{G,t} + MB_{t+1} = \left(1-\tau_{N,t}\right)W_t N_t + \left(1+i_{t-1}\right)B_{G,t-1} + MB_t - T_t + \Gamma_t \tag{A.2}$$

Thus, the household purchases the final consumption good (at a price of P_t) and subject to a sales tax $\tau_{C,t}$. Each household earns after-tax labor income $\left(1-\tau_{N,t}\right)W_t N_t$ ($\tau_{N,t}$ denotes the tax rate), pays a lump-sum tax T_t (this may be regarded as net of any transfers), and receives a proportional share of the profits Γ_t of all intermediate firms.

In every period t, the household maximizes the utility functional (B.8) with respect to its consumption, labor supply and bond holdings. Forming the Lagrangian and computing the first-

[A.1] In deriving the model, we also include a sales tax to complement the discussion in Section 2.4 on the composition of the tax base.

order conditions w.r.t. $\left[\begin{array}{ccc} C_t & N_t & B_{G,t} \end{array}\right]$, we obtain

$$\begin{aligned}
(C_t - C\nu_t)^{-\frac{1}{\sigma}} - \lambda_t P_t (1 + \tau_{C,t}) &= 0, \\
-N_t^{\chi} + \lambda_t (1 - \tau_{N,t}) W_t &= 0, \\
-\lambda_t + \beta (1 + i_t) \mathrm{E}_t \lambda_{t+1} &= 0,
\end{aligned}$$

and by defining $\Lambda_t \equiv \lambda_t P_t$ as the pre-tax cost of consumption in utility units, we can rewrite the first-order conditions as

$$\begin{aligned}
\Lambda_t &= \frac{(C_t - C\nu_t)^{-\frac{1}{\sigma}}}{(1 + \tau_{C,t})}, \\
N_t^{\chi} &= \Lambda_t (1 - \tau_{N,t}) \frac{W_t}{P_t}, \\
\Lambda_t &= \beta \mathrm{E}_t \frac{(1 + i_t)}{1 + \pi_{t+1}} \Lambda_{t+1},
\end{aligned}$$

where we have introduced the notation $1 + \pi_{t+1} = P_{t+1}/P_t$.

By substituting out for Λ_t, we derive the consumption Euler equation

$$\frac{(C_t - C\nu_t)^{-\frac{1}{\sigma}}}{(1 + \tau_{C,t})} = \beta \mathrm{E}_t \frac{(1 + i_t)}{1 + \pi_{t+1}} \frac{(C_{t+1} - C\nu_{t+1})^{-\frac{1}{\sigma}}}{(1 + \tau_{C,t+1})}, \tag{A.3}$$

and the following labor supply schedule

$$mrs_t \equiv \frac{N_t^{\chi}}{(C_t - C\nu_t)^{-\frac{1}{\sigma}}} = \frac{(1 - \tau_{N,t})}{(1 + \tau_{C,t})} \frac{W_t}{P_t}. \tag{A.4}$$

(A.3) and (A.4) are the key equations for the household side of the model.

A.1.2. Firms

We assume a familiar setting with a continuum of monopolistically competitive firms to rationalize Calvo-style price stickiness. The framework in the stylized model is identical to that described below in the full model with capital (Appendix B.1.1), with two important exceptions. First, aggregate capital is assumed to be fixed, so that aggregate production is given by

$$Y_t = K^{\alpha} N_t^{1-\alpha}. \tag{A.5}$$

Despite the fixed aggregate stock, shares of the aggregate capital stock can be freely allocated across the f firms, implying that real marginal cost, $MC_t(f)/P_t$ is identical across firms and equal to

$$\frac{MC_t}{P_t} = \frac{W_t/P_t}{MPL_t} = \frac{W_t/P_t}{(1 - \alpha) K^{\alpha} N_t{}^{-\alpha}}. \tag{A.6}$$

The second notable difference relative to the setup in the full model with capital is that here we do not allow for dynamic indexation to lagged inflation. Instead, all firms which are not allowed to reoptimize their prices in period t (which is the case with probability ξ_p), update their prices according to the following formula

$$\tilde{P}_t = (1+\pi) P_{t-1}, \tag{A.7}$$

where π is the steady-state (net) inflation rate and $\tilde{P}_t$ is the updated price.

Given Calvo-style pricing frictions, firm f that is allowed to reoptimize its price ($P_t^{opt}(f)$) solves the following problem

$$\max_{P_t^{opt}(f)} \mathrm{E}_t \sum_{j=0}^{\infty} \xi_p^j \psi_{t,t+j} \left[(1+\pi)^j P_t^{opt}(f) - MC_{t+j} \right] Y_{t+j}(f)$$

where $\psi_{t,t+j}$ is the stochastic discount factor (the conditional value of future profits in utility units, i.e. $\beta^j \mathrm{E}_t \frac{\lambda_{t+j}}{\lambda_t}$, recalling that the household is the owner of the firms), θ_p the net markup and the demand for firm f is given by $Y_{t+j}(f) = \left[\frac{P_t^*(f)}{P_t}\right]^{\frac{-(1+\theta_p)}{\theta_p}} Y_t$. The first-order condition is given by

$$\mathrm{E}_t \sum_{j=0}^{\infty} \xi_p^j \psi_{t,t+j} \left[(1+\pi)^j \frac{-1}{\theta_p} - \frac{-(1+\theta_p)}{\theta_p} \frac{1}{P_t^{opt}(f)} MC_{t+j} \right] Y_{t+j}(f) = 0,$$

which after multiplying through by -$\frac{\theta_p}{1+\theta_p} P_t^{opt}(f)$ can be rewritten as

$$\mathrm{E}_t \sum_{j=0}^{\infty} \xi_p^j \psi_{t,t+j} \left[\frac{(1+\pi)^j P_t^{opt}(f)}{1+\theta_p} - MC_{t+j} \right] Y_{t+j}(f) = 0 \tag{A.8}$$

By implication of equations (B.2) and (A.7), the evolution of the final goods price is given by

$$P_t = \left[(1-\xi_p) \left(P_t^{opt} \right)^{\frac{-1}{\theta_p}} + \xi_p \left((1+\pi) P_{t-1} \right)^{\frac{-1}{\theta_p}} \right]^{-\theta_p} \tag{A.9}$$

where we have used the fact that all firms that reoptimize will set the same price (because they face the same costs for labor and capital), and that the updating price for the non-optimizing firms equals the past aggregate price (as we consider a continuum of firms which does not re-optimize).

A.1.3. Government

The evolution of nominal government debt is determined by the following equation

$$B_{G,t} = (1+i_{t-1}) B_{G,t-1} + P_t G_t - \tau_{C,t} P_t C_t - \tau_{N,t} W_t N_t - T_t - MB_{t+1} + MB_t \tag{A.10}$$

where G_t denotes real government expenditures on the final good Y_t. Scaling with $1/(P_t Y)$, we obtain

$$\frac{B_{G,t}}{P_t Y} = \frac{(1+i_{t-1})}{(1+\pi_t)} \frac{B_{G,t-1}}{P_{t-1} Y} + \frac{G_t}{Y} - \tau_{C,t} \frac{C_t}{Y} - \tau_{N,t} \frac{W_t N_t}{P_t Y} - \frac{T_t}{P_t Y} - \frac{MB_{t+1}}{P_t Y} + \frac{MB_t}{P_t Y}. \tag{A.11}$$

In the benchmark model where lump-sum taxes stabilize the evolution of government debt (as share of nominal trend GDP, $b_{G,t} \equiv \frac{B_{G,t}}{P_t Y}$), we assume that lump-sum taxes as share of nominal trend GDP, $\tau_t \equiv \frac{T_t}{P_t Y}$, follows the rule (11). In the variant of the model with distortionary labor-income taxes, the lump-sum tax rule is replaced by the rule (23).

Turning to the central bank, it is assumed to adhere to the non-linear Taylor-type policy rule (in log-linearized form) in equation (3), where i denotes the steady-state (net) nominal interest rate, which is given by $r + \pi$ where $r \equiv 1/\beta - 1$.

A.1.4. The Aggregate Resource Constraint

We now turn to discuss the derivation of the aggregate resource constraint. Let Y_t^* denote the unweighted average (sum) of output for each firm f, i.e.

$$Y_t^* = \int_0^1 Y_t(f)df$$

Recalling that $Y_{t+j}(f) = \left[\frac{P_t^*(f)}{P_t}\right]^{\frac{-(1+\theta_p)}{\theta_p}} Y_t$, it follows that

$$\begin{aligned} Y_t^* &= \int_0^1 Y_t(f)df = \int_0^1 \left[\frac{P_t(f)}{P_t}\right]^{\frac{-(1+\theta_p)}{\theta_p}} Y_t df \\ &= \left(\frac{1}{P_t}\right)^{\frac{-(1+\theta_p)}{\theta_p}} \left[\left(\int_0^1 P_t(f)^{\frac{-(1+\theta_p)}{\theta_p}} df\right)^{\frac{-\theta_p}{(1+\theta_p)}}\right]^{\frac{-(1+\theta_p)}{\theta_p}} Y_t \\ &= \left(\frac{P_t^*}{P_t}\right)^{\frac{-(1+\theta_p)}{\theta_p}} Y_t, \end{aligned}$$

where Y_t is aggregate output of the final good sector, as defined above, and P_t^* is the indicated weighted average of the individual prices, defined as

$$P_t^* \equiv \left(\int_0^1 P_t(f)^{\frac{-(1+\theta_p)}{\theta_p}} df\right)^{\frac{-\theta_p}{(1+\theta_p)}}. \tag{A.12}$$

Notice how the weights for P_t^* differ from what they are for the aggregate price level P_t (see eq. B.2). Now, actual output is Y_t, and this is what is available to be divided into private consumption and government spending:

$$Y_t = C_t + G_t. \tag{A.13}$$

Using the definition of the production function (A.5), we can write the resource constraint in real terms as follows:

$$\underbrace{C_t + G_t}_{\equiv Y_t} \leq \left(\frac{P_t^*}{P_t}\right)^{\frac{(1+\theta_p)}{\theta_p}} \underbrace{K^\alpha N_t{}^{1-\alpha}}_{\equiv Y_t^*}. \tag{A.14}$$

The sticky price distortion clearly introduces a wedge between input use and the output available for consumption (including by the government). Even so, this term vanishes in the log-linearized version of the model.

A.1.5. Equilibrium

We now collect the equilibrium relationships in the model and derive a log-linear approximation of the model.

Collecting the equations First, we may regard the households equations (A.3) and (A.4) as determining C_t and N_t, and marginal cost relation equation (A.6) as determining MC_t/P_t, and the aggregate resource constraint (A.14) as determining the real wage W_t/P_t. The Taylor-type policy rule determines the nominal interest rate i_t, and the firms pricing equations (A.8) and (A.9) determines the evolution of the aggregate price level P_t, whereas the (shadow) gross real interest rate $1 + r_t$ is determined by the Fisher relationship

$$1 + r_t = \mathrm{E}_t \frac{(1 + i_t)}{(1 + \pi_{t+1})} \tag{A.15}$$

Finally, the fiscal budget constraint (A.11) determines the evolution of government debt $B_{G,t}$, and the final goods resource constraint (A.13) relate consumption and government spending to final output Y_t. The other fiscal variables, $G_t, \tau_{C,t}, \tau_{N,t}$ and τ_t, are exogenous or determined by policy rules.

Log-linear Approximation of Model We will now derive the equations in Section 2 in turn. We start with the sticky price equilibrium conditions, and then discuss the flex-price equilibrium. In general, a log-linearized variable is denoted with lower case letters, and derived as

$$x_t = \frac{dX_t}{X}, \tag{A.16}$$

except in the special case $X = 0$ when the log-linearized variable is simply given by dX_t (e..g government debt as share of nominal trend GDP, and the lump-sum tax rate). Moreover, for inflation and interest rates, we use the approximation that $d(1 + x_t) \approx x_t$ because x_t is small. Finally, notice that for distortionary tax rates, we use $d\tau_{X,t} \equiv \tau_{X,t}$ (thus, rather than introducing new notation, the tax rates are henceforth understood to be in deviations from their steady state level; this is also the case for the preference shock ν_t).

Totally differentating the government debt evolution equation (A.11) ,we obtain (dropping the seignorage term which is assumed to be arbitrarily small)

$$b_{G,t} = (1 + r)\, b_{G,t-1} + g_y g_t - c_y (\tau_{C,t} + \tau_C c_t) - \frac{1-\alpha}{1+\theta_p} (\tau_{N,t} + \tau_N \zeta_t + \tau_N n_t) - \tau_t + b_G (1+r)(i_{t-1} - \pi_t), \tag{A.17}$$

where we have introduced the notation that ζ_t represents the real wage (as percent deviation from steady state, i.e. $d(W_t/P_t)/(W/P)$), defined $g_y \equiv G/Y$, and used that $\frac{WN}{PY} = \frac{1-\alpha}{1+\theta_p} \equiv s_N$ and our simplifying assumption that $b_G = 0$. Assuming that the labor income tax is the only tax which balances the budget in steady state, it then follows that:

$$g_y = \frac{1-\alpha}{1+\theta_p} \tau_N, \tag{A.18}$$

implying that the log-linearized budget constraint in the benchmark model with lump-sum taxes can be written as (10) in Section 2. However, in the model in Section 2.5 where dynamic adjustments in $\tau_{N,t}$ stabilizes government debt, (A.17) is the budget constraint when setting $\tau_{C,t} = \tau_t = 0$ for all t and $\tau_C = 0$.

To derive a log-linearized representation for real marginal cost, we work from the equation (A.6), which implies

$$mc_t = \zeta_t - y_t + n_t = \zeta_t + \frac{\alpha}{1-\alpha} y_t,$$

where the second equality follows from (A.5). By noting that real marginal cost is constant in the flex-price equilibrium, we have

$$\zeta_t^{pot} - y_t^{pot} + n_t^{pot} = \zeta_t^{pot} + \frac{\alpha}{1-\alpha} y_t^{pot} = 0. \tag{A.19}$$

Accordingly, we can write (log-linearized) real marginal cost as

$$mc_t = \left(\zeta_t - \zeta_t^{pot}\right) + \frac{\alpha}{1-\alpha}\left(y_t - y_t^{pot}\right). \tag{A.20}$$

In order to write this equation solely in terms of the output gap,

$$x_t \equiv y_t - y_t^{pot}, \tag{A.21}$$

we need to derive a log-linearized equation for the real wage. To obtain such a measure, we log-linearize equation (A.4) to obtain

$$\chi n_t + \frac{1}{\sigma(1-\nu)}\left(c_t - \nu\nu_t\right) = \zeta_t - \frac{\tau_{N,t}}{1-\tau_N} - \frac{\tau_{C,t}}{1+\tau_C},$$

again recalling that $\tau_{j,t}$ for $j = [N, C]$ and ν_t are to be interpreted as percentage point deviations. By log-linearizing and substituting the aggregate resource constraint in (A.13) into this expression, we obtain

$$\zeta_t = \chi n_t + \frac{1}{\sigma(1-\nu)}\left(\frac{1}{1-g_y}\left(y_t - g_y g_t\right) - \nu\nu_t\right) + \frac{\tau_{N,t}}{1-\tau_N} + \frac{\tau_{C,t}}{1+\tau_C},$$

and using (A.5), i.e. that $n_t = \frac{1}{1-\alpha} y_t$, we finally derive the following expression for the log-linearized real wage:

$$\zeta_t = \left(\frac{\chi}{1-\alpha} + \frac{1}{\sigma(1-\nu)(1-g_y)}\right) y_t - \frac{g_y}{\sigma(1-\nu)(1-g_y)} g_t - \frac{\nu}{\sigma(1-\nu)}\nu_t + \frac{1}{1-\tau_N}\tau_{N,t} + \frac{1}{1+\tau_C}\tau_{C,t}. \tag{A.22}$$

Next, we log-linearize the consumption Euler equation, (A.3), to get

$$-\frac{c_t - \nu v_t}{\sigma(1-\nu)} = \mathrm{E}_t\left[i_t - \pi_{t+1} - \frac{1}{1+\tau_c}\Delta\tau_{C,t+1} - \frac{c_{t+1} - \nu\nu_{t+1}}{\sigma(1-\nu)}\right],$$

where we have used that

$$1 = \beta\frac{1+i}{1+\pi} = \beta(1+r).$$

By substituting the log-linearized aggregate resource constraint (A.13) into this expression, and defining:

$$\hat{\sigma} \equiv \sigma (1-\nu)(1-g_y). \tag{A.23}$$

we obtain after some re-arranging:

$$y_t = \mathrm{E}_t y_{t+1} - \hat{\sigma}(i_t - \mathrm{E}_t \pi_{t+1}) - g_y \mathrm{E}_t \Delta g_{t+1} - (1-g_y)\nu \mathrm{E}_t \Delta \nu_{t+1} + \frac{\hat{\sigma}}{1+\tau_c} \mathrm{E}_t \Delta \tau_{C,t+1}, \tag{A.24}$$

which is the log-linearized IS curve equation. Using the labor supply equation (A.22) and labor demand equation (A.19) under flexible prices, we get

$$\left(\frac{\chi}{1-\alpha} + \frac{1}{\hat{\sigma}} + \frac{\alpha}{1-\alpha}\right) y_t^{pot} = \left[\frac{g_y}{\hat{\sigma}} g_t + \frac{\nu}{\sigma(1-\nu)}\nu_t - \frac{1}{1-\tau_N}\tau_{N,t}^{pot} - \frac{1}{1+\tau_C}\tau_{C,t}\right],$$

where we use the notation z_t^{pot} for endogenous variables, and simply z_t for exogenous variables. Notice that $\tau_{N,t}^{pot}$ for the moment is treated as an endogenous variable as it potentially depends on other endogenous variables via (23). Using the notation

$$\phi_{mc} \equiv \frac{\chi}{1-\alpha} + \frac{1}{\hat{\sigma}} + \frac{\alpha}{1-\alpha}, \tag{A.25}$$

the solution for potential output can be written

$$y_t^{pot} = \frac{1}{\phi_{mc}\hat{\sigma}}\left[g_y g_t + (1-g_y)\nu\nu_t - \frac{\hat{\sigma}}{1-\tau_N}\tau_{N,t}^{pot} - \frac{\hat{\sigma}}{1+\tau_C}\tau_{C,t}\right]. \tag{A.26}$$

To get a tractable solution for the potential real interest rate, we use the definition in (A.23) to rearrange (A.24) as:

$$r_t^{pot} = \frac{1}{\hat{\sigma}}\mathrm{E}_t\Delta y_{t+1}^{pot} - \frac{g_y}{\hat{\sigma}}\mathrm{E}_t\Delta g_{t+1} - \frac{1-g_y}{\hat{\sigma}}\nu\mathrm{E}_t\Delta\nu_{t+1} + \frac{1}{1+\tau_c}\mathrm{E}_t\Delta\tau_{C,t+1},$$

and by substituting the expression for y_t^{pot} in (A.26) into this equation, we obtain

$$r_t^{pot} = \frac{1}{\hat{\sigma}\phi_{mc}}\mathrm{E}_t\left[\begin{array}{c} \frac{g_y}{\hat{\sigma}}\Delta g_{t+1} + \frac{1-g_y}{\hat{\sigma}}\nu\Delta\nu_{t+1} - \\ \frac{1}{1-\tau_N}\Delta\tau_{N,t+1}^{pot} - \frac{1}{1+\tau_C}\Delta\tau_{C,t+1} \end{array}\right] - \frac{g_y}{\hat{\sigma}}\mathrm{E}_t\Delta g_{t+1} - \frac{1-g_y}{\hat{\sigma}}\nu\mathrm{E}_t\Delta\nu_{t+1} + \frac{1}{1+\tau_c}\mathrm{E}_t\Delta\tau_{C,t+1},$$

which can be rearranged as

$$r_t^{pot} = \tfrac{1}{\hat{\sigma}}\left(1 - \tfrac{1}{\hat{\sigma}\phi_{mc}}\right)\mathrm{E}_t\left[-g_y\Delta g_{t+1} - (1-g_y)\nu\Delta\nu_{t+1}\right] - \tfrac{1}{\hat{\sigma}\phi_{mc}(1-\tau_N)}\mathrm{E}_t\Delta\tau_{N,t+1}^{pot} + \left(1 - \tfrac{1}{\hat{\sigma}\phi_{mc}}\right)\tfrac{1}{1+\tau_c}\mathrm{E}_t\Delta\tau_{C,t+1}, \tag{A.27}$$

which is the general solution for the potential real interest rate.

The Benchmark Model With Lump-sum Taxes From the equations above, it is an easy task to derive the benchmark model with lump-sum taxes. In this version of the model,

$\tau_{N,t} = \tau_{C,t} = 0$ for all t. Accordingly, equation (5) follows from (A.27), and (4) follows from (A.26). The IS-curve (1) obtains from (A.24) which holds for actual and potential output, so that:

$$y_t - y_t^{pot} = \begin{array}{c} \left(\mathrm{E}_t y_{t+1} - \hat{\sigma}\left(i_t - \mathrm{E}_t \pi_{t+1}\right) - g_y \mathrm{E}_t \Delta g_{t+1} - \left(1 - g_y\right) \nu \mathrm{E}_t \Delta \nu_{t+1}\right) \\ - \left(\mathrm{E}_t y_{t+1}^{pot} - \hat{\sigma} r_t^{pot} - g_y \mathrm{E}_t \Delta g_{t+1} - \left(1 - g_y\right) \nu \mathrm{E}_t \Delta \nu_{t+1}\right), \end{array}$$

which can be written as equation (1) by using the definitions (A.21) and (A.23).

As is well-known, log-linearization of (A.8) and (A.9) around the inflation target π results in the following Phillips curve

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} = \frac{\left(1 - \xi_p\right)\left(1 - \beta \xi_p\right)}{\xi_p} mc_t. \tag{A.28}$$

To write the model in terms of the output gap x_t instead of mc_t as in the text, we use (A.20) and (A.22), which in the model with time-varying lump-sum taxes simplifies to

$$\begin{aligned} mc_t &= \left(\zeta_t - \zeta_t^{pot}\right) + \frac{\alpha}{1-\alpha}\left(y_t - y_t^{pot}\right) \\ &= \left(\frac{\chi}{1-\alpha} + \frac{1}{\sigma\left(1-\nu\right)\left(1-g_y\right)}\right)\left(y_t - y_t^{pot}\right) + \frac{\alpha}{1-\alpha}\left(y_t - y_t^{pot}\right) \\ &= \phi_{mc} x_t, \end{aligned}$$

where x_t is defined accordingly with (A.21) and ϕ_{mc} is defined as in (A.25). Using this in (A.28), we obtain (2) with κ_p defined as in (7).

As mentioned previously, (10) obtains from (A.17) by using $\tau_C = 0$ and (A.18). Apart from the equations stated in the main text, we use (A.26) to compute y_t^{pot}, which enables us to compute actual output as $y_t = x_t + y_t^{pot}$. To get hours worked and real wage in (10), we use (A.22) and $n_t = \frac{1}{1-\alpha} y_t$.

The Model With Exogenous Distortionary Labor-Income Taxes In this variant of the model we have that $\tau_{N,t}$ varies *exogenously*, but we still assume that $\tau_{C,t} = \tau_C = 0$ for all t. Since labor income taxes are exogenous, all the model equations are identical, except that the labor income tax affects r_t^{pot} according to (A.27) and y_t^{pot} according to (A.26). In this version of the model, the IS-curve (1) is identical to the benchmark model. Finally, $\tau_{N,t}$ enters the government budget constraint (A.17).

The Model With Endogenous Distortionary Labor-Income Taxes In this variant of the model $\tau_{N,t}$ varies endogenously according to the rule given by equation (23 in Section 2.5, but we still assume $\tau_{C,t} = \tau_C = 0$ for all t. Also in this version of the model, the IS-curve (1) is identical to the benchmark model. However, the expression for marginal costs changes, because it follows from (A.22) that the following wedge between the actual and potential labor tax-rate will enter into marginal costs:

$$
\begin{aligned}
mc_t &= \left(\zeta_t - \zeta_t^{pot}\right) + \frac{\alpha}{1-\alpha}\left(y_t - y_t^{pot}\right) \\
&= \left(\frac{\chi}{1-\alpha} + \frac{1}{\sigma\left(1-\nu\right)\left(1-g_y\right)}\right)\left(y_t - y_t^{pot}\right) + \frac{1}{1-\tau_N}\left(\tau_{N,t} - \tau_{N,t}^{pot}\right) + \frac{\alpha}{1-\alpha}\left(y_t - y_t^{pot}\right) \\
&= \phi_{mc} x_t + \frac{1}{1-\tau_N}\left(\tau_{N,t} - \tau_{N,t}^{pot}\right),
\end{aligned}
$$

implying that a negative gap between the actual and potential labor income tax rate will put downward pressure on marginal costs and hence inflation. In all other aspects, this variant is identical to the model with exogenous $\tau_{N,t}$, with the exception that the exogenous process for $\tau_{N,t}$ is replaced with the rule (23) and that $\tau_t = 0$ for all t.

A.2. Additional Results in Simple Benchmark Model

Below, we report and discuss briefly some additional results referred to in Section 2 of the main text.

A.2.1. Complete Stabilization when Policy Unconstrained (Section 2.2)

As indicated in Section 2.2, Woodford (2003) shows that the monetary policy rule in equation (3), if unconstrained, can fully stabilize inflation at target and output at potential if the coefficients on inflation and/or the output gap are sufficiently large (i.e., inflation and the output gap become arbitrarily close to zero). Here we reproduce this result in our augmented model that includes exogenous shocks to the labor and sales tax rates to illustrate that other shocks that generate the same path of r_t^{pot} as the taste shock would have identical implications for the duration of the liquidity trap, the path of policy rates, and the multiplier.

To show the full stabilization result, substitute the unconstrained monetary policy rule into the IS curve equation (1)to get:

$$x_t = \mathrm{E}_t x_{t+1} - \hat{\sigma}(\gamma_\pi \pi_t + \gamma_x x_t - \mathrm{E}_t \pi_{t+1} - r_t^{pot}), \tag{A.29}$$

which can be rearranged as:

$$(1 + \hat{\sigma}\gamma_x)x_t = \mathrm{E}_t x_{t+1} - \hat{\sigma}\gamma_\pi \pi_t + \hat{\sigma}\mathrm{E}_t \pi_{t+1} + \hat{\sigma} r_t^{pot}. \tag{A.30}$$

Taking the lead of this equation, multiplying by β, and subtracting the result from equation (A.30) yields:

$$(1+\hat{\sigma}\gamma_x)[x_t - \beta \mathrm{E}_t x_{t+1}] = [x_{t+1} - \beta \mathrm{E}_t x_{t+2}] - \hat{\sigma}\gamma_\pi[\pi_t - \beta \mathrm{E}_t \pi_{t+1}] + \hat{\sigma}[\pi_{t+1} - \beta \mathrm{E}_t \pi_{t+2}] + \hat{\sigma}[r_t^{pot} - \beta \mathrm{E}_t r_{t+1}^{pot}]. \tag{A.31}$$

Grouping terms in x_t and its leads on the left hand side of the equality, using the Phillips curve (2), and moving the exogenous shocks to the right, yields:

$$\mathrm{E}_t x_{t+2} - \frac{1+\beta(1+\hat{\sigma}\gamma_x)+\hat{\sigma}\kappa_p}{\beta}\mathrm{E}_t x_{t+1} + \frac{1+\hat{\sigma}\gamma_x+\hat{\sigma}\kappa_p\gamma_\pi}{\beta}x_t = \frac{\hat{\sigma}}{\beta}r_t^{pot} - \hat{\sigma}\mathrm{E}_t r_{t+1}^{pot}, \tag{A.32}$$

This equation can be written in the alternative form in the lead operator:

$$\mathrm{E}_t\Psi(L^{-1})x_t = \lambda_{x,t} = \frac{\hat{\sigma}}{\beta}r_t^{pot} - \hat{\sigma}\mathrm{E}_t r_{t+1}^{pot}, \tag{A.33}$$

where

$$\Psi(L^{-1}) = L^{-2} - \frac{1+\beta(1+\hat{\sigma}\gamma_x)+\hat{\sigma}\kappa_p}{\beta}L^{-1} + \frac{1+\hat{\sigma}\gamma_x+\hat{\sigma}\kappa_p\gamma_\pi}{\beta} = (L^{-1}-v_1)(L^{-1}-v_2), \tag{A.34}$$

and $\lambda_{x,t}$ in equation (A.33) depends only on the exogenous shocks, and the parameters v_1 and v_2 in equation (A.34) have the following relation to the structural parameters:

$$v_1+v_2 = \frac{1+\beta(1+\hat{\sigma}\gamma_x)+\hat{\sigma}\kappa_p}{\beta}, \quad v_1v_2 = \frac{1+\hat{\sigma}\gamma_x+\hat{\sigma}\kappa_p\gamma_\pi}{\beta} \tag{A.35}$$

The exogenous shocks g_t, ν_t, $\tau_{N,t}$ and $\tau_{C,t}$ are assumed to follow first order autoregressions with persistence coefficients $(1-\rho_G)$, $(1-\rho_v)$, $(1-\rho_n)$, and $(1-\rho_c)$, respectively (as a slight generalization of the model in the main text, which imposes $\rho_G = \rho_v$). Accordingly, equation (A.27) for the potential real rate may be expressed (noting $\mathrm{E}_t\Delta g_{t+1} = -\rho_G g_t$, and similarly for the other shocks):

$$r_t^{pot} = \tfrac{1}{\hat{\sigma}}\left(1-\tfrac{1}{\hat{\sigma}\phi_{mc}}\right)\left[g_y\rho_G g_t + (1-g_y)\,\nu\rho_v\nu_t + \tfrac{1}{\hat{\sigma}\phi_{mc}(1-\tau_N)}\rho_n\tau_{N,t} + \left(1-\tfrac{1}{\hat{\sigma}\phi_{mc}}\right)\tfrac{1}{1+\tau_c}\rho_c\tau_{C,t}\right. \tag{A.36}$$

Using equation (A.36), λ_{xt} may be expressed in terms of the exogenous shocks as:

$$\lambda_{x,t} = \frac{1}{\beta}\left(1-\tfrac{1}{\hat{\sigma}\phi_{mc}}\right)\left[g_y\rho_G(1-\beta+\beta\rho_G)g_t + (1-g_y)\,\nu\rho_v(1-\beta+\beta\rho_v)\nu_t + \tfrac{\hat{\sigma}}{1+\tau_c}\rho_c\tau_{C,t}\right] + \tfrac{1}{\beta\phi_{mc}(1-\tau_N)}\rho_n\tau_{N,t} \tag{A.37}$$

or equivalently:

$$\lambda_{x,t} = \phi_{xg}g_t + \phi_{xv}\nu_t + \phi_{xc}\tau_{C,t} + \phi_{xn}\tau_{N,t}. \tag{A.38}$$

We next premultiply equation (A.33) by $\frac{1}{v_1v_2}$ to yield:

$$\frac{1}{v_1v_2}\mathrm{E}_t\Psi(L^{-1})x_t = \mathrm{E}_t(\frac{1}{v_1}L^{-1}-1)(\frac{1}{v_2}L^{-1}-1)x_t = \frac{1}{v_1v_2}\lambda_{x,t}. \tag{A.39}$$

As shown in Woodford (2003) Chapter 4 (and proved in his Appendix C.2), the roots v_1 and v_2 of the characteristic equation (A.35) lie outside the unit circle, and hence have a determinate solution, provided that $\gamma_\pi + \frac{1-\beta}{\kappa_p}\gamma_x > 1$ (the output gap is not annualized here, and hence not divided by 4 as in Woodford's equation C.18). Solving equation (A.39) forward, the output gap may be expressed in the form:

$$x_t = \frac{\phi_{xg}}{m_g} g_t + \frac{\phi_{xv}}{m_v} \nu_t + \frac{\phi_{xc}}{m_c} \tau_{C,t} + \frac{\phi_{xn}}{m_n} \tau_{N,t}, \tag{A.40}$$

where the coefficients m_g is given by:

$$m_g = v_1 v_2 - (1-\rho_G)(v_1+v_2)(1-\rho_G)^2 = \rho_G(\frac{1}{\beta} - (1-\rho_G)) + \frac{\hat{\sigma}\gamma_x}{\beta}(1-(1-\rho_G)\beta) + \frac{\hat{\sigma}\kappa_p}{\beta}(\gamma_\pi - (1-\rho_G)) > 0 \tag{A.41}$$

The coefficients m_v, m_c, and m_n are identical in form, but with different persistence parameters ρ_v, ρ_c, and ρ_n, respectively. Because m_g depends linearly on the monetary rule coefficients γ_π and γ_x, while the parameters $\phi_{xg}, \phi_{xv}, \phi_{xc}, \phi_{xn}$ do not, equation (A.40) implies that the output gap may be kept arbitrarily close to zero for a rule that reacts aggressively enough either to inflation, the output gap, or both. Moreover, given the form of the price-setting equation (2), inflation can also be fully stabilized. With inflation and the output gap at target, the IS curve given by equation (1) implies that the policy rate i_t simply tracks r_t^{pot}.

As seen from equation (A.36), shocks to the labor tax rate $\tau_{N,t}$ or sales tax rate $\tau_{C,t}$ can be scaled to have the same effect on the *path* of r_t^{pot} (provided the persistence parameters are identical). This would also be true for productivity shocks (in a slight generalization of the model). Since r_t^{pot} is the only shock in the model in a reduced form sense, an alternative configuration of structural shocks (to taste, tax rates, or productivity) that produced the same path for r_t^{pot} would have the same implications for the duration of the liquidity trap and for the fiscal multiplier.

A.2.2. Allowing for a Price Markup Shock (Section 2.3)

Complementing our discussion in Section 2.3, Figure A.1 shows the effects of an equally-sized government spending hike under a baseline that incorporates a price markup shock in addition to the taste shock. The price markup shocks induces a trade-off between inflation and output gap stabilization. Specifically, the aggregate supply curve (2) now incorporates a price markup shock $\theta_{p,t}$:

$$\pi_t = \beta \pi_{t+1|t} + \kappa_p (x_t + \theta_{p,t})$$

where $\theta_{p,t} = 0.5\theta_{p,t-1} + \varepsilon_{p,t}$. In all other respects, the model is unchanged.

We consider two alternative monetary policy rules that differ in the relative weight that they put on stabilizing the output gap compared to inflation. At one extreme, we assume that the Taylor rule is very aggressive in stabilizing the output gap, setting $\gamma_x = 1000$ and $\gamma_\pi = 1$. In this case, the path of the policy rate is essentially identical to that in the baseline with only taste shocks, reflecting that the markup shock has very small effects on inflation two or more years ahead. Accordingly, a temporary rise in government spending has the same impact as in our benchmark case which excludes markup shocks. Alternatively, we assume that the Taylor rule coefficients impose $\gamma_\pi = 1000$ and $\gamma_x = 0$, so that the rule focuses exclusively on stabilizing inflation. In this case, the upward pressure on inflation due to the markup shock shortens the duration of the liquidity trap by several quarters (even assuming no fiscal shock), reflecting that policy only cares about keeping inflation at target under this rule. With a shorter liquidity trap, fiscal stimulus causes a larger and faster adjustment of policy rates as seen in the figure. As a consequence, the multiplier is considerably smaller in this case than under strict output gap targeting.

A.2.3. History-Dependent Policy Rules (Section 2.3)

Following the discussion at the end of Section 2.3, we consider the implications of rules that allow for history dependence in the conduct of monetary policy. Specifically, we add a price level gap term so that the policy rule assumes the form:

$$i_t = \max\left(-i, \gamma_\pi \pi_t + \gamma_x x_t + \gamma_p p_t\right),$$

where the price level gap variable p_t is defined as

$$p_t = \pi_t + p_{t-1}.$$

In the benchmark formulation of our rule, we assume $\gamma_\pi = \gamma_x = 1000$ and $\gamma_p = 0$. In the alternative monetary rule that includes a response to p_t, we assume that $\gamma_p = 10$. In addition, we study the implications of a more standard Taylor rule which sets $\gamma_\pi = 1.5$ and $\gamma_x = .125$, and then add p_t by setting $\gamma_p = .125$. Under each of the rules, we adjust the size of the negative taste shock to imply an eight quarter liquidity trap as in the baseline scenario (this is the only type of shock considered).

The effects of spending hike under the four alternative rules are shown in Figure A.2. Under our benchmark policy rule, the spending multiplier is damped modestly with the inclusion of a weight on the price level gap. However, the differences are considerably larger under the standard Taylor rule. Because the standard Taylor rule is not very effective in keeping output and inflation near baseline in our model, there is a very large benefit to fiscal stimulus – the fiscal multiplier exceeds 5. But when the Taylor rule is modified to include a price level response, the adverse effects of the taste shock are diminished, and the benefits to fiscal stimulus substantially reduced. Clearly, history dependent rules that imply a credible commitment to stabilizing the price level can have important implications for the spending multiplier.

A.2.4. Marginal Multiplier Schedule: Distortionary vs. Lump-Sum Taxes (Section 2.5)

This discussion complements that in Section 2.5 of the main text by presenting (marginal) multiplier and government debt schedules under different rules for the tax reaction function. In particular, the upper panels of Figure A.3 shows how the government spending multiplier and government debt response vary with the duration of the liquidity trap both under our benchmark of lump-sum taxes (the solid blue lines) and several alternatives in which the distortionary tax rate reacts to the lagged stock of debt ("Simple Debt Labor-Tax Rule"), the inertial version of the same rule with a lag of the tax rate ("Labor Tax Rule with Smoothing"), and the balanced budget rule. As discussed in Section 2.5, it is apparent from the upper left panel that the disparity between the multiplier under lump-sum taxes and the simple debt rule (the green dashed line) is quite small for a liquidity trap duration of less than eight quarters; however, as the liquidity trap duration is more prolonged, the disparity increases markedly, reflecting mainly that inflation is much less responsive under distortionary taxes. While the multiplier exceeds 7 in a 12 quarter liquidity trap under lump-sum tax adjustment, the multiplier is only slightly above 2 under labor tax financing.

With the highly inertial form of the debt targeting rule (the dashed black lines), the multiplier and government spending debt response are nearly identical to that under lump-sum taxes for an eight quarter liquidity trap (hence, as noted in the text, this case was omitted from Figure 4). For even longer-lived traps, the multiplier grows somewhat more slowly with the liquidity trap duration under the labor tax rule with smoothing, but the multiplier still remains pretty close to its level under lump-sum taxes.

Finally, Figure A.3 also considers a balanced budget rule. As seen in the upper panel, the multiplier under the balanced budget rule lies uniformly below the lump-sum tax multiplier (and in particular, increases a bit more gradually as the liquidity trap duration lengthens). However, as seen in the bottom panel – and as discussed in the main text – the balanced budget multiplier actually grows more quickly than the lump-sum tax multiplier if the government spending share is relatively low (since the sensitivity of inflation to the output gap is nearly as large as under lump-sum taxes in this case). Accordingly, the multiplier under the balanced budget rule rises above the lump-sum tax multiplier if the liquidity trap duration exceeds six quarters.

A.2.5. Endogenous Government Spending (Section 2.6)

Complementing the discussion in Section 2.6, Panel A of Figure A.4 shows how automatic stabilizers reduce the government spending multiplier by comparing the effects of a one percent of baseline GDP increase in the exogenous component of government spending under our benchmark with $\mu = 0$ to an alternative specification in which μ is set to unity. The improvement in the debt/GDP ratio is also somewhat smaller under automatic stabilizers. As discussed in the text, the dampening of the multiplier and government debt response would be even more pronounced if automatic stabilizers were stronger, i.e., for higher values of μ.

To illustrate the interaction between automatic stabilizers and uncertainty about the path of shocks, it is insightful to consider a simple framework in which fiscal spending decisions must be made before the government and public become fully informed about the severity of the adverse shock(s) affecting the economy. In this vein, the lower panels of Figure A.4 compare the effects of a 1 percent of GDP rise in government spending under perfect foresight with the expected responses to the same spending hike when information about the taste shock becomes revealed only after the spending decision has been made; in the latter case, agents assign a 50 percent probability to a taste shock that would generate an 8 quarter liquidity trap absent fiscal stimulus (as in the benchmark), a 25 percent probability to a more severe shock that would produce a 12 quarter liquidity trap, and a 25 percent probability to a less severe shock that would generate a 4 quarter liquidity trap. Panel B shows the case for an economy without automatic stabilizers, while panel C shows a corresponding case with stabilizers (with $\mu = 1$).[A.2]

Without automatic stabilizers, the expected multiplier lies substantially above the multiplier under perfect foresight, and there is a much larger expected decline in government debt. These implications reflect the relatively high degree of convexity of the multiplier schedule in the absence of automatic stabilizers, which makes the payoff to fiscal expansion if the bad state materializes especially high. By contrast, with automatic stabilizers as in the lower panel, the difference between the expected multiplier and multiplier under perfect foresight is comparatively smaller; the gap would narrow further if the stabilizers were even more responsive (i.e., for higher μ).

[A.2] The size of the taste shocks in each of the three states is the same with and without the automatic stabilizers.

Figure A.1: Effects of Spending Hike in Simple Model For Alternative Policy Rules Under Baseline Generated by Both Taste and Markup Shocks.

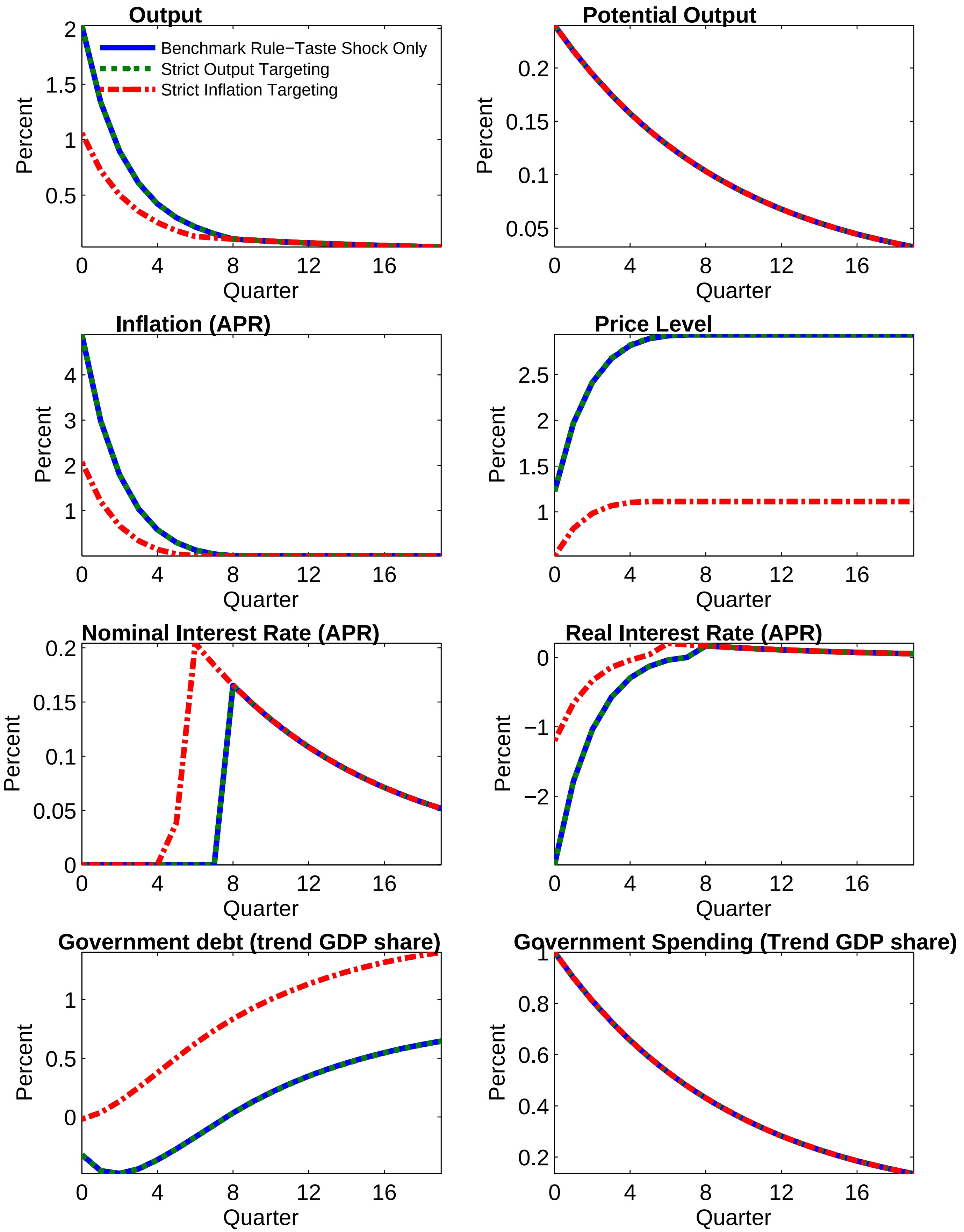


Figure A.2: Immediate Government Spending Rise in Simple Model Under Monetary Policy Rules With and Without History Dependence.

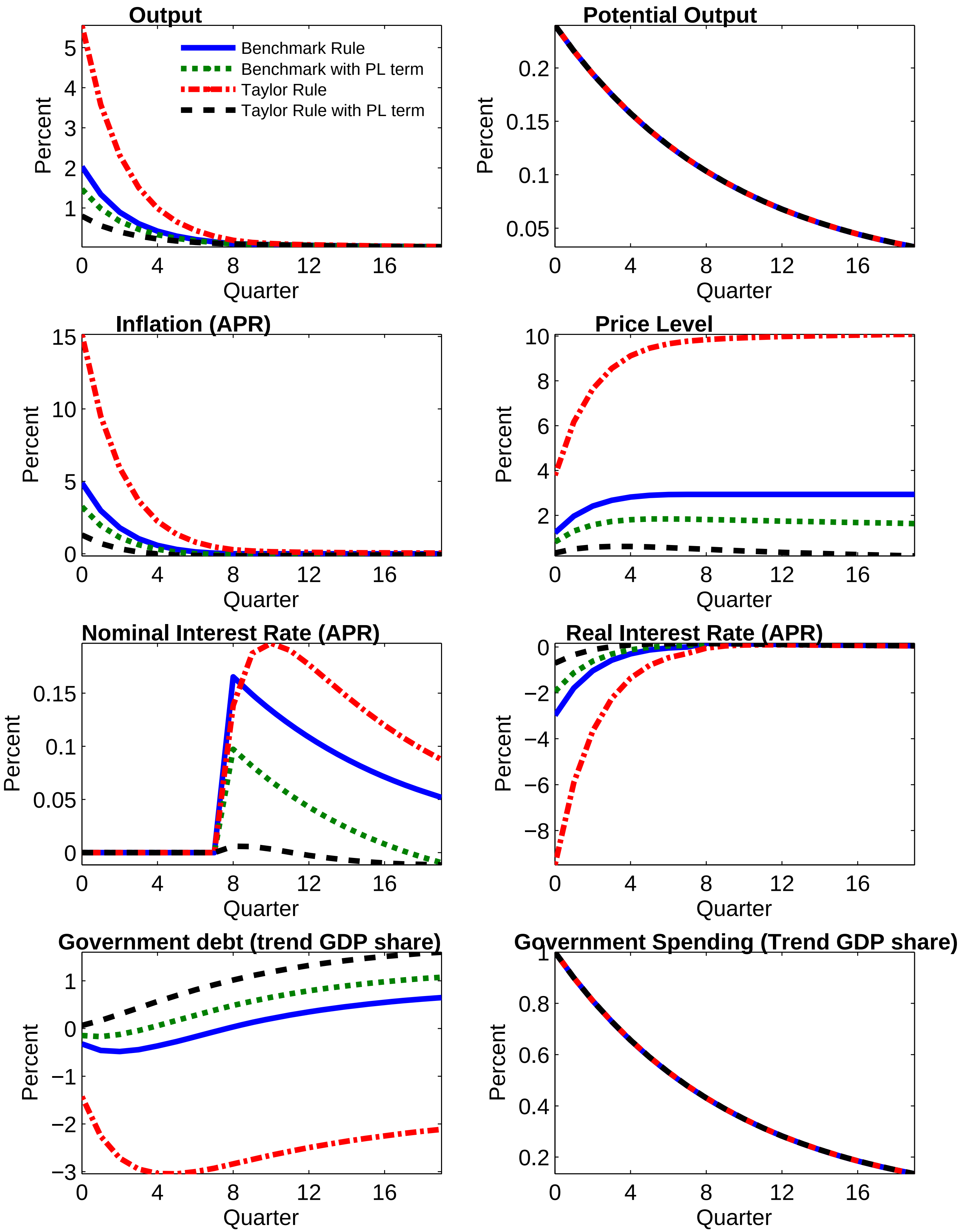


Figure A.3: Marginal Output and Government Debt Multipliers Under Alternative Financing Assumptions: Lump−Sum Vs. Distortionary Labor−Income Taxes

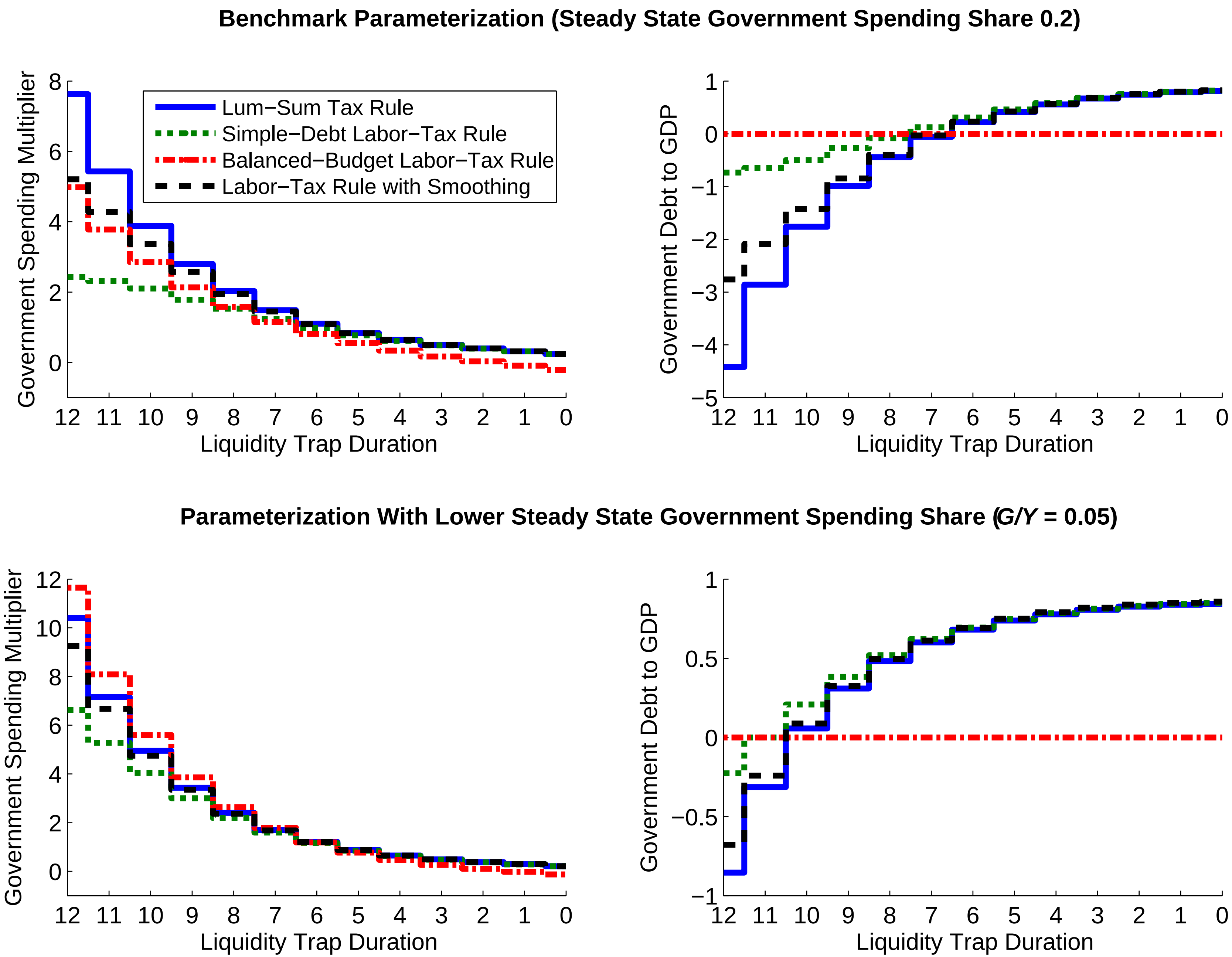
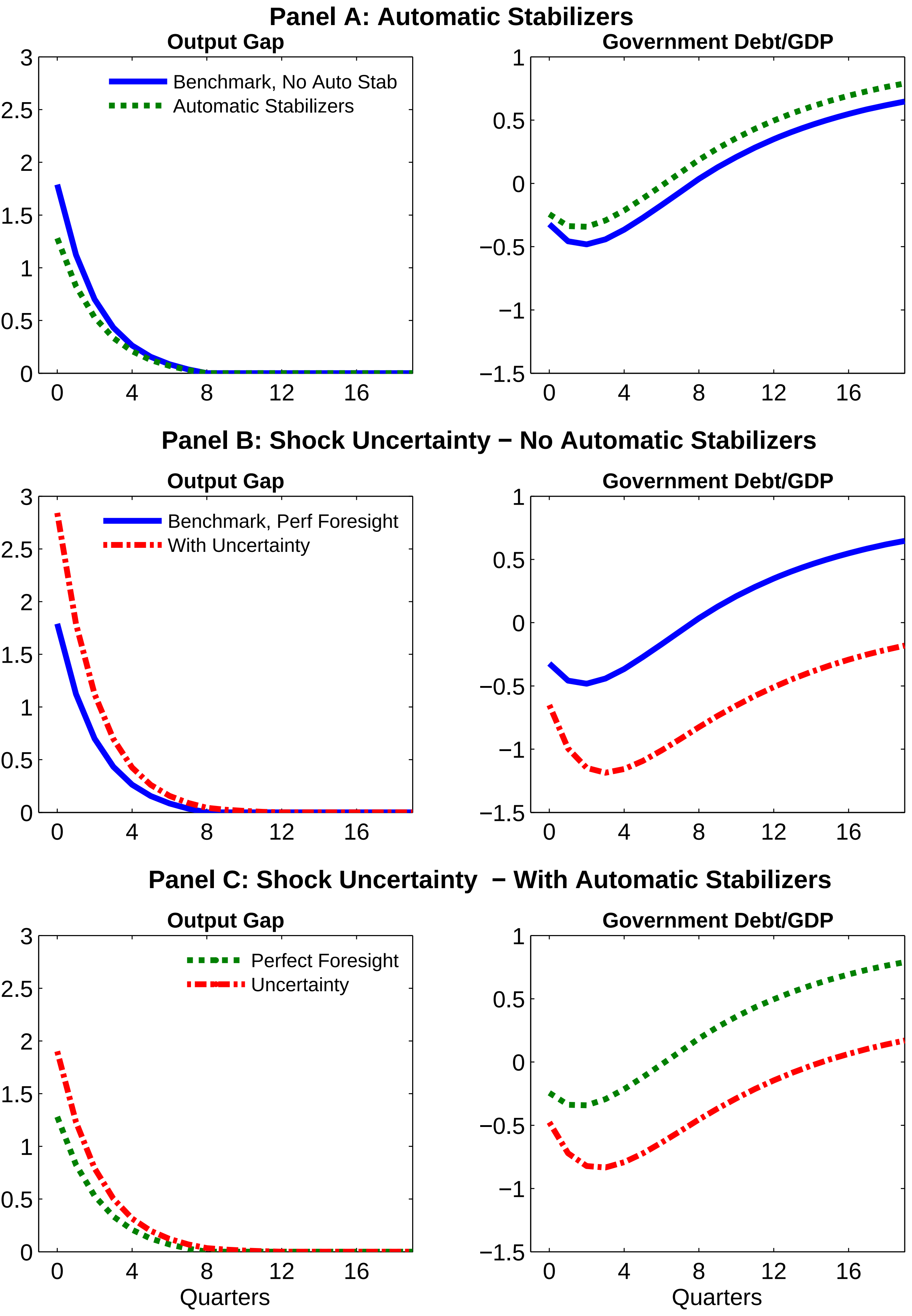


Figure A.4: Immediate Government Spending Rise: Assessing the Impact of Automatic Stabilizers and Shock Uncertainty



Appendix B. The New-Keynesian Model with Keynesian Agents and Financial Frictions

This appendix contains two parts. Section B.1 describes the model used in Section 3. Section B.2 discusses some additional results referred to in the main text, including the construction of the baseline for our simulations, and a decomposition of the sources of improvement in the debt/GDP ratio that underlie a "fiscal free lunch."

B.1. The Model

The model is essentially a variant of the CEE/SW model augmented with "Keynesian" households, as in Erceg, Guerrieri and Gust (2006), and financial frictions, following Bernanke, Gertler and Gilchrist (1999). As such, our model incorporates nominal rigidities by assuming that labor and product markets exhibit monopolistic competition, and that wages and prices are determined by staggered nominal contracts of random duration (following Calvo (1983) and Yun (1996)). In addition, the model includes an array of real rigidities, including habit persistence in consumption, and costs of changing the rate of investment. Monetary policy follows a Taylor rule, and fiscal policy specifies that taxes respond to government debt.

B.1.1. Firms and Price Setting

Final Goods Production We assume that a single final output good Y_t is produced using a continuum of differentiated intermediate goods $Y_t(f)$. The technology for transforming these intermediate goods into the final output good is constant returns to scale, and is of the Dixit-Stiglitz form:

$$Y_t = \left[\int_0^1 Y_t(f)^{\frac{1}{1+\theta_p}} df \right]^{1+\theta_p} \tag{B.1}$$

where $\theta_p > 0$.

Firms that produce the final output good are perfectly competitive in both product and factor markets. Thus, final goods producers minimize the cost of producing a given quantity of the output index Y_t, taking as given the price $P_t(f)$ of each intermediate good $Y_t(f)$. Moreover, final goods producers sell units of the final output good at a price P_t that can be interpreted as the aggregate price index:

$$P_t = \left[\int_0^1 P_t(f)^{\frac{-1}{\theta_p}} df \right]^{-\theta_p} \tag{B.2}$$

Intermediate Goods Production A continuum of intermediate goods $Y_t(f)$ for $f \in [0,1]$ is produced by monopolistically competitive firms, each of which produces a single differentiated good. Each intermediate goods producer faces a demand function for its output good that varies inversely with its output price $P_t(f)$, and directly with aggregate demand Y_t :

$$Y_t(f) = \left[\frac{P_t(f)}{P_t} \right]^{\frac{-(1+\theta_p)}{\theta_p}} Y_t \tag{B.3}$$

Each intermediate goods producer utilizes capital services $K_t(f)$ and a labor index $L_t(f)$ (defined below) to produce its respective output good. The form of the production function is Cobb-Douglas:

$$Y_t(f) = K_t(f)^{\alpha} L_t(f)^{1-\alpha} \tag{B.4}$$

Firms face perfectly competitive factor markets for hiring capital and the labor index. Thus, each firm chooses $K_t(f)$ and $L_t(f)$, taking as given both the rental price of capital R_{Kt} and the aggregate wage index W_t (defined below). Firms can costlessly adjust either factor of production. Thus, the standard static first-order conditions for cost minimization imply that all firms have identical marginal cost per unit of output.

The prices of the intermediate goods are determined by Calvo-Yun style staggered nominal contracts. In each period, each firm f faces a constant probability, $1-\xi_p$, of being able to reoptimize its price $P_t(f)$. The probability that any firm receives a signal to reset its price is assumed to be independent of the time that it last reset its price. If a firm is not allowed to optimize its price in a given period, we follow Christiano, Eichenbaum and Evans (2005) by assuming that it adjusts its price by a weighted combination of the lagged and steady state rate of inflation, i.e., $P_t(f) = \pi_{t-1}^{\iota_p} \pi^{1-\iota_p} P_{t-1}(f)$ where $0 \leq \iota_p \leq 1$. A positive value of ι_p introduces structural inertia into the inflation process.

B.1.2. Households and Wage Setting

We assume a continuum of monopolistically competitive households (indexed on the unit interval), each of which supplies a differentiated labor service to the production sector; that is, goods-producing firms regard each household's labor services $N_t(h)$, $h \in [0,1]$, as an imperfect substitute for the labor services of other households. It is convenient to assume that a representative labor aggregator combines households' labor hours in the same proportions as firms would choose. Thus, the aggregator's demand for each household's labor is equal to the sum of firms' demands. The labor index L_t has the Dixit-Stiglitz form:

$$L_t = \left[\int_0^1 N_t(h)^{\frac{1}{1+\theta_w}} \, dh \right]^{1+\theta_w} \tag{B.5}$$

where $\theta_w > 0$. The aggregator minimizes the cost of producing a given amount of the aggregate labor index, taking each household's wage rate $W_t(h)$ as given, and then sells units of the labor index to the production sector at their unit cost W_t:

$$W_t = \left[\int_0^1 W_t(h)^{\frac{-1}{\theta_w}} \, dh \right]^{-\theta_w} \tag{B.6}$$

It is natural to interpret W_t as the aggregate wage index. The aggregator's demand for the labor hours of household h – or equivalently, the total demand for this household's labor by all goods-producing firms – is given by

$$N_t(h) = \left[\frac{W_t(h)}{W_t} \right]^{-\frac{1+\theta_w}{\theta_w}} L_t \tag{B.7}$$

The utility functional of a typical member of household h is

$$\mathbb{E}_t \sum_{j=0}^{\infty} \beta^j \{ \frac{1}{1-\sigma} C_{t+j}(h) - \varkappa C_{t+j-1} - \nu_c \nu_t \}^{1-\sigma} - \frac{\chi_0}{1+\chi} N_{t+j}(h)^{1+\chi} \} \tag{B.8}$$

where the discount factor β satisfies $0 < \beta < 1$. The period utility function depends on household h's current consumption $C_t(h)$, as well as lagged aggregate per capita consumption to allow for the possibility of external habit persistence (Smets and Wouters 2003). As in the simple model considered in the previous section, a positive taste shock ν_t raises the marginal utility of consumption associated with any given consumption level. The period utility function also depends inversely on hours worked $N_t(h)$.

Household h's budget constraint in period t states that its expenditure on goods and net purchases of financial assets must equal its disposable income:

$$\begin{aligned} & P_t C_t(h) + P_t I_t(h) + \frac{1}{2} \psi_I P_t \frac{(I_t(h) - I_{t-1}(h))^2}{I_{t-1}(h)} + \\ & P_{B,t} B_{G,t+1} - B_{G,t} + \int_s \xi_{t,t+1} B_{D,t+1}(h) - B_{D,t}(h) \\ = \; & (1-\tau_{N,t}) W_t(h) N_t(h) + (1-\tau_K) R_{K,t} K_t(h) + \delta \tau_K P_t K_t(h) + \Gamma_t(h) - T_t(h) \end{aligned} \tag{B.9}$$

Thus, the household purchases the final output good (at a price of P_t), which it chooses either to consume $C_t(h)$ or invest $I_t(h)$ in physical capital. The total cost of investment to each household h is assumed to depend on how rapidly the household changes its rate of investment (as well as on the purchase price). Our specification of investment adjustment costs as depending on the square of the change in the household's gross investment rate follows Christiano, Eichenbaum, and Evans (2005). Investment in physical capital augments the household's (end-of-period) capital stock $K_{t+1}(h)$ according to a linear transition law of the form:

$$K_{t+1}(h) = (1-\delta) K_t(h) + I_t(h) \tag{B.10}$$

In addition to accumulating physical capital, households may augment their financial assets through increasing their government bond holdings ($P_{B,t} B_{G,t+1} - B_{G,t}$), and through the net acquisition of state-contingent bonds. We assume that agents can engage in frictionless trading of a complete set of contingent claims. The term $\int_s \xi_{t,t+1} B_{D,t+1}(h) - B_{D,t}(h)$ represents net purchases of state-contingent domestic bonds, with $\xi_{t,t+1}$ denoting the state price, and $B_{D,t+1}(h)$ the quantity of such claims purchased at time t. Each member of household h earns after-tax labor income $(1-\tau_{N,t}) W_t(h) N_t(h)$, after-tax capital rental income of $(1-\tau_K) R_{K,t} K_t(h)$, and a depreciation allowance of $\delta \tau_K P_t K_t(h)$. Each member also receives an aliquot share $\Gamma_t(h)$ of the profits of all firms, and pays a lump-sum tax of $T_t(h)$ (this may be regarded as taxes net of any transfers).

In every period t, each member of household h maximizes the utility functional (B.8) with respect to its consumption, investment, (end-of-period) capital stock, bond holdings, and holdings of contingent claims, subject to its labor demand function (B.7), budget constraint (B.9), and transition equation for capital (B.10). Households also set nominal wages in Calvo-style staggered contracts that are generally similar to the price contracts described above. Thus, the probability that a household receives a signal to reoptimize its wage contract in a given period is denoted by $1-\xi_w$. In addition, we specify a dynamic indexation scheme for the adjustment of the wages of those households that do not get a signal to reoptimize, i.e., $W_t(h) = \omega_{t-1}^{\iota_w}\pi^{1-\iota_w}W_{t-1}(h)$, where ω_{t-1} is gross nominal wage inflation in period $t-1$. Dynamic indexation of this form introduces some structural persistence into the wage-setting process.

B.1.3. Fiscal and Monetary Policy and the Aggregate Resource Constraint

Government purchases G_t are assumed to follow an exogenous AR(1) process with a persistence coefficient of 0.9. Government purchases have no effect on the marginal utility of private consumption, nor do they serve as an input into goods production. Government expenditures are financed by a combination of labor, capital, and lump-sum taxes. The government does not need to balance its budget each period, and issues nominal debt to finance budget deficits according to:

$$P_{B,t}B_{G,t+1} - B_{G,t} = P_tG_t - T_t - \tau_{N,t}W_tL_t - \tau_K\left(R_{K,t} - \delta P_t\right)K_t. \tag{B.11}$$

In eq. (B.11), all quantity variables are aggregated across households, so that $B_{G,t}$ is the aggregate stock of government bonds and K_t is the aggregate capital stock, and $T_t = (\int_0^1 T_t\left(h\right)dh)$ aggregate lump-sum taxes. In our benchmark specification, the lump-sum and capital tax rate is held fixed, and lump-sum taxes adjust endogenously according to a tax rate reaction function that allows taxes to respond to debt (as in Section 2.5) subject to smoothing. In log-linearized form:

$$\tau_{N,t} - \tau_N = \left(\varphi_\tau\right)\left(\tau_{N,t-1} - \tau_N\right) + \left(1-\varphi_\tau\right)\varphi_b\left(\tilde{b}_{G,t} - \tilde{b}_G\right), \tag{B.12}$$

where $\tilde{b}_{G,t} \equiv \frac{B_{G,t}}{4P_tY}$. In the working paper version of the paper – Erceg and Linde (2010) - we assumed lump-sum taxes were used to stabilize debt, but as Section 2.5 highlighted that the difference between lump-sum and distortionary tax financing can potentially be important in long-lived liquidity traps, we choose to work with distortionary tax financing – which we think is more empirically plausible – in this version of the paper.[B.1]

[B.1] In the working paper version (Erceg and Linde, 2010), we compare the effects with alternative financing schedules (lump-sum and distortionary labor-income taxes), and find that the effects are modest for an 8 quarter liquidity trap. In this version of the paper, it turns out that the financing assumption does not matter much even in 10-11 quarter liquidity trap, as we consider a tax rule that is somewhat less responsive to debt (following the empirical evidence in Traum and Yang, 2011, and our own simple regression of quarterly federal income tax rate on debt over the 1960-2011 period). It is worth pointing out that the form of the tax rate reaction function for lump-sum taxes has no effect on equilibrium allocations when that all agents are Ricardian, but when a share of households are "Keynesian" and consume directly their current after-tax disposable income, even the coefficients in a lump-sum rule matter for equilibrium allocations.

Monetary policy is assumed to be given by a Taylor-style interest rate reaction function similar to equation (3) except allowing for a smoothing coefficient γ_i:

$$i_t = \{\max\left(-i, (1-\gamma_i)\left(\gamma_\pi \pi_t + \gamma_x x_t\right) + \gamma_i i_{t-1}\right)\} \tag{B.13}$$

Finally, total output of the service sector is subject to the resource constraint:

$$Y_t = C_t + I_t + G_t + \psi_{I,t} \tag{B.14}$$

where $\psi_{I,t}$ is the adjustment cost on investment aggregated across all households (from eq. B.9, $\psi_{I,t} \equiv \frac{1}{2}\psi_I \frac{(I_t(h)-I_{t-1}(h))^2}{I_{t-1}(h)}$).

B.1.4. Keynesian Households

In the full with non-Ricardian households, we assume that a fraction s_{kh} of the population consists of "Keynesian" households whose members consume their current after-tax income each period, and set their wage equal to the average wage of the optimizing households. Because all households face the same labor demand schedule, each Keynesian household works the same number of hours as the average optimizing household. Thus, the consumption of Keynesian households $C_t^K(h)$ is simply determined as

$$P_t C_t^K(h) = (1-\tau_{Nt}) W_t(h) N_t(h) - T_t,$$

where T_t denotes (net) lump-sum taxes. Consumption of the non-Keynesian households is given the consumption Euler equation derived by maximizing (B.8) subject to (B.9).

B.1.5. Production of capital services

We build on the model described above by incorporating a financial accelerator mechanism following the basic approach of Bernanke, Gertler and Gilchrist (1999). Thus, the intermediate goods producers rent capital services from entrepeneurs (at the price R_{Kt}) rather than directly from households. Entrepeneurs purchase capital from competitive capital goods producers, with the latter employing the same technology to transform investment goods into finished capital goods as described by equations B.10) and B.9). To finance the acquisition of physical capital, each entrepreneur combines his net worth with a loan from a bank, for which the entrepreneur must pay an external finance premium (over the risk-free interest rate set by the central bank) due to an agency problem. We follow Christiano, Motto and Rostagno (2008) by assuming that the debt contract between entrepreneurs and banks is written in nominal terms (rather than real terms as in Bernanke, Gertler and Gilchrist, 1999). Banks obtain funds to lend to the entrepreneurs by issuing deposits to households at the interest rate set by the central bank. By assuming perfect competition and free entry among banks and that all bank portfolios are well diversified (i.e., that each bank lends out to a continuum of entrepreneurs, whose default risk is independently distributed), it follows that banks make zero profits in each state of the economy and that there is no credit risk to households associated with bank deposits.[B.2]

B.1.6. Solution and Calibration

To analyze the behavior of the model, we log-linearize the model's equations around the non-stochastic steady state. Nominal variables, such as the contract price and wage, are rendered stationary by suitable transformations. To solve the unconstrained version of the model, we compute the reduced-form solution of the model for a given set of parameters using the numerical algorithm of Anderson and Moore (1985), which provides an efficient implementation of the solution method proposed by Blanchard and Kahn (1980).

When we solve the model subject to the non-linear monetary policy rule (B.13), we use the techniques described in Hebden, Lindé and Svensson (2009). An important feature of the Hebden, Lindé and Svensson algorithm is that the duration of the liquidity trap is endogenous, and is affected by the size of the fiscal impetus. Their algorithm consists of adding a sequence of current and future innvoations to the linear component of the policy rule to guarantee that the zero bound constraint is satisfied given the economy's state vector. The sequence of innovations is assumed to be correctly anticipated by private agents at each date. This solution method is easy to use, and well-suited to examine the implications of the zero bound constraint in models with large dimensional state spaces; moreover, it yields identical results to the method of Jung, Terinishi, and Watanabe (2005).

As in Section 2, we set the discount factor $\beta = 0.995$, and steady state (net) inflation $\pi = .005$, implying a steady state nominal interest rate of $i = .01$ at a quarterly rate. The subutility function over consumption is logarithmic, so that $\sigma = 1$, and the parameter determining the degree of habit persistence in consumption $\varkappa$ is set at 0.6 (similar to the empirical estimate of Smets and Wouters 2003). The Frisch elasticity of labor supply $\frac{1}{\chi}$ of 0.4 is well within the range of most estimates from the empirical labor supply literature (see e.g. Domeij and Flodén, 2006).

The capital share parameter α is set to 0.35. The quarterly depreciation rate of the capital stock $\delta = 0.025$, implying an annual depreciation rate of 10 percent. We set the cost of adjusting investment parameter $\psi_I = 3$, which is somewhat smaller than the value estimated by Christiano, Eichenbaum, and Evans (2005) using a limited information approach; however, the analysis of Erceg, Guerrieri, and Gust (2006) suggests that a lower value may be better able to capture the unconditional volatility of investment.

We maintain the assumption of a relatively flat Phillips curve by setting the price contract duration parameter $\xi_p = 0.9$. As in Christiano, Eichenbaum and Evans (2005), we also allow for a fair amount of intrinsic persistence by setting the price indexation parameter $\iota_p = 0.9$. It bears emphasizing that our choice of ξ_p does not necessarily imply an average price contract duration

[B.2] We refer to Bernanke, Gertler and Gilchrist (1999) and Christiano, Motto and Rostagno (2008) for further details. An excellent exposition is also provided in Christiano, Trabandt and Walentin (2007).

of 10 quarters. Altig et al. (2011) show in a model very similar to ours that a low slope of the Phillips curve can be consistent with frequent price reoptimization if capital is firm-specific, at least provided that the steady-state markup is not too high, and it is costly to vary capital utilization; both of these conditions are satisfied in our model, as the steady state markup is 10 percent ($\theta_p = .10$), and capital utilization is fixed. Specifically, our choice of ξ_p implies a Phillips curve slope of about 0.007. Given strategic complementarities in wage-setting across households, the wage markup influences the slope of the wage Phillips curve. Our choices of a wage markup of $\theta_W = 1/3$ and a wage contract duration parameter of $\xi_w = 0.85-$ along with a wage indexation parameter of $\iota_w = 0.9$ - imply that wage inflation is about as responsive to the wage markup as price inflation is to the price markup.

The parameters of the monetary policy rule are set as $\gamma_i = 0.7$, $\gamma_\pi = 3$ and $\gamma_x = 0.25$. These parameter choices are supported by simple regression analysis using instrumental variables over the 1993:Q1-2008:Q4 period. This analysis suggests that the response of the policy rate to inflation and the output gap has increased in recent years, which helps account for somewhat higher response coefficients than typically estimated when using sample periods which include the 1970s and 1980s. Overall, as noted in the main text, our calibration of the monetary policy rule and the Phillips Curve slope parameters tilts in the direction of reducing the sensitivity of inflation to macroeconomic shocks.

We set the population share of the Keynesian households to optimizing households, s_{kh}, to 0.47, which implies that the Keynesian households' share of total consumption is about 1/3. This calibration perhaps overstates the role of non-Ricardian households in affecting consumption behavior, but seems useful to help put plausible bounds on how the multiplier may vary with the degree of non-Ricardian behavior in consumption (recognizing that the CEE/SW workhorse model is a special case in which $s_{kh} = 0$ and there are no financial frictions). Our calibration of the parameters affecting the financial accelerator follow BGG (1999). Thus, the monitoring cost, μ, expressed as a proportion of entrepreneurs' total gross revenue, is 0.12. The default rate of entrepeneurs is 3 percent per year, and the variance of the idiosyncratic productivity to entrepreneurs is 0.28.

The share of government spending of total expenditure is set equal to 20 percent. The government debt to GDP ratio is 0.5, close to the total estimated U.S. federal government debt to output ratio at end-2009. The steady state capital income tax rate, τ_K, is set to 0.2, while the lump-sum tax revenue to GDP ratio is set to 0.02. For simplicity, we set the depreciation allowance $\delta\tau_K = 0$. Given these choices, the government's intertemporal budget constraint implies that labor income tax rate τ_N equals 0.27 in steady state. The parameters in the fiscal policy rule in equation (B.12) are set to $\varphi_\tau = 0.92$, $\varphi_b = 0.1$ following the evidence in Traum and Yang (2011), implying that the tax rule is not very aggressive. Importantly, given the low share of government revenue accounted for by lump-sum taxes, most of the variation in the government budget deficit reflects fluctuations in revenue from the capital and labor income tax (due to variations in the tax base), and the service cost of debt.

B.2. Additional Results in the Large-Scale Model

Here we discuss some additional results referred to in Section 3 of the main text.

B.2.1. Initial Economic Conditions (Section 3.0)

The effects of the government spending increases considered in Sections 3.1 and 3.2 clearly depend on initial conditions which determine the depth and duration of the underlying liquidity trap. Under our baseline setting for these initial conditions, we assume that taste shocks (phased in over three quarters) generate a sharp fall in output and inflation as shown by the solid lines in Figure B.1.a, and cause the policy rate to decline to its lower bound of zero for eight quarters.[B.3] The taste shocks are scaled to induce a maximum output contraction of 8 percent relative to baseline, which is similar to the fall in U.S. GDP relative to trend that occurred during the Great Recession The implication that the liquidity trap lasts eight quarters seems reasonably consistent with market perceptions of the likely duration of the liquidity trap in early 2009 when large-scale fiscal stimulus was proposed. For example, the "projected"path for the federal funds rate implied by overnight indexed swap rates in early 2009 – shown in the lower left panel labelled B.1.b – was below 1 percent for a horizon extending eight quarters.

One feature of the baseline that merits additional discussion is the behavior of inflation. Under our benchmark calibration, Figure B.1.a shows that baseline inflation path falls more than 2 percentage points below its steady state level of 2 percent for over a year. The magnitude of the inflation response is somewhat greater than what occurred during the financial crisis. During that episode, even short-run inflation expectations – for instance, as proxied by the path of expected inflation over the next six quarters from the Survey of Professional Forecasters (shown in the lower right panel) – remained remarkably stable. Moreover, inflation expectations have continued to remain very stable since that time.

Although such evidence raises the possibility that our benchmark may overstate the responsiveness of inflation to large and highly persistent shocks, and thus perhaps exaggerate the fiscal multiplier, it bears emphasizing that our benchmark calibration implies much less movement in inflation than other commonly-adopted calibrations. To highlight this, Figure B.1.a also reports results for two alternative calibrations. In the case labelled "more flexible p and w," the mean duration of price and wage contracts is reduced to four and five quarters, respectively (i.e., $\xi_P = .75$ and $\xi_W = .80$), while another alternative labelled "looser rule" adopts the standard Taylor rule coefficients in the monetary policy rule (i.e., $\gamma_\pi = 1.5$ and $\gamma_x = 0.125$). Inflation declines by considerably more under either of these alternative calibrations.

[B.3] The scenario is generated by a sequence of three unanticipated negative taste shocks ν_t that cause the policy rate to fall to zero after three quarters.

B.2.2. Sources of Fiscal Free Lunch (Section 3.2)

Complementing the discussion in Section 3.2, Figure B.2 provides additional detail useful for understanding the channels through which higher government spending can generate a fiscal free lunch in a long-lived 10 quarter liquidity trap. In particular, the figure shows how a 1 percent of baseline GDP hike in spending affects the government/debt GDP ratio at horizons of 1-5 years, and provides a decomposition of the effects on government debt. The improvement in the overall debt/GDP ratio of a bit more than 1.5 percent after 12 quarters – the black dash-dotted line in the figure – is consistent with the *average* response of the debt/GDP ratio shown in the upper right panel in Figure 6.

Turning to the sources of improvement, the contribution of government spending – the red solid bars – to the debt/GDP ratio is positive as expected, and continues to rise through time given that the spending increase in persistent (recalling that positive bars in the figure imply a source of upward pressure on debt). Even so, higher labor income tax revenue alone (the light blue bars, holding the labor tax rate at steady state) is nearly sufficient to keep debt from expanding at a horizon of three years (the same horizon as shown in Figure 5); with higher capital income tax revenue and lower debt-servicing costs, government debt falls about 1.5 percentage points below its initial level after 12 quarters as noted previously. The debt/GDP ratio eventually moves back toward its initial level as the spending contribution grows a bit more, and because the labor tax rate actually *falls* (accounting for the positive contribution in the figure).

Fig. B.1.a: Simulated and Actual Paths for Key Macroeconomic Variables in Full Model

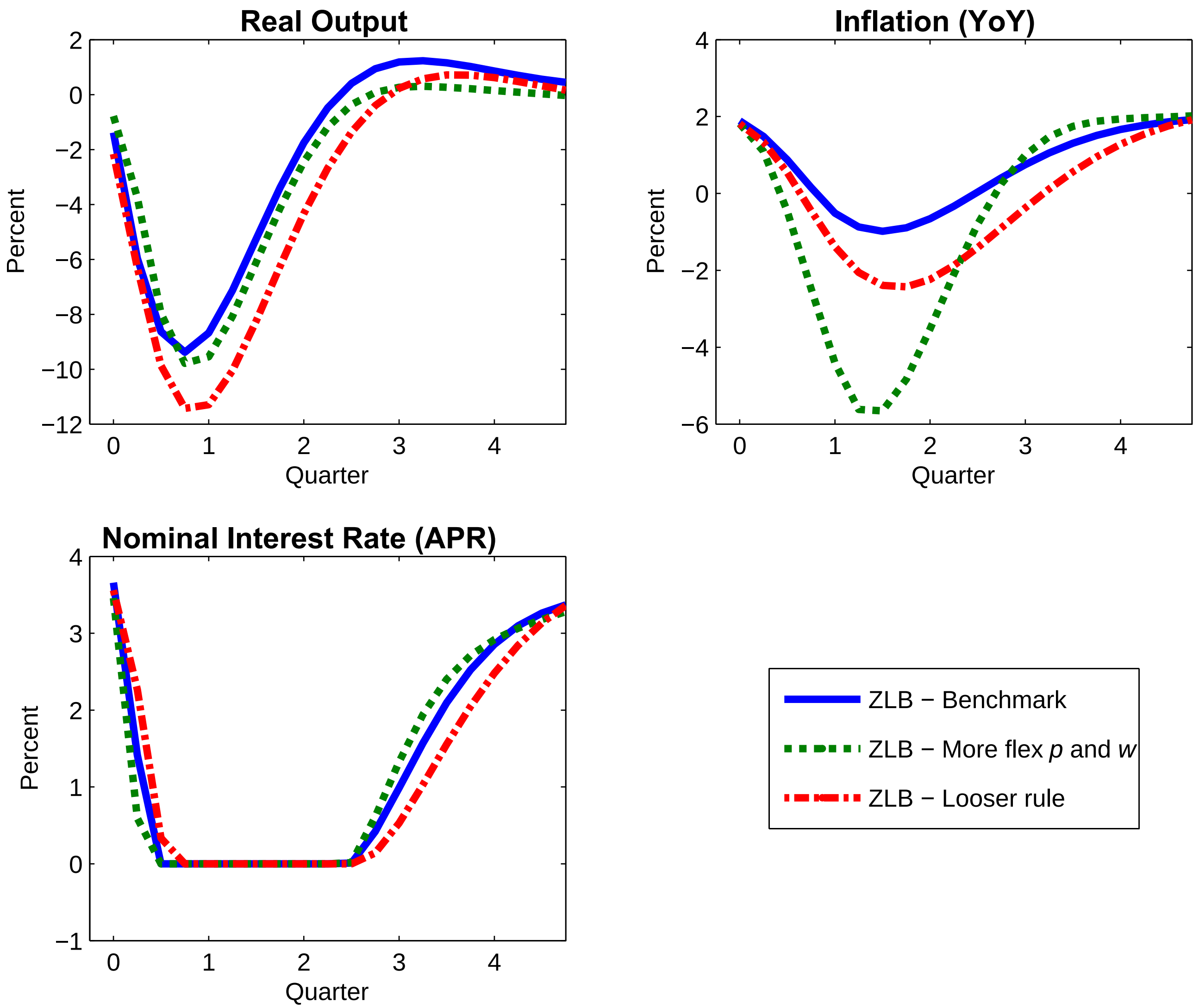


Figure B.1.b: Actual and Expected FFR and Core Inflation Rates

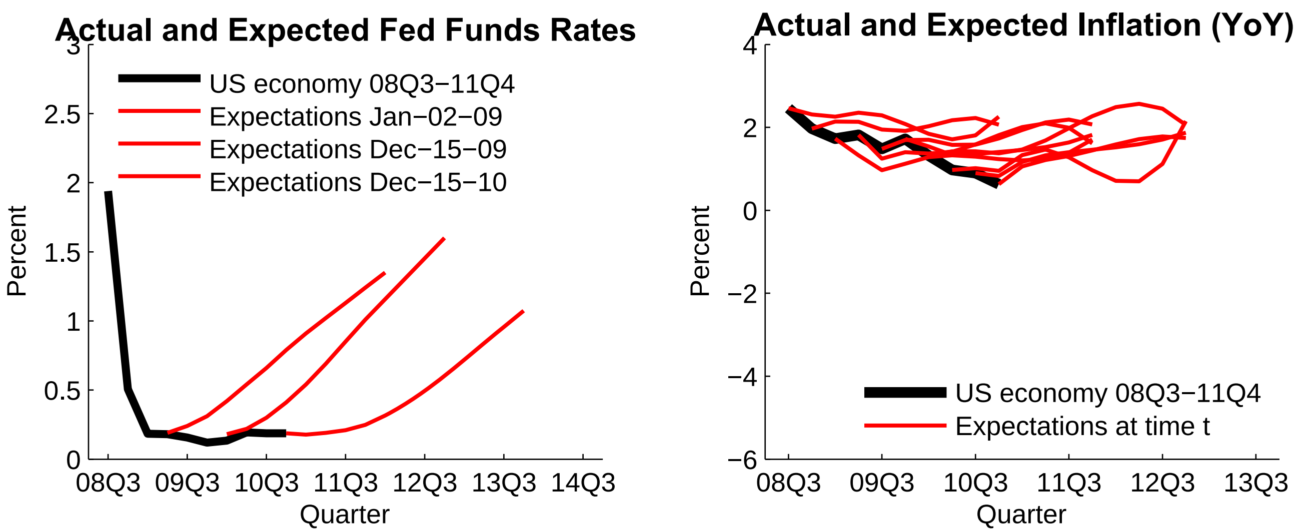


Figure B.2: Contribution to Debt Dynamics in Full Model in a Deep (10 quarter) Liquidity Trap