Offshore Production and Business Cycle Dynamics

with Heterogeneous Firms

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Offshore Production and Business Cycle Dynamics with Heterogeneous Firms*

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Abstract

Cross-country variation in production costs encourages the relocation of production facilities to other countries, a process known as offshoring through vertical foreign direct investment. I examine the effect of offshoring on the international transmission of business cycles. Unlike the existing macroeconomic literature, I distinguish between fluctuations in the number of offshoring firms (the extensive margin) and in the value added per offshoring firm (the intensive margin) as separate transmission mechanisms. The firms’ decision to produce offshore depends on the firm-specific level of labor productivity, on fluctuations in the relative cost of effective labor, and on the fixed and trade costs of offshoring. The model replicates the procyclical pattern of offshoring and the dynamics along its two margins, which I document using data from U.S. manufacturing and Mexico’s maquiladora sectors. Offshoring enhances the synchronization of business cycles, and dampens the real exchange rate appreciation generated by aggregate productivity differentials across countries.

JEL classification: F23, F41

Keywords: offshore production, extensive margin, heterogeneous firms, firm entry, business cycle dynamics, terms of labor, real exchange rate.

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1 Introduction

Firms often follow strategies that involve the establishment of production affiliates at foreign locations with relatively lower production costs, a process known in the international trade literature as offshoring through vertical foreign direct investment (FDI).¹ Unlike production under horizontal FDI - which means that foreign affiliates aim to gain market access by replicating the operations of their parent firms at the location where final consumption takes place - the type of offshoring that I model is primarily motivated by cross-country differences in the cost of effective labor, as foreign affiliates add value to goods consumed in the multinationals’ country of origin.² The number of offshoring firms (the extensive margin) and the real value added per offshoring firm (the intensive margin) fluctuate over the business cycle, and thus affect output, prices and wages in both the parent and the host countries.³

I document the business cycle features of offshoring motivated by lower production costs using data from U.S. manufacturing and Mexico’s maquiladora sector.⁴ Using the number of maquiladora plants to reflect the extensive margin, I show that the total value added and the number of plants in Mexico are strongly procyclical with the U.S. manufacturing output (Figure 3). In addition, the business cycle dynamics of the maquiladora sector differ across the total value added and its extensive margin: The total value added co-moves almost contemporaneously with U.S. manufacturing, whereas the number of plants lags the expansions and contractions in U.S. manufacturing by about four quarters, a result which highlights the inter-temporal dynamics of the extensive margin of offshoring.

Despite this empirical evidence, the international macroeconomics literature does not

¹"Offshoring" refers to the activity of firms that relocate certain stages of production to foreign countries; firms can either become integrated across borders through vertical or/and horizontal FDI, or purchase intermediate goods and services from unaffiliated foreign suppliers. In contrast, "outsourcing" applies to firms that purchase intermediates from unaffiliated suppliers - either at home or abroad - rather than producing them in house (see Helpman, 2006).


³Bergin, Feenstra and Hanson (2008) show that the extensive margin of offshoring accounts for more than one third of the adjustment of industry-level employment, and for nearly half of the adjustment of total employment in Mexico’s maquiladora sector.

⁴The maquiladora sector consists of manufacturing plants that import intermediate goods, process them, and export the resulting output.
fully capture the business cycle dynamics of offshoring along its extensive margin. Burstein, Kurz, and Tesar (2009) examine the role of production sharing in the transmission of business cycles in a two-country model in which the location of plants is fixed over time.\(^5\) Bergin, Feenstra, and Hanson (2007) focus on the importance of offshore production in amplifying the transmission of shocks across countries, in a model in which the number of offshoring firms adjusts instantly - rather than gradually over time as in the data - in response to simultaneous aggregate shocks in the parent and the host countries.

To address this issue, I model the endogenous creation of offshore production plants as a firm-specific decision, in a two-country (North and South), dynamic stochastic general equilibrium framework. The key features of the model include endogenous firm entry in the parent country and firm heterogeneity in labor productivity. Firm entry is subject to a sunk cost reflecting headquarter activities at home. Following entry in the North, each firm can use either domestic or foreign labor in the production of a different variety of goods. The use of foreign labor involves the establishment of an offshore plant, and is subject to fixed and trade costs every period.\(^6\) Since firms are heterogeneous in productivity, the decision to produce offshore is firm-specific: Despite the lower cost of effective labor abroad (i.e. the lower wage relative to aggregate productivity), only the more productive firms can afford the fixed and trade costs of offshoring. The cross-country asymmetry in the cost of effective labor also implies that offshoring takes place one-way; only some of the Northern firms have an incentive to produce offshore, whereas all Southern firms produce at home.

The key results of the paper are as follows. First, the model generates a procyclical pattern of offshoring that is consistent with the data from U.S. manufacturing and Mexico’s maquiladora sector. In the model, the number of offshoring firms every period depends on the fluctuations in the relative cost of effective labor across countries. A positive shock to aggregate productivity in the North encourages domestic firm entry, and causes the domestic wage to rise above aggregate productivity as labor demand increases to accommodate firm entry requirements. Notably, the increase in the cost of effective labor in the North is gradual (because the number of firms is a stock variable), and causes a gradual increase in

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\(^5\)In Burstein, Kurtz and Tesar (2008), the low elasticity of substitution between the domestic and foreign goods enhances the cross-country co-movement of output.

\(^6\)I maintain a one-to-one correspondence between an offshoring firm, a variety, and an offshore plant.
the number of offshoring firms (the extensive margin), as in the data. Second, offshoring
enhances the co-movement of output across countries. The increase in output in the North
(generated by a country-specific shock to aggregate productivity) and also in the South
(caused by the immediate jump in Northern demand for offshored varieties, and also by
the subsequent relocation of production by the Northern firms to the South) enhance the
co-movement of output across the two economies.\(^7\) The result is consistent with the em-
pirical regularity documented in Burstein, Kurz, and Tesar (2009) that country pairs with
larger shares of offshoring-related bilateral trade exhibit larger correlations of manufacturing
output. Third, offshoring narrows price dispersion across countries, as it reduces the appre-
ciation of the real exchange rate that follows a domestic increase in aggregate productivity in
the framework with firm entry and endogenously traded varieties. Thus, offshoring dampens
the Harrod-Balassa-Samuelson effect through a number of channels, including the transfer
of upward pressure from the domestic to the foreign wage, the reduced size of the domestic
non-traded sector, and the decline in import prices that occurs as offshoring crowds out the
less productive foreign exporters.

This paper is related to a growing body of macroeconomic literature that focuses on
endogenous firm entry and adjustments along the extensive margin of exports (but not of
offshoring).\(^8\) For example, Ghironi and Melitz (2005) study the export decision of firms in
the presence of fixed exporting costs, in a framework with firm entry and firm heterogeneity.
Alessandria and Choi (2007) analyze the extensive margin of exports in a model with sunk
costs and continuation fixed costs that explains the "exporter hysteresis" behavior.\(^9\) Corsetti,
Martin, and Pesenti (2007) examine the terms-of-trade implications of productivity improve-
ments in the sectors of firm entry and production, in a model in which the extensive margin

\(^7\) In the traditional international real business cycle literature, in contrast, a domestic increase in aggregate
productivity leads to increased production at home but not offshore, as in Backus, Kehoe and Kydland (1992).

\(^8\) Recent empirical literature highlights the role of the extensive margin in international trade in the
presence of fixed exporting costs: Baldwin and Harrigan (2007) show that the number of traded goods (the
extensive margin) decreases with distance and increases with the size of the importing country. Besedes and
Prusa (2006) find that the survival rate of exports for differentiated good varieties increases with the initial
transaction size and also with the length of the relationship. Hummels and Klenow (2005) show that larger
economies have larger exports, and that the extensive margin accounts for as much as 60 percent of this
difference.

\(^9\) "Exporter hysteresis" refers to the behavior of firms that continue to serve the foreign market even after
a real exchange rate appreciation reduces their export competitiveness.
of exports is endogenous. Finally, Mejean (2006) studies the effect of endogenous firm entry in the tradable sector on the real exchange rate dynamics and the Harrod-Balassa-Samuelson effect.

The assumptions of the model considered in this paper are consistent with the empirical evidence on the determinants of offshore production provided by recent studies. Hanson, Mataloni, and Slaughter (2005) show that U.S. multinational firms attract larger shares of the sales of their foreign affiliates when the latter benefit from lower trade costs and lower wages abroad. Kurz (2006) shows that the U.S. plants and firms using imported components in production are larger and more productive than their domestically-oriented counterparts, as the larger productivity allows them to cover the fixed costs of offshoring.

The study of offshoring motivated by lower production costs is important to understand the macroeconomic interdependence between country pairs and economic areas separated by persistent differences in the cost of effective labor, such as the U.S. and Latin America, or Western Europe and the new member countries of the European Union (Marin, 2006; Meyer, 2006). In 2005, offshore production through vertical integration was responsible for as much as 50 percent of the manufacturing sales of the U.S. affiliates in Mexico, and for 26 percent of the sales of the U.S. affiliates in Latin America as a whole, shares which were directed towards the U.S. parent firms (BEA, 2007).

The rest of this paper is organized as follows: Section 2 introduces the model of offshoring with heterogeneous firms that allows for fluctuations in offshoring along its extensive and intensive margins. Section 3 translates the model with firm heterogeneity into an equivalent framework with two representative firms that produce domestically and offshore. Section 4 describes the model calibration. Section 5 presents the results; it shows the business cycle dynamics of offshoring in the presence of aggregate productivity shocks, and also compares the empirical moments of offshoring from the U.S. to Mexico with their model counterparts. Section 6 concludes with a summary and possible extensions of the model.
2 Model of Offshoring with Heterogeneous Firms

2.1 Markets and Production Strategies

The model consists of two economies, North and South. Each economy includes one representative household, and also a continuum of firms that are monopolistically competitive and heterogeneous in labor productivity, with each firm producing a different variety of goods. Every period, the existing firms choose the destination market(s) that they serve and the location of production, as follows:

1. All firms serve their domestic market. For this purpose, the Northern firms produce their varieties using either domestic or foreign labor. The use of foreign labor involves the establishment of an offshore production plant, and offers the advantage of a lower production cost. However, offshoring is subject to per-period fixed and trade costs, as described below.\textsuperscript{10}

2. Some firms from each economy also serve the foreign market. They use exclusively domestic labor in production, and export their varieties subject to a per-period fixed cost, as in Ghironi and Melitz (2005).

Because the number of firms in each economy varies over time, and also because the existing firms re-optimize their offshoring and exporting strategies every period, the composition of consumption baskets in each economy changes over time.

2.2 Firms Serving the Domestic Market: Domestic vs. Offshore Production

This section illustrates the mechanisms of domestic and offshore production as alternative choices for the Northern firms that produce for their domestic market. Every period, the firm with idiosyncratic labor productivity $z$ must choose one of the two possible production strategies:

\textsuperscript{10}All Southern firms produce domestically because the higher cost of effective labor in the North offers them no incentive to produce offshore.
(a) Domestic production, with output being a function of the aggregate productivity in
the North $Z_t$, the firm-specific labor productivity $z$, and domestic labor $l_t$:

$$ y_{D,t}(z) = Z_t z l_t. \quad (1) $$

(b) Alternatively, the firm with idiosyncratic labor productivity $z$ may choose to produce
offshore using Southern labor $l_t^*$:

$$ y_{V,t}(z) = Z_t^* z l_t^*. \quad (2) $$

The Northern firm producing offshore becomes subject to the Southern aggregate labor
productivity $Z_t^*$, but is able to carry its own idiosyncratic labor productivity $z$ to the South.\(^{11}\)

The monopolistically-competitive firm with idiosyncratic productivity $z$ solves the profit-
maximization problem for the alternative scenarios of domestic and offshore production:

$$ \max_{\{\rho_{D,t}(z)\}} d_D(z) = \rho_{D,t}(z) y_{D,t}(z) - \frac{w_t}{Z_t z} y_{D,t}(z), \quad (3) $$

$$ \max_{\{\rho_{V,t}(z)\}} d_V(z) = \rho_{V,t}(z) y_{V,t}(z) - \frac{w_t^* Q_t}{Z_t^* z} y_{V,t}(z) - f_V \frac{w_t^* Q_t}{Z_t^*}, \quad (4) $$

where $\rho_{D,t}(z)$ and $\rho_{V,t}(z)$ are the prices associated with each of the two production strategies,
\(w_t\) and $w_t^*$ are the real wages in the North and the South, and $Q_t$ is the real exchange rate.

The cost of producing one unit of output either domestically or offshore varies not only with
the cost of effective labor $\frac{w_t}{Z_t}$ and $\frac{w_t^* Q_t}{Z_t^*}$ across countries, but also with the level of idiosyncratic
labor productivity $z$ across firms.\(^{12}\) In addition, the Northern firms producing offshore incur
a fixed offshoring cost equal to $f_V$ units of Southern effective labor\(^{13}\) - that reflects the

\(^{11}\)Strategies (1) and (2) are the special cases of a more general framework of offshoring, in which the
offshoring firm with idiosyncratic labor productivity $z$ uses a combination of Northern and Southern labor,
$l_t$ and $l_t^*$. The output of firm $z$ is a Cobb-Douglas function of domestic and foreign inputs,
$y_{V,t}(z) = \left[ \frac{Z_t z l_t}{\alpha} \right]^\alpha \left[ \frac{Z_t^* z l_t^*}{1-\alpha} \right]^{1-\alpha}$, as in Antras and Helpman (2004).
In this paper, I explore two special cases: At one extreme, I set $\alpha = 1$ to shut down offshore production, a case which replicates Ghironi and Melitz (2005). At the other extreme, I set $\alpha = 0$ so that the firms choosing to produce offshore use exclusively foreign inputs.
The smaller $\alpha$, the larger the range of operations that the offshoring firms relocate abroad. I use the l'Hôpital
rule and $\omega = 1/\alpha$ to obtain: \( \lim_{\alpha \to 0} \left( \frac{1}{\alpha} \right)^\alpha = \lim_{\omega \to \infty} \omega^1/\omega = \lim_{\omega \to \infty} e^{\ln \omega} = e^{\lim_{\omega \to \infty} \left( \frac{\ln \omega}{\omega} \right) \text{Hôpital}} = e^{\lim_{\omega \to \infty} (1/\omega)} = 1. \)

\(^{12}\)The cost of effective labor is the ratio between the real wage and aggregate productivity in each country.
The real exchange rate $Q_t = \frac{P_t}{P_t^*}$ is the ratio between the price indexes in the South and the North expressed
in the same currency, where $\varepsilon_t$ is the nominal exchange rate.

\(^{13}\)The cost of $f_V$ units of Southern effective labor is equivalent to $f_V w_t^*/Z_t^*$ units of the Southern con-
building and maintenance of the offshore production facility - and also an iceberg trade cost \( \tau > 1 \) associated with the shipping of goods produced offshore back to the parent country. For every \( \tau \) units produced offshore, only one unit arrives in the North for consumption, as the difference is lost due to trade barriers, transportation and insurance costs (Anderson and Wincoop, 2004). The demand functions for the variety produced by firm \( z \) domestically or offshore are

\[
y_{D,t}(z) = \rho_{D,t}(z)^{-\theta}C_t \quad \text{and} \quad y_{V,t}(z) = \rho_{V,t}(z)^{-\theta}C_t,
\]

where \( C_t \) is the aggregate consumption in the North.

The profit-maximization problem implies the equilibrium prices

\[
\rho_{D,t}(z) = \frac{\theta}{\theta - 1} \frac{w_t}{Z_t z} \quad \text{and} \quad \rho_{V,t}(z) = \frac{\theta}{\theta - 1} \frac{w_t^* Q_t}{Z_t^* z},
\]

for the alternative scenarios of domestic and offshore production. The corresponding profits, expressed in units of the aggregate consumption basket \( C_t \), are:

\[
d_{D,t}(z) = \frac{1}{\theta} \rho_{D,t}(z)^{1-\theta}C_t, \quad \quad (5)
\]

\[
d_{V,t}(z) = \frac{1}{\theta} \rho_{V,t}(z)^{1-\theta}C_t - f_V \frac{w_t^* Q_t}{Z_t^*}. \quad \quad (6)
\]

When deciding upon the location of production every period, the firm with productivity \( z \) compares the profit \( d_{D,t}(z) \) that it would obtain from domestic production with the profit \( d_{V,t}(z) \) from producing the same variety offshore. As a particular case, I define the productivity cutoff level \( z_{V,t} \) on the support interval \([z_{\text{min}}, \infty)\), so that the firm at the cutoff obtains equal profits from producing domestically or offshore:

\[
z_{V,t} = \{ z \mid d_{D,t}(z_{V,t}) = d_{V,t}(z_{V,t}) \}. \quad \quad (7)
\]

The productivity cutoff \( z_{V,t} \) is a variable that reacts to fluctuations in the relative cost of effective labor across countries, and thus reflects the behavior of the extensive margin of offshoring over the business cycle.

The model implies that only the relatively more productive Northern firms find it profitable to produce their varieties offshore. Despite the lower cost of effective labor in the South, only firms with idiosyncratic productivity above a certain cutoff (\( z > z_{V,t} \)) obtain benefits from offshoring that are large enough to cover the fixed and iceberg trade costs.
This feature of the model is consistent with the empirical evidence in Kurz (2006), that the U.S. plants and firms using imported components in production are larger and more productive than their domestically-oriented counterparts, as the larger idiosyncratic productivity levels allow them to cover the fixed costs of offshoring.\footnote{A useful implication of model with firm heterogeneity is that the more productive firms have larger output and revenue. Given two firms with idiosyncratic productivity $z_2 > z_1$, the ratios of output and profits are $\frac{y(z_2)}{y(z_1)} = \left(\frac{z_2}{z_1}\right)^\theta > 1$ and $\frac{d(z_2)}{d(z_1)} = \left(\frac{z_2}{z_1}\right)^{\theta-1} > 1$ (see Melitz, 2003). Empirical studies show that firms using imported inputs in production are not only more productive, but also have larger revenues and employ more workers (Kurz, 2006).}

**Existence of the equilibrium productivity cutoff** Next I show that the existence of the equilibrium productivity cutoff $z_{V,t}$ requires a cross-country asymmetry in the cost of effective labor, so that some of the Northern firms will always have an incentive to produce offshore. I begin by re-writing the per-period profits that would be obtained from domestic and offshore production as $d_{D,t}(z) = M_t \left(\frac{w_t}{Z_t}\right)^{1-\theta} z^{\theta-1}$ and $d_{V,t}(z) = M_t \left(\tau \frac{w_t Q_t}{Z_t}\right)^{1-\theta} z^{\theta-1} - f_V \frac{w_t Q_t}{Z_t}$, where $M_t \equiv \frac{1}{\theta} \left(\frac{\theta}{\theta-1}\right)^{1-\theta} C_t$ measures the size of the Northern market. In Figure 1, I plot the two profits as functions of the idiosyncratic productivity parameter $z^{\theta-1}$ over the support interval $[z_{\min}, \infty)$. The vertical intercept is zero for the case of domestic production; it is equal to the negative of the fixed cost in the case of offshoring ($- f_V \frac{w_t Q_t}{Z_t}$).

![Figure 1. The firm-specific productivity cutoff $z_{V,t}$.

\[ - f_V \frac{w_t Q_t}{Z_t} \]
The existence of the equilibrium productivity cutoff $z_{V,t}$ in Figure 1 requires that the profit function from offshoring must be steeper than the profit from domestic production, $\text{slope} \{d_{V,t}(z)\} > \text{slope} \{d_{D,t}(z)\}$. When this condition is met, offshoring generates greater profits than domestic production for the subset of firms with idiosyncratic productivity $z$ along the upper range of the support interval ($z > z_{V,t}$). The slope inequality is equivalent to $\tau \frac{w_t Q_t / Z_t}{w_t / Z_t} < 1$, which implies that the effective wage in the South must be sufficiently lower than that in the North, so that the difference covers both the fixed cost and the iceberg trade cost ($\tau > 1$), and thus provides an incentive for some of the Northern firms to produce offshore. The model calibration and the magnitude of macroeconomic shocks ensure that this condition is satisfied every period.\footnote{A second condition necessary to avoid the corner solution when all firms would produce offshore is that $d_{D,t}(z_{\text{min}}) > d_{V,t}(z_{\text{min}})$. It ensures that $z_{V,t} > z_{\text{min}}$ in all periods.}

2.3 Exporting Firms

Firms from each economy can choose to serve the foreign market through exports as in Ghironi and Melitz (2005), in addition to producing for their domestic market. In the North, the firm with idiosyncratic productivity $z$ would use an amount of domestic labor $l_{H,t}$ to produce for the Southern market, $y_{H,t}(z) = Z_t z_l H_t$. The Southern firms that choose to export to the North face a similar problem.

Profit maximization implies the following equilibrium price and profit functions for the Northern exporter with productivity factor $z$: $\rho_{H,t}(z) = \frac{\theta}{\theta - 1} \tau^* \frac{w_t Q_t / Z_t}{Z_t z_l H_t}$ and $d_{H,t}(z) = \frac{1}{\theta} \rho_{H,t}(z)^{1-\theta} C_t^* Q_t - f_H \frac{w_t}{Z_t}$, where $C_t^*$ is the aggregate consumption in the South. Producing for the foreign market generates additional profits, but involves a fixed exporting cost equal to $f_H$ units of Northern effective labor, and also an iceberg trade cost $\tau^*$. Thus, the model implies that only the subset of Northern firms with idiosyncratic labor productivity above a productivity cutoff $z_{H,t}$ find it profitable to export to the Southern market, as they can afford the fixed \footnote{I view exporting as a special case within a more general framework, in which firms serve the foreign market using a mix of domestic and foreign inputs in production: $y_{H,t}(z) = \left( \frac{Z_t z_l H_t}{\eta} \right)^{\alpha} \left( \frac{Z_t z_l H_t}{1-\eta} \right)^{1-\eta}$, where a larger $\eta$ is equivalent to a smaller content of Southern inputs used in the production of goods sold in the South. In my model, I incorporate the special case with endogenous exports as in Ghironi and Melitz (2005) by setting $\eta = 1$. Alternatively, I would model the case in which Northern firms serve the Southern market exclusively through their foreign affiliates (as in Contessi, 2006) by setting $\eta = 0.$}
and iceberg trade costs of exporting. The time-varying productivity cutoff is:

$$z_{H,t} = \inf \{ z \mid d_{H,t}(z_{V,t}) > 0 \}.$$

### 2.4 Households and Consumption Baskets

The representative household in the North maximizes the expected lifetime utility, which is

$$\max_{\{B_{t+1}, x_{t+1}\}} \left[ \mathbb{E}_t \sum_{s=t}^{\infty} \beta^{s-t} C_t^{1-\gamma} \right],$$

subject to the budget constraint:

$$(\tilde{v}_t + \tilde{d}_t)N_t x_t + (1 + r_t)B_t + w_t L \geq \tilde{v}_t (N_t + N_{E,t}) x_{t+1} + B_{t+1} + C_t,$$

where $C_t$ is the amount of aggregate consumption, $\beta \in (0, 1)$ is the subjective discount factor, and $\gamma > 0$ is the inverse of the inter-temporal elasticity of substitution. (The representative household in the South faces a similar problem.) The Northern household starts every period with share holdings $x_t$ in a mutual fund of $N_t$ firms whose average market value is $\tilde{v}_t$, and also with real bond holdings $B_t$. It receives dividends equal to the average firm profit $\tilde{d}_t$ in proportion with the number of firms and with its share holdings, interest $r_t B_t$ on bond holdings, and the real wage $w_t$ for the amount of labor $L \equiv 1$ supplied inelastically.

The Northern household purchases two types of assets every period. First, it purchases $x_{t+1}$ shares in a mutual fund of Northern firms that includes: (i) $N_t$ firms already producing at time $t$, either domestically or offshore, and (ii) $N_{E,t}$ new firms that enter the market in period $t$. Each share is worth its market value $\tilde{v}_t$, equal to the net present value of the expected stream of future profits of the average firm. Second, the household also purchases the risk-free bond $B_{t+1}$ denominated in units of the Northern consumption basket. In addition, the household purchases the consumption basket $C_t$, which includes varieties produced by the Northern firms either domestically ($\omega \in \Omega_t^{NN}$) or offshore ($\omega \in \Omega_t^{NS}$); it also includes varieties produced by Southern firms and imported by the North ($\omega \in \Omega_t^{SS}$):
\[ C_t = \left[ \int_{z_{\min}}^{z_{\text{V,t}}} y_{D,t}(\omega)^{\gamma - 1} \omega d\omega + \int_{\omega \in \Omega_t^{NN}} y_{V,t}(\omega)^{\gamma - 1} \omega d\omega + \int_{\omega \in \Omega_t^{NS}} y_{H,t}(\omega)^{\gamma - 1} \omega d\omega \right]^{\frac{\theta}{\gamma - 1}} \tag{10} \]

where \( \theta > 1 \) is the symmetric elasticity of substitution across varieties. I use the home consumption basket \( C_t \) as the numeraire good, so that the price index in the North is \( 1 = \left[ \int \rho_t(\omega)^{1-\theta} d\omega \right]^{\frac{1}{1-\theta}} \), where \( \omega \in \Omega_t^{NN} \cup \Omega_t^{NS} \cup \Omega_t^{SS} \), and \( \rho_t(\omega) \) is the real price of each variety expressed in units of the Northern consumption basket. The first-order conditions generate the following Euler equations for bonds and stocks:

\[ C_t^{-\gamma} = \beta (1 + r_{t+1}) E_t \left[ C_{t+1}^{-\gamma} \right], \tag{11} \]

\[ \tilde{v}_t = \beta (1 - \delta) E_t \left[ \left( \frac{C_{t+1}}{C_t} \right)^{-\gamma} (\tilde{d}_{t+1} + \tilde{v}_{t+1}) \right], \tag{12} \]

where \( \delta \) is the exogenous rate of firm exit every period.

### 2.5 Firm Entry and Exit

Firm entry takes place in both countries every period, as in Ghironi and Melitz (2005). An unbounded pool of potential entrant firms face a trade-off between the sunk entry cost (reflecting headquarter activities in the parent country, such as research and development, management, marketing) and the expected stream of future monopolistic profits (discounted by the probability of exit very period). In the North, firm entry requires a sunk entry cost equal to \( f_E \) units of Northern effective labor.\(^{18}\) After paying the sunk entry cost, each firm is randomly assigned an idiosyncratic labor productivity factor \( z \) that is drawn independently from a common distribution \( G(z) \) with support over the interval \( [z_{\min}, \infty) \), and which the firm keeps for the entire duration of its life.

The \( N_{E,t} \) firms entering at time \( t \) do not produce until period \( t + 1 \). Irrespective of their idiosyncratic productivity, all firms - including the new entrants - are subject to a random

\(^{18}\)The sunk entry cost is equivalent to \( f_E w_t/Z_t \) units of the Northern consumption basket.
exit shock that occurs with probability $\delta$ at the end of every period. Thus, the law of motion for the number of producing firms is $N_{t+1} = (1 - \delta)(N_t + N_{E,t})$, where $N_t = N_{D,t} + N_{V,t}$ is the total number of Northern firms that produce either domestically or offshore at period $t$.

The potential entrant firms anticipate their expected post-entry value $\tilde{v}_t$, which depends on the expected stream of future profits $\tilde{d}_t$, the stochastic discount factor, and the exogenous probability $\delta$ of exit every period. The forward iteration of the Euler equation for stocks (12) generates the following expression for the expected post-entry value of the average firm:

$$
\tilde{v}_t = E_t \left\{ \sum_{s=t+1}^{\infty} [\beta(1 - \delta)]^{s-t} \left( \frac{C_s}{C_t} \right)^{-\gamma} \tilde{d}_s \right\}.
$$

(13)

In equilibrium, firm entry takes place until the expected value of the average firm is equal to the sunk entry cost expressed in units of the Northern consumption basket:

$$
\tilde{v}_t = f_E \frac{w_t}{Z_t}.
$$

(14)

3 Aggregation over Heterogeneous Firms

This section translates the model with firm heterogeneity into an equivalent framework with two representative Northern firms that produce domestically and offshore for their domestic market. Since offshoring takes place one-way, there is only one representative Southern firm that produces for the domestic market. In addition to the firm producing for the domestic market, one representative firm in each economy produces domestically for the export market.

3.1 Average Firm Productivity Levels

**Domestic vs. offshore production** First I describe the average productivity levels of the two representative Northern firms that produce domestically and offshore for the Northern market. In Figure 2, I plot the density of the firm-specific labor productivity levels $z$ over the support interval $[z_{min}, \infty)$. Every period $t$, there are $N_{D,t}$ firms from the North with idiosyncratic productivity levels below the offshoring cutoff ($z < z_{V,t}$) that produce domestically; their average productivity is $\bar{z}_{D,t}$. There are also $N_{V,t}$ firms with productivity
factors above the cutoff \((z > z_{V,t})\) that choose to produce offshore; their average productivity is \(\bar{z}_{V,t}\).\(^{19}\) Since the firm-specific labor productivity levels \(z\) are random draws from a common distribution \(G(z)\) with density \(g(z)\), I compute the two average productivity levels as:

\[
\begin{align*}
\bar{z}_{D,t} &= \left[ \frac{1}{G(z_{V,t})} \int_{z_{\min}}^{z_{V,t}} z^{\theta-1} g(z) \, dz \right] \frac{1}{\theta-1} \\
\bar{z}_{V,t} &= \left[ \frac{1}{1 - G(z_{V,t})} \int_{z_{V,t}}^{\infty} z^{\theta-1} g(z) \, dz \right] \frac{1}{\theta-1}.
\end{align*}
\] \(15\)

Figure 2. Average labor productivity levels of Northern firms that produce domestically (\(\bar{z}_{D,t}\)) and offshore (\(\bar{z}_{V,t}\)) for the Northern market.

I assume that the firm-specific labor productivity draws \(z\) are Pareto-distributed, with p.d.f. \(g(z) = k z_{min}^\theta z^{k-1}\) and c.d.f. \(G(z) = 1 - (z_{min}/z)^k\) over the support interval \([z_{min}, \infty)\). Using this assumption, I derive analytical solutions for the average productivity levels of the two representative Northern firms that produce domestically and offshore as functions of the time-variant productivity cutoff \(z_{V,t}\):\(^{20}\)

\[
\begin{align*}
\bar{z}_{D,t} &= \nu z_{\min} z_{V,t} \left[ \frac{z_{V,t}^{k-(\theta-1)} - z_{\min}^{k-(\theta-1)}}{z_{V,t}^k - z_{\min}^k} \right] \frac{1}{\theta-1} \\
\bar{z}_{V,t} &= \nu z_{V,t},
\end{align*}
\] \(16\)

where the productivity cutoff is \(z_{V,t} = z_{min}(N_t/N_{V,t})^{(1/k)}\), and the parameters are \(\nu \equiv \ldots\)

\(^{19}\)Note that \(\bar{z}_{V,t}\) is the average productivity of offshoring firms, whereas \(z_{V,t}\) is the productivity cutoff above which firms produce offshore.

\(^{20}\)See Appendix A.3.
\[ \left[ \frac{k}{k-(\theta-1)} \right]^{1/k} \] and \( k > \theta - 1. \) \(^{21}\) Since offshoring takes place one-way, from the North to the South, the Southern firms serve their domestic market exclusively through domestic production. Their average productivity is constant, \( \bar{z}_D^* = \nu z_{\text{min}}^* \), as it covers the entire support interval \([z_{\text{min}}^*, \infty)\).

**Exporting firms** Under the assumption of Pareto-distributed productivity draws, the average productivity levels of the exporting firms in each economy are as in Ghironi and Melitz (2005):

\[
\bar{z}_{H,t} = \nu \tilde{z}_{\text{min}} \left( \frac{N_t}{N_{H,t}} \right)^{1/k} \quad \text{and} \quad \bar{z}_{H,t} = \nu \tilde{z}_{\text{min}} \left( \frac{N_{D,t}^*}{N_{H,t}^*} \right)^{1/k} . \tag{17}
\]

### 3.2 Average Prices and Profits

Using the average productivity levels for the domestic, offshoring and exporting firms described above, I translate the model of offshoring in terms of *three representative Northern firms*: one produces domestically, another produces offshore (each serving the Northern market), while a third firm produces domestically and exports to the Southern market. There are only *two representative Southern firms*: one produces for the local market, and the other exports to the North. I describe the average prices and profits for each representative firm in Table 1.

Using the property that the Northern firm at the productivity cutoff \( z_{V,t} \) is indifferent between the two production strategies, I derive the following relationship between the average profits of the two representative firms that produce domestically and offshore: \(^{22}\)

\[
\tilde{d}_{V,t} = \frac{k}{k-(\theta-1)} \left( \frac{z_{V,t}}{\bar{z}_{D,t}} \right)^{\theta-1} \tilde{d}_{D,t} + \frac{\theta - 1}{k-(\theta-1)} f_{V} \frac{w_{t}^* Q_{t}}{Z_{t}} . \tag{18}
\]

In addition, using the property that the firm at the productivity cutoff \( z_{H,t} \) obtains zero

\(^{21}\)I use the Pareto c.d.f. \( G(z_{V,t}) = 1 - (z_{\text{min}}/z_{V,t})^k \) and the share of Northern firms producing offshore \( N_{V,t}/N_t = 1 - G(z_{V,t}) \) to write the productivity cutoff as \( z_{V,t} = z_{\text{min}} (N_t/N_{V,t})^{(1/k)} \). The share of Northern firms producing domestically is \( N_{D,t}/N_t = G(z_{V,t}) \). Parameter \( k \) reflects the dispersion of the productivity draws: A relatively larger \( k \) implies a smaller dispersion and a higher concentration of productivities \( z \) towards the lower productivity bound \( z_{\text{min}} \).

\(^{22}\)See Appendix A.4.
profits from exporting, the average profits from exports are:

\[ \tilde{d}_{H,t} = \frac{\theta - 1}{k - (\theta - 1)} f_H \frac{w^*_t}{Z^*_t}, \text{ and } \tilde{d}^*_{H,t} = \frac{\theta - 1}{k - (\theta - 1)} f^*_H \frac{w^*_t}{Z^*_t}. \] (19)

### Table 1. Average prices and profits

<table>
<thead>
<tr>
<th>Firm</th>
<th>Origin</th>
<th>Production</th>
<th>Market</th>
<th>Average price</th>
<th>Average profit</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>North</td>
<td>North</td>
<td>North</td>
<td>( \hat{\rho}<em>{D,t} = \frac{\theta}{\theta - 1} \frac{w_t}{Z_t} \hat{d}</em>{D,t} )</td>
<td>( \tilde{d}<em>{D,t} = \frac{1}{\theta} (\hat{\rho}</em>{D,t})^{1-\theta} C_t )</td>
</tr>
<tr>
<td>2.</td>
<td>South</td>
<td>South</td>
<td>South</td>
<td>( \hat{\rho}^<em>_{D,t} = \frac{\theta}{\theta - 1} \frac{w^</em><em>t}{Z^<em>_t} \hat{d}^</em></em>{D,t} )</td>
<td>( \tilde{d}^<em>_{D,t} = \frac{1}{\theta} (\hat{\rho}^</em>_{D,t})^{1-\theta} C^*_t )</td>
</tr>
<tr>
<td>3.</td>
<td>North</td>
<td>South</td>
<td>North</td>
<td>( \hat{\rho}_{V,t} = \frac{\theta}{\theta - 1} \frac{w^<em>_t}{Z^</em><em>t} \hat{d}</em>{V,t} )</td>
<td>( \tilde{d}<em>{V,t} = \frac{1}{\theta} (\hat{\rho}</em>{V,t})^{1-\theta} C_t - f_v \frac{w^<em>_t}{Z^</em>_t} )</td>
</tr>
<tr>
<td>4.</td>
<td>North</td>
<td>North</td>
<td>South</td>
<td>( \hat{\rho}_{H,t} = \frac{\theta}{\theta - 1} \frac{w^<em>_t}{Z^</em><em>t} \hat{d}</em>{H,t} )</td>
<td>( \tilde{d}<em>{H,t} = \frac{1}{\theta} (\hat{\rho}</em>{H,t})^{1-\theta} C_t Q_t - f_H \frac{w^<em>_t}{Z^</em>_t} )</td>
</tr>
<tr>
<td>5.</td>
<td>South</td>
<td>South</td>
<td>North</td>
<td>( \hat{\rho}^<em>_{H,t} = \frac{\theta}{\theta - 1} \frac{w^</em><em>t}{Z^<em>_t} \hat{d}^</em></em>{H,t} )</td>
<td>( \tilde{d}^<em>_{H,t} = \frac{1}{\theta} (\hat{\rho}^</em>_{H,t})^{1-\theta} C_t Q_t^{-1} - f^<em>_H \frac{w^</em>_t}{Z^*_t} )</td>
</tr>
</tbody>
</table>

**Price indexes**  The consumption price index in the Northern economy is a function of the average prices of varieties produced domestically and offshore by the Northern firms, as well as the average price of the varieties imported from the South:

\[ 1 = N_{D,t} (\hat{\rho}_{D,t})^{1-\theta} + N_{V,t} (\hat{\rho}_{V,t})^{1-\theta} + N_{H,t}^* (\hat{\rho}_{H,t})^{1-\theta}. \] (20)

In the South, there is no representative firm producing offshore. The consumption price index depends on the average price of varieties produced domestically by the Southern firms, and also on that of varieties imported from the North:

\[ 1 = N_{D,t}^* (\hat{\rho}_{D,t})^{1-\theta} + N_{H,t} (\hat{\rho}_{H,t})^{1-\theta}. \] (21)

**Total profits**  The total profits of the Northern firms include the average profits from domestic and offshore production, as well as those from exports:

\[ N_t \tilde{d}_t = N_{D,t} \tilde{d}_{D,t} + N_{V,t} \tilde{d}_{V,t} + N_{H,t} \tilde{d}_{H,t}. \] (22)
The total profits of the Southern firms combine the average profits from domestic sales and exports:

\[ N_{D,t} d_t = N_{D,t} \tilde{d}_{D,t} + N_{H,t} \tilde{d}_{H,t}. \]  

(23)

### 3.3 Aggregate Accounting and the Current Account Balance

I measure the aggregate income as the sum of the wage bill and the amount of stock dividends that households in each economy obtain every period, \( Y_t = w_t + N_t \tilde{d}_t \) and \( Y_t^* = w_t^* + N_{D,t} \tilde{d}_t^* \).

The value added offshore, defined as the wage bill of the Southern workers employed for production in the offshoring sector, is part of the Southern output. The profits of the Northern firms producing offshore are part of the Northern output.

Under financial autarky in the markets for bonds and stocks, aggregate accounting implies that households spend their income from labor and stock holdings on consumption and investment in new firms, \( C_t + N_{E,t} \tilde{v}_t = Y_t \) and \( C_t^* + N_{E,t}^* \tilde{v}_t^* = Y_t^* \).

The current account in the North is:

\[
CA_t = N_{H,t} (\tilde{p}_{H,t})^{1-\theta} C_t + \underbrace{N_{V,t} \tilde{d}_{V,t}}_{\text{(a) Exports}} - \underbrace{N_{V,t} (\tilde{p}_{V,t})^{1-\theta} C_t}_{\text{(b) Repatriated profits}} - \underbrace{N_{H,t} (\tilde{p}_{H,t})^{1-\theta} C_t}_{\text{(c) Value added offshore}} - \underbrace{N_{H,t}^* (\tilde{p}_{H,t}^*)^{1-\theta} C_t}_{\text{(d) Imports of Southern varieties}}
\]

Under financial autarky, the balanced current account condition (\( CA_t = 0 \)) states that the sum of (a) exports by Northern firms to the South and (b) repatriated profits of offshore affiliates must be equal to the sum of (c) the value added by offshore affiliates imported in the North and (d) the imports of varieties produced by the Southern firms.\(^{23}\)

### 3.4 Model Summary

The baseline model with financial autarky for the Northern economy is characterized by 16 equations in 16 endogenous variables: \( N_t, N_{D,t}, N_{V,t}, N_{H,t}, N_{E,t}, \tilde{d}_t, \tilde{d}_{D,t}, \tilde{d}_{V,t}, \tilde{d}_{H,t}, \tilde{z}_{D,t}, \tilde{z}_{V,t}, \tilde{z}_{H,t}, \tilde{v}_t, r_t, w_t \) and \( C_t \). Since the Southern firms do not produce in the high-cost North, the Southern economy is described by only 11 equations in 11 endogenous variables; there are

\(^{23}\)In the case with international trade in bonds, the current account balance equals the change in bond holdings, which is the negative of the financial account balance (Appendix A.2).
no Southern counterparts for \( N_t, N_{VT}, \tilde{d}_{VT}, \tilde{z}_{DT} \) and \( \tilde{z}_{VT} \). In particular, the average labor productivity of the representative Southern firm producing for the domestic market \( (\tilde{z}_D^*) \) is constant over time. Variables \( N_{DT}, r_t, N_t^* \) and \( r_t^* \) are predetermined.\(^{24}\)

### 4 Calibration

I use a standard quarterly calibration by setting the subjective rate of time discount \( \beta = 0.99 \) to match an average annualized interest rate of 4 percent. The coefficient of relative risk aversion is \( \gamma = 2 \). Following Ghironi and Melitz (2005), I set the intra-temporal elasticity of substitution \( \theta = 3.8 \), and the probability of firm exit \( \delta = 0.025 \) to match the annual 10 percent job destruction in the U.S.

As summarized in Table 2, I calibrate the fixed costs of offshoring \( (f_V) \) and exporting \( (f_H \) and \( f_H^*) \) as well as the Pareto distribution parameter \( k \), so that the model matches the importance of offshoring and trade for the Mexican economy in steady state, as illustrated by four empirical moments: (1) The maquiladora value added represents about 20 percent of Mexico’s manufacturing GDP (INEGI, 2008), compared to 25 percent in the model; (2) The maquiladora exports represent about half of Mexico’s total exports (Bergin, Feenstra, and Hanson, 2008), vs. 60 percent in the model; (3) Employment in the maquiladora sector accounts for approximately 25 percent of Mexico’s total manufacturing employment (Bergin, Feenstra, and Hanson, 2008), compared to 22 percent in the model; (4) Total imports represent the equivalent of 33 percent of Mexico’s GDP (INEGI, 2008), and 32 percent in the model. To this end, I set \( f_V = 0.0057 \) (the fixed cost of offshoring for Northern firms), \( f_H = 0.032 \) and \( f_H^* = 0.018 \) (the fixed costs of exporting for the Northern and Southern firms, respectively), as well as \( k = 4.2 \) (the Pareto distribution coefficient).\(^{25}\)

Without loss of generality, I set the lower bound of the support interval for firm-specific productivity in the North and the South at \( z_{min} = z_{min}^* = 1.\)

\(^{24}\)The model summary is in Appendix A.1. The steady-state solution is available in Appendix A.6.

\(^{25}\)In the alternative model with exports only, I set \( f_H = 0.0260 \) and \( f_H^* = 0.0226 \) so that the fraction of Northern exporting firms (10 percent) and that of Southern exporting firms (63 percent) match the corresponding steady state values from the model with offshoring.
Table 2. Calibration parameters and steady-state targets

<table>
<thead>
<tr>
<th>Calibration parameters:</th>
<th>Steady-state targets:</th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fixed cost of offshoring</td>
<td>$f_V = 0.0057$</td>
<td>Maquila. VA in Mex. manufacturing</td>
<td>20%</td>
</tr>
<tr>
<td>Fixed cost of exporting, North</td>
<td>$f_H = 0.0320$</td>
<td>Maquila. share in Mexican exports</td>
<td>50%</td>
</tr>
<tr>
<td>Fixed cost of exporting, South</td>
<td>$f_H^* = 0.0180$</td>
<td>Maquila. share in manuf. employment</td>
<td>25%</td>
</tr>
<tr>
<td>Pareto distribution coefficient</td>
<td>$k = 4.2$</td>
<td>Mexico’s imports as % of GDP</td>
<td>33%</td>
</tr>
</tbody>
</table>

In order to obtain a steady-state asymmetry in the cost of effective labor across countries, I set the sunk entry cost - which reflects headquarter activities sensitive to the regulation of starting a business in the firms’ country of origin - to be larger in the South than in the North ($f_E = 4f_E$ and $f_E = 1$). As a result, the steady state number of firms, the labor demand and the real wage are relatively lower in the South. The calibration reflects the considerable variation in the cost of starting a business across countries: The corresponding monetary cost is 3.3 times higher in Mexico than in the U.S. or Canada; it is 6.2 times higher in Hungary than in the U.K. (World Bank, 2007). The asymmetric sunk entry costs, along with the iceberg trade cost ($\tau = 1.3$) and the values for $f_V$, $f_H$ and $f_H^*$ reported above, generate a steady state value for the terms of labor that is less than one ($TOL = \frac{Qw^*/Z^*}{w/Z} = 0.76$).26

In other words, the steady state cost of effective labor in the South, defined as real wage divided by aggregate productivity, is 76 percent of the corresponding level in the North. The calibration provides an incentive for the Northern firms to produce offshore in steady state.

The resulting steady-state fraction of the Northern firms that use foreign labor ($N_V/N$) is 1.4 percent; the fraction of exporting firms ($N_H/N$) is 10.1 percent. Since I model offshoring in an asymmetric two-country framework that abstracts from the exchanges of U.S. firms with the rest of the world (other than Mexico), the steady state values reported above are less than their empirical counterparts. In the data, approximately 14 percent of the U.S. firms (other than domestic wholesalers) used imported inputs from both Mexico and the rest of the world in 1997 (Bernard, Jensen, Redding and Schott, 2007). Out of that, intra-firm imports represented half of the total amount, while the rest was accounted by arm’s

26 The terms of labor is the ratio between the cost of effective labor in the South and the North expressed in units of the same consumption basket. The calibration ensures that the condition $\tau TOL < 1$ from Section 2.2 is satisfied in steady state.
length transactions (Bardhan and Jafee, 2004). Approximately 21 percent of the U.S. manufacturing plants were exporters in 1992 (Bernard, Eaton, Jensen and Kortum, 2003).

The calibration also implies that, in the North, the steady-state expenditure share of the varieties produced by Northern firms domestically (66.0 percent) - firms which are relatively less productive than the average - is less than their fraction in the total number of varieties available in the North (89.2 percent). In contrast, since the offshored varieties are produced by the relatively more productive Northern firms, their expenditure share (21.2 percent) is more than their fraction in the total number of varieties available in the North (1.2 percent). The pattern is consistent with the more productive firms having larger market shares than their less productive counterparts.

5 Results

5.1 Offshoring to Mexico’s Maquiladora Sector

In this section I describe the cyclicality of offshoring motivated by lower production costs using data from U.S. manufacturing and Mexico’s maquiladora sector. In particular, I document the business cycle dynamics of the extensive margin of offshoring from the U.S. to Mexico, which will be useful to assess the implications of the model with offshoring that are described in the following sections.

Mexico’s maquiladora sector The maquiladora sector represents an appropriate empirical setup to study the cyclicality of offshoring by U.S. manufacturing firms motivated by lower production costs, due to the absence of local consumption in Mexico and its direct links to U.S. manufacturing. The plants operating under Mexico’s maquiladora program import inputs, process them, and ship the resulting goods back to the country of origin.

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27 The value of 14 percent would understate the fraction of plants that use foreign inputs if the offshoring firms tend to operate multiple plants that produce different varieties.

28 In the South, the share of Southern varieties in total spending (61.7 percent) is less than their fraction in the total number of varieties (62.8 percent), since Northern exporters are more productive than the average Southern firm.
Although only a subset of the maquiladora plants are U.S.-owned, most of them accommodate the offshoring operations of U.S. firms: They import most of their inputs from the U.S. (82 percent), and export most of their output (90 percent) back to the U.S. (Hausman and Kaytko, 2003; Burstein, Kurz and Tesar, 2009). The value added of the maquiladora sector is part of Mexico’s manufacturing output.

**Empirical cross-correlations**  Mexico’s total manufacturing output and, in particular, the maquiladora value added are strongly correlated with U.S. manufacturing. In Figure 3 (panel 1), I plot the detrended series for Mexico’s maquiladora value added (the dashed line) and Mexico’s total manufacturing output (the dotted line) against the manufacturing component of U.S. industrial production (henceforth U.S. IP, the solid line), for the interval between 1990:Q1 and 2006:Q4. The chart shows that the U.S. recessions in 1990 and 2001, as well as the expansion throughout the late 1990s, were associated with similar patterns in the maquiladora value added. During the 1994-95 financial crisis in Mexico, the decline in the maquiladora value added was less pronounced than the drop in Mexico’s total manufacturing output, as the offshoring sector in Mexico benefited from the direct links with U.S. manufacturing. The cross-correlations in panel 2 show that Mexico’s maquiladora value added moves closely together with the U.S. manufacturing output, and that its correlation with U.S. manufacturing is larger than that of Mexico’s total manufacturing output.

In panel 3 (bottom left), I plot the detrended series for the number of maquiladora plants in Mexico (the dashed line) - which reflects the extensive margin of offshoring - against the U.S. IP for manufacturing (the solid line). The cross-correlations in panel 4 show that U.S. manufacturing leads the number of maquiladora plants by about four quarters. The result suggests that the extensive margin of offshoring adjusts gradually over time, whereas the maquiladora value added is contemporaneously correlated with the U.S. manufacturing output.

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29 The maquiladora sector accounts for about 20 percent of Mexico’s manufacturing value added (INEGI), for 50 percent of manufacturing exports, and for 25 percent of Mexico’s employment (Bergin, Feenstra and Hanson, 2007, 2008).

30 The seasonally adjusted data in natural logs is expressed in deviations from a Hodrick-Prescott trend. The data for U.S. manufacturing IP is provided by the Federal Reserve Board. The data for Mexico’s manufacturing IP and the maquiladora sector (real value added and the number of plants) is provided by INEGI (2008). I aggregate the maquiladora data into quarters (from the original monthly frequency), and seasonally adjust it using the X-12-ARIMA method of the U.S. Census Bureau.
output.

**Figure 3.** Business cycle dynamics of offshoring to Mexico.

### 5.2 Impulse Responses of Offshoring

I log-linearize the baseline model of offshore production under financial autarky around the steady state, and compute the impulse responses to a transitory one-percent increase in aggregate productivity in the North. I assume that aggregate productivity is described by the autoregressive process \( \log Z_{t+1} = \rho \log Z_t + u_t \), with the persistence parameter \( \rho = 0.9 \).

Figure 4 shows the impulse responses of the baseline model of offshoring (thick solid lines), and contrasts them to those from two alternative frameworks: (i) a model of offshoring in which the productivity cutoff is fixed, so that the fraction of offshoring firms is constant over the business cycle (thin solid lines)\(^{31}\); and (ii) the extreme case with no offshoring, a case in

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\(^{31}\)In the alternative model with fixed productivity cutoff, the fraction of offshoring firms is constant, but the number of offshoring firms varies over time due to firm entry in the country of origin. During expansions in the North, the new entrants that draw idiosyncratic productivity factors above the cutoff start by producing directly offshore. However, none of the firms that initially produce at home can relocate offshore when the terms of labor appreciate.
which I replicate the model with exports in Ghironi and Melitz (2005) (dashed lines). For each variable, the horizontal axis illustrates quarters after the initial shock, and the vertical axis shows the percent deviations from the original steady state in each quarter.

![Graphs showing impulse responses](image)

Figure 4. Impulse responses to a one-percent shock to aggregate productivity in the North, baseline model of offshoring with adjustable productivity cutoff (thick solid line); alternative models with fixed productivity cutoff (thin solid line) and no offshoring (dashed line).

The intensive margin  In the baseline model (thick solid lines), on impact, the increase in aggregate labor productivity in the North generates a proportional increase in the real wage \( (w_t) \). The rising demand for varieties produced both domestically and offshore causes
an immediate increase in offshoring along its intensive margin (i.e. offshore value added per firm). Since the increase in aggregate productivity in the North is not replicated in the South, the excess demand for Southern effective labor causes the real wage in the South \( w_t^* \) and the terms of labor \( TOL_t = \frac{Q_t w_t^* / Z_t}{u_t / Z_t} \) to jump. As a result, the number of Northern firms that produce offshore \( (N_{V,t}) \) drops on impact due to: (i) the increase in the cost of effective labor offshore, and (ii) the increase in the fixed cost of offshoring, both of which are sensitive to the effective wage in the South.

The extensive margin As the aggregate labor productivity in the North persists above its initial steady state, the larger market size encourages firm entry, as shown by the gradual increase in the number of incumbent firms \( (N_t) \). In turn, firm entry leads to an increase in the demand for Northern labor, which causes the cost of effective labor to appreciate gradually in the North relative to the South. In Figure 4, this appreciation is visible as the real wage in the North declines more slowly after the initial shock than aggregate productivity, and thus the terms of labor persist below their initial steady state level. Following the appreciation of the terms of labor, the number of offshoring firms \( (N_{V,t}) \) increases, as some of the more productive Northern firms relocate production to the South. Notably, the increase in the number of offshoring firms is gradual, as it mirrors the gradual appreciation of the terms of labor.

The total value added offshore \( (VA_R) \) increases by more under the baseline model of offshoring (thick solid line) than in the alternative model of offshoring in which the productivity cutoff is fixed (thin solid line). Thus, 20 quarters after the shock, more than half of the increase in the total value added offshore is due to the adjustment along the extensive margin.

In the South, the initial jump in the real wage - caused by the spike of the intensive margin of offshoring - is followed by an additional increase which occurs gradually over time, as some of the more productive Northern firms relocate production to the South. Since the increase in offshoring along its extensive margin transfers some of the upward pressure from the domestic to the foreign wage, the terms of labor appreciate by less \( (TOL \) declines by less) in the baseline model of offshoring (thick solid line) than in the alternative models with
no extensive margin adjustment (thin solid line) and no offshoring (dashed line).

5.3 The Extensive and Intensive Margins of Offshoring

In this section I provide evidence in support of the model of offshoring with extensive margin adjustments over the business cycle. Thus, I analyze the empirical cross-correlations between lags and leads of the U.S. manufacturing output and two empirical indicators of offshore production in Mexico: (i) the number of maquiladora establishments, as an empirical proxy for the extensive margin; and (ii) the value added per establishment, as a proxy for the intensive margin. Then I compare the empirical correlations to their model counterparts.

In the model, the total value added offshore, \( VA_t = N_{V,t} \left( \frac{\eta^\theta \varphi Q_t}{Z_t^{1-\theta}} \right)^{1-\theta} C_t, \) is a function of the number of offshoring firms, their average idiosyncratic productivity, the foreign cost of effective labor, and the aggregate consumption in the North. The number of offshoring firms \( (N_{V,t}) \) measures the extensive margin of offshore production, and constitutes the counterpart of the number of maquiladora plants in Mexico. The real value added per offshoring firm \( (VA_{R,t}/N_{V,t}) \) represents the intensive margin of offshoring; it is the model counterpart of the value added per maquiladora plant.\(^{32}\)

**Model vs. empirical cross-correlations** Figure 5 (panels 1 and 2) shows the empirical correlations between U.S. manufacturing and the two margins of the maquiladora sector (black line), together with their 95 percent confidence intervals. It also shows the model correlations generated by the baseline model of offshoring under financial autarky (red solid line), as well as those generated by the baseline model augmented with elastic labor supply (green dotted line). Aggregate productivity follows the bivariate autoregressive process:

\[
\begin{bmatrix}
\log Z_t \\
\log Z^*_t \\
\end{bmatrix} = 
\begin{bmatrix}
\rho_Z & \rho_{ZZ^*} \\
\rho_{Z^*Z} & \rho_{Z^*} \\
\end{bmatrix} 
\begin{bmatrix}
\log Z_{t-1} \\
\log Z^*_{t-1} \\
\end{bmatrix} + 
\begin{bmatrix}
\xi_t \\
\xi^*_t \\
\end{bmatrix},
\]

\(^{24}\)

\(^{32}\)I deflate the value added offshore by the average price index of the varieties produced offshore, \( VA_{R,t} = P_tVA_t / \bar{P}_{V,t} \). To this end, I decompose the price index for the offshore varieties into components reflecting (a) variety and (b) average price as \( P_t = (N_{V,t})^{1-\theta} \bar{P}_{V,T} \), to obtain \( VA_{R,t} = (N_{V,t})^{1-\theta} VA_t \).
where the persistence parameters are $\rho_Z = \rho_{Z^*} = 0.906$, and the spillovers are $\rho_{ZZ^*} = \rho_{Z^*Z} = 0.088$, as in Backus, Kehoe, and Kydland (1992). The variance of the shocks is $0.00852^2$ and the covariance is $0.18728 \times 10^{-4}$, values which correspond to a correlation of innovations of 0.258.

Regarding the extensive margin (panel 1), the data shows a strong and positive correlation between the number of maquiladora plants and the past U.S. manufacturing output. As discussed, expansions in U.S. manufacturing tend to lead the number of offshore plants by about four quarters. The model is successful in capturing this pattern; the correlation between the number of offshoring firms and the past output in the North is positive, and reaches a peak for the Northern output lagged by four quarters. The result is explained by the fact that, following a productivity improvement in the North, the increase in the number of offshoring firms is gradual, as it mirrors the gradual appreciation of the terms of labor caused by domestic firm entry. Although the contemporaneous correlation between the number of offshoring firms and Northern output is slightly negative\(^{33}\) (rather than positive as in the data), the model replicates the inter-temporal dynamics of the extensive margin.

\(^{33}\)Following a positive shock to aggregate productivity in the North, the initial drop in the number of offshoring firms - caused by a spike in the Southern wage - is followed by a gradual increase above the initial steady state level, as the terms of labor appreciate over time.
Turning towards the intensive margin (panel 2), the empirical correlation between the maquiladora value added per plant and the past U.S. manufacturing output is negative and statistically significant. The model is successful in replicating this pattern as well. Following a positive technology shock in the North, the number of offshoring firms increases faster than the total value added offshore due to the appreciation of the terms of labor. As a result, the value added per offshoring firm declines below its initial level several quarters after the shock, and the correlation between the intensive margin of offshoring and past output in the North is negative.

5.4 The Co-Movement of Output and Offshore Production

In this section I illustrate the cross-country correlations of Northern output and the value added offshore generated by the baseline model of offshoring, and examine their sensitivity to the trade cost and the persistence of aggregate productivity.

The co-movement of output and the value added offshore I assume that aggregate productivity follows the bivariate autoregressive process described by equation (24). The persistence parameters are asymmetric across the two economies ($\rho_Z = 0.996$ and $\rho_{Z*} = 0.951$), there are no spillovers ($\rho_{ZZ*} = \rho_{Z*Z} = 0$), and the technology shocks are less volatile in the North than in the South (i.e. variances $0.0050939^2$ vs. $0.0139570^2$), with the covariance $0.1898 \times 10^{-4}$ implying a correlation of shocks of 0.27. These assumptions are based on the estimates of the bivariate productivity process for the U.S. and Mexico in Mandelman and Zlate (2008), that use data on total factor productivity (TFP) for the two countries.

Table 3 shows the cross-country correlation of output $\text{Corr}(Y_R, Y_{R*})$, the correlation of output in the country of origin with the value added offshore $\text{Corr}(Y_R, VA_{R*})$, and also the cross-country correlation of consumption $\text{Corr}(C_R, C_{R*})$ generated by the model of offshoring.\textsuperscript{34} It also reports the correlations generated by the alternative framework in which I

\textsuperscript{34}In order to compute the cross-country correlations, I deflate output and consumption by the average price indexes in each country, since the empirical price deflators are best represented by the average price index $\bar{P}_t$ rather than the welfare-based price index $P_t$. For instance, I use $P_t = (N_{D,t} + N_{V,t} + N_{H,t})^{1/3} \bar{P}_t$ to deflate output in the North as $Y_{R,t} = P_t Y_t / \bar{P}_t = (N_{D,t} + N_{V,t} + N_{H,t})^{1/3} Y_t$. I deflate the value added offshore by
shut down offshoring, and thus replicate the model with endogenous exports in Ghironi and Melitz (2005).

Table 3. Cross-country contemporaneous correlations

<table>
<thead>
<tr>
<th>Model:</th>
<th>Financial autarky</th>
<th>International bond trading</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Offshoring</td>
<td>No offshoring</td>
</tr>
<tr>
<td>Corr($Y_R, Y^*_R$)</td>
<td>0.35</td>
<td>0.34</td>
</tr>
<tr>
<td>Corr($Y_R, VA_R$)</td>
<td>0.99</td>
<td>n/a</td>
</tr>
<tr>
<td>Corr($C_R, C^*_R$)</td>
<td>0.40</td>
<td>0.44</td>
</tr>
</tbody>
</table>

Two notable results emerge from Table 3. First, the correlation of Northern output with the value added offshore is larger than the cross-country correlation of total output, $\text{Corr}(Y_R, VA_R) > \text{Corr}(Y_R, Y^*_R)$, a result which is consistent with the empirical correlations documented above (i.e. the maquiladora value added co-moves more closely with the U.S. manufacturing output than does Mexico’s total manufacturing output.) In the model, the value added offshore is closely related to aggregate productivity, net firm entry and the appreciation of the cost of effective labor in the North, and thus is strongly correlated with the Northern output. In contrast, the total output in the South, on one hand, receives the positive contribution of the offshoring sector, which enhances its co-movement with the Northern output. On the other hand, the relative productivity increase in the North dampens firm entry in the South, and thus partially offsets the additional co-movement generated by offshoring. In addition, offshoring reduces the competitiveness of the Southern exporters, as the relocation of production transfers some of the upward pressure from the Northern wage to the Southern one, which further dampens output co-movement. Nonetheless, under international bond trading, the cross-country co-movement of total output is further reduced by the resource-shifting effect that occurs as households lend across the border to finance firm entry in the country receiving the favorable productivity shock.

Second, the model with offshoring enhances the co-movement of output relative to the special case with no offshoring. Intuitively, in the model with no offshoring in Ghironi and Melitz (2005), the positive shock to aggregate productivity in the North generates an increase in the demand for all varieties - produced both in the North and in the South - the average price index of the varieties produced offshore as $VA_{R,t} = P_t VA_t / \bar{P}_{V,t} = (N_{V,t})^{1/2} VA_t$.  

28
as well as an immediate increase in the relative price of Southern varieties. The resulting substitution away from Southern varieties is offset by the increase in net firm entry and the gradual appreciation of the cost of effective labor in the North, which enhances the export competitiveness of the Southern firms. Therefore, the model with no offshoring still generates positive output co-movement across the two economies.

Offshoring introduces an additional transmission mechanism that enhances the co-movement of output. Following an increase in aggregate productivity in the North, firm entry causes a gradual appreciation of the terms of labor, which in turn provides an incentive for some of the more productive Northern firms to relocate production to the South. The increase of output in the North (generated by the positive shock to aggregate productivity) and also in the South (generated by the initial jump in Northern demand for offshored varieties and, subsequently, the gradual relocation of production by the Northern firms to the South), enhance the co-movement of total output across the two economies. The effect is dampened under international trade in bonds, as the Southern households lend to finance firm entry during expansions in the North.

The remainder of this section examines the sensitivity of cross-country correlations to: (a) variation in the iceberg trade cost \( \tau \) and (b) variation in the persistence of the bivariate autoregressive productivity process \( \rho_Z \). Figures 6 and 7 show that the model with offshoring generates larger cross-country correlations for both output and consumption relative to the special case with no offshoring, results which hold for a wide range of possible values for the trade cost \( \tau \in [1.21, 1.34] \) and the persistence parameter \( \phi_Z \in [0.9, 1) \).

**Sensitivity to the trade cost \( \tau \)** The cross-country correlation of output is greater for lower values of the trade cost (Figure 6). During expansions in the North, a lower trade cost (that applies to both offshoring and non offshoring-related trade) enhances the demand for varieties produced in the South (either by Northern offshoring firms or by Southern exporters). A lower trade cost also facilitates the relocation of production offshore over the business cycle, and thus enhances the cross-country co-movement of output.
Figure 6. Cross-country correlations of output and consumption, sensitivity to $\tau$.

The result is consistent with the empirical regularity documented in Burstein, Kurz, and Tesar (BKT, 2009), namely that country pairs with (i) larger shares of offshoring-related trade in bilateral trade and (ii) larger bilateral trade flows relative to output also exhibit larger correlations of manufacturing output. In BKT (2009), the regression of output correlations between the U.S. and foreign economies on (i) the production-sharing intensity of foreign exports and (ii) the share of exports in foreign output generates OLS coefficients that are positive and statistically significant (0.746 and 0.140, respectively). In the model considered in this paper, a decline in the trade cost from its baseline calibration value to the lower extreme (i.e. from 1.3 to 1.21) is associated with an 11 percentage point increase in the correlation of output (Figure 6, panel 1 for financial autarky). The same reduction in the trade cost is linked to (i) an increase in the steady-state share of offshoring-related trade in the Southern exports (from 60 to 92 percent), and also (ii) an increase in the share of exports in the Southern output (from 40 to 86 percent). The resulting slope between the output correlation and the steady-state share of offshoring-related trade in Southern exports (0.344) is roughly half the corresponding OLS coefficient in BKT (2009). The slope between the output correlation and the steady-state share of exports in Southern output (0.240) is slightly larger than its empirical counterpart.
**Sensitivity to the aggregate productivity persistence $\rho_Z$** The model with offshoring generates larger cross-country correlations of output than the alternative model with no offshoring for the entire range of values of the persistence parameter $\rho_Z$.\textsuperscript{35} More, the additional co-movement generated by offshoring increases with the persistence parameter. Under financial autarky (panel 1 in Figure 7), the additional correlation brought by offshoring increases from 6 to 12 percentage points as the persistence parameter rises from $\rho_Z = 0.9$ to $\rho_Z = 0.995$. Following a positive technology shock in the North, the larger persistence of aggregate productivity leads to a larger increase in domestic demand, and thus to a larger increase in offshoring along its intensive margin on impact. The larger persistence also enhances firm entry and generates a larger appreciation of the terms of labor over the business cycle, which in turn provides a greater incentive for firms to relocate production offshore. Under financial integration (panel 2), the result is dampened by the resource-shifting effect that occurs when households lend to the country that receives a positive technology shock.

1. Cross-country correlations, autarky
2. Cross-country correlations, trade in bonds

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\textsuperscript{35}I assume that the persistence parameter is symmetric across the North and the South, that there are zero spillovers, but maintain the variance-covariance matrix of shocks from Mandelman and Zlate (2008).
5.5 Real Exchange Rate Dynamics

Offshoring narrows the price dispersion across countries, as it dampens the appreciation of the real exchange rate that follows a domestic increase in aggregate productivity. In the absence of offshoring, the framework with firm entry and endogenously traded varieties in Ghironi and Melitz (2005) generates the Harrod-Balassa-Samuelson effect (i.e. more productive economies exhibit higher average prices), as the country that receives a favorable shock to aggregate productivity also experiences an appreciation of the terms of labor and a rise in import prices. However, offshoring dampens this effect though a number of channels, including the transfer of upward pressure from the domestic to the foreign wage, the reduced size of the domestic non-traded sector, and the decline in import prices as offshoring crowds out the less productive foreign exporters.

**Average prices and product variety** I use the consumer price index (CPI)-based real exchange rate \( Q_{CPI,t} = \varepsilon_t \widehat{P}_t / \widehat{P}_t \) as the theoretical counterpart for the empirical real exchange rate, since the average prices \( \widehat{P}_t \) and \( \widehat{P}_t^* \) best represent the corresponding empirical CPI levels in the presence of endogenous product variety (Broda and Weinstein, 2003). To this end, I break down the welfare-based price indexes \( P_t \) and \( P_t^* \) into components reflecting (a) product variety and (b) average prices as \( P_t = (N_{D,t} + N_{V,t} + N_{H,t}^*)^{1/\theta} \widehat{P}_t \) for the North and \( P_t^* = (N_{D,t}^* + N_{H,t}^*)^{1/\theta} \widehat{P}_t^* \) for the South. The resulting expression for the CPI-based real exchange rate is:

\[
Q_{CPI,t}^{1-\theta} = \left( \frac{N_{D,t} + N_{V,t} + N_{H,t}^*}{N_{D,t}^* + N_{H,t}^*} \right) \left( \frac{N_{D,t}^* \left( \frac{TOL_t}{z_{D,t}} \right)^{1-\theta} + N_{H,t} \left( \frac{\tau_t}{z_{H,t}} \right)^{1-\theta}}{N_{D,t} \left( \frac{1}{z_{D,t}} \right)^{1-\theta} + N_{V,t} \left( \frac{\tau_t TOL_t}{z_{V,t}} \right)^{1-\theta} + N_{H,t} \left[ \frac{\tau_t TOL_t}{z_{H,t}} \right]^{1-\theta}} \right)
\]

where the terms of labor \( TOL_t = \frac{Q_t w_t}{w_t} \) measures the cost of effective labor in the South relative to the North; the iceberg trade costs \( \tau_t \) and \( \tau_t^* \) affect the imports of the North and the South, respectively.
Analytical results  The log-linearized version of (25) is:

\[ Q_{CPI,t} = \left[ s_D - s_V + s_D^* - 1 \right] TOL_t + \]

\[ + (s_D - s_V) \tilde{z}_{D,t} + s_V \tilde{z}_{V,t} - (1 - \alpha) s_V \tilde{\tau}_t + \]

\[ + (1 - s_D) \left( \tilde{z}_{H,t}^* - \tilde{\tau}_t \right) - \left( 1 - s_D^* \right) \left( \tilde{z}_{H,t} - \tilde{\tau}_t^* \right) + \]

\[ + \frac{1}{\theta - 1} \left( s_V - \frac{N_V}{N_D + N_V + N_H} \right) \left( \tilde{N}_{V,t} - \tilde{N}_{H,t}^* \right) + \]

\[ + \frac{1}{\theta - 1} \left[ - \left( \frac{N_D^*}{N_D + N_V + N_H} - s_D^* \right) \left( \tilde{N}_{D,t} - \tilde{N}_{H,t}^* \right) - \right. \]

\[ \left. \left( \frac{N_D^*}{N_D + N_V + N_H} - (s_D - s_V) \right) \left( \tilde{N}_{D,t} - \tilde{N}_{H,t}^* \right) \right], \]

where the variables marked with a hat denote percent deviations from their steady states. Parameter \( s_D \) is the steady-state share of spending in the North on varieties produced by Northern firms both domestically and offshore; \( s_V \) is the steady-state share of spending in the North only on varieties produced by Northern firms offshore (I shut down offshoring when \( s_V = 0 \) and \( N_V = 0 \)); \( s_D^* \) is the steady-state share of spending in the South on varieties produced by Southern firms. The calibration ensures that: (a) \( (s_D - s_V) + s_D^* > 1 \), as the domestically-produced varieties represent more than 50 percent of consumption spending in each country; (b) \( \frac{N_D}{N_D + N_V + N_H} - (s_D - s_V) > 0 \) and \( \frac{N_D^*}{N_D + N_V + N_H} - s_D^* > 0 \), i.e. the market shares of varieties produced domestically by the less productive firms are smaller than their fraction in the total number of varieties; and (c) \( s_V - \frac{N_V}{N_D + N_V + N_H} > 0 \), i.e. the market share of varieties produced offshore by the more productive Northern firms is larger than their fraction in the total number of varieties available in the North. The model implies that the relatively more productive offshoring firms have larger market shares than their less productive domestic counterparts, which is in line with the empirical evidence in Kurz (2006).

The log-linearized form of (25) outlines five channels (labeled C1-C5 in the log-linearized equation above) through which the CPI-based real exchange rate is affected by: (1) changes in the price of non-traded varieties induced by fluctuations in the terms of labor \( (TOL_t) \); (2) changes in the price of offshored varieties caused by fluctuations in the average productivity of offshoring firms \( \left( \tilde{z}_{V,t} \right) \) and in the magnitude of trade costs \( (\tilde{\tau}_t) \); (3) changes in relative import prices triggered by fluctuations in the average productivity of Southern exporters.
(\(\tilde{z}_{H,t}\)) relative to that of their Northern counterparts (\(\tilde{z}_{H,t}\)); (4) changes in the number of varieties produced by the Northern firms offshore (\(\tilde{N}_{v,t}\)) relative to the number of imported varieties produced by Southern firms (\(\tilde{N}_{H,t}^{*}\)); and (5) changes in the number of domestic varieties (\(\tilde{N}_{D,t}\)) relative to the number of imported Southern varieties (\(\tilde{N}_{H,t}^{*}\)).

**Impulse responses** Offshoring dampens the appreciation of the real exchange rate that follows an increase in aggregate productivity in the North. The effect occurs through channels C1, C3 and C4 defined above. Figure 8 describes the effect of each channel on the real exchange rate in the baseline model of offshoring with financial autarky: It shows the impulse responses of the real exchange rate and related variables to a transitory one-percent increase in aggregate productivity in the North, when productivity follows the autoregressive process

\[
\log Z_{t+1} = \rho \log Z_t + u_t \quad \text{(with persistence } \rho = 0.9)\]

**(C1) Changes in the price of non-traded varieties.** In the model with no offshoring (dashed lines), a productivity increase in the North encourages firm entry and leads to the appreciation of the terms of labor in the medium run (i.e. \(TOL_t\) decreases). This causes the average price of non-traded varieties in the North to increase relative to that in the South, and thus leads to the appreciation of the CPI-based real exchange rate (i.e. \(Q_{CPI,t}\) decreases).

Offshoring (thick solid line) dampens the appreciation of the real exchange rate through this channel in two ways: (a) Offshoring dampens the appreciation of the terms of labor (i.e. causes \(TOL_t\) to decrease by less) relative to the alternative models with no offshoring (dashed line) or to the model with a fixed productivity cutoff (thin solid line), because the relocation of production transfers upward pressure from the domestic to the foreign wage. (b) Offshoring also reduces the impact of the terms of labor on the real exchange rate, since it reduces the share of non-traded varieties in total spending. The effect is illustrated by the coefficient on \(\tilde{TOL}_t\) in channel C1 (\(s_D - s_V + s_D - 1\), which decreases with the share of offshored varieties in total spending (\(s_V\)).

**(C2) Changes in the price of offshored varieties.** On impact, the increase in the Southern wage causes the number of offshoring firms to drop and their average productivity to increase. However, offshoring becomes an increasingly profitable option in the medium run
due to the gradual appreciation of the terms of labor. As a result, the number of offshoring firms rises, their average productivity $\tilde{z}_{V,t}$ declines, and their average price increases over time. Thus, offshoring contributes to the appreciation of the real exchange rate in the medium run through this channel.\footnote{Exogenous policy changes can also affect the price of goods produced offshore. For instance, tariff cuts for the varieties produced offshore, reflected by a decrease in $\tau_t$, would dampen the appreciation of the CPI-based real exchange rate.}

(C3) Changes in relative import prices. In the absence of offshoring, the appreciation of the terms of labor reduces the export profitability of the Northern firms relative to that of their Southern counterparts. As a result, the average productivity of the surviving Northern exporters ($\tilde{z}_{H,t}$) increases relative to that of Southern exporters ($\tilde{z}_{H,t}$), and their average price declines. This causes an increase in the average price of imports in the North relative to the South, which leads to the appreciation of the real exchange rate.

Offshoring reverses this effect. As the relocation of production places upward pressure on the Southern wage, offshoring harms the export competitiveness of Southern firms, and causes the average productivity of the surviving Southern exporters ($\tilde{z}_{V,t}$) to increase relative to that of their Northern counterparts ($\tilde{z}_{H,t}$). In contrast to the model with exports only, offshoring causes a decline in the average price of imports in the North relative to the South, and therefore dampens the appreciation of the real exchange rate.

(C4) Expenditure switching from imports towards offshored varieties. As offshoring reduces the competitiveness of Southern exports, consumers in the North switch their expenditure away from the less competitive Southern varieties ($N_{H,t}^*$ decreases) and towards the relatively cheaper varieties produced offshore ($N_{V,t}$ increases). The result dampens the appreciation of the real exchange rate.

(C5) Expenditure switching from imports towards domestic varieties. Firm entry in the North causes the number of domestic varieties ($N_{D,t}$) to increase relative to that of imported foreign varieties ($N_{H,t}^*$). Thus, consumers switch their expenditure away from imported varieties and towards the varieties produced domestically by the relatively less productive firms, which are available at relatively higher prices. This channel works
towards the appreciation of the real exchange rate.

Figure 8. Impulse responses to a one-percent shock to aggregate productivity in the North, baseline model of offshoring with adjustable productivity cutoff (thick solid line); alternative models with fixed productivity cutoff (thin solid line) and no offshoring (dashed line).

6 Conclusion

I study the effect of offshoring on the cross-country transmission of business cycles, while focusing on its extensive and intensive margins as separate transmission mechanisms. The
paper considers a model of offshoring with heterogeneous firms that is consistent with the empirical patterns of offshoring from U.S. manufacturing to Mexico’s maquiladora sector. First, following an aggregate productivity increase in the country of origin (North), the value added per offshoring firm jumps on impact and then returns to its initial steady state. However, domestic firm entry causes a gradual increase in the relative cost of effective labor (i.e. the wage adjusted by aggregate productivity), which in turn generates a gradual increase in the number of offshoring firms (the extensive margin), as in the data. Second, offshoring enhances the cross-country co-movement of output relative to the model with endogenous exports. The result is consistent with the empirical regularity documented in Burstein, Kurz, and Tesar (2009) that country pairs with larger shares of offshoring-related trade in bilateral trade also exhibit larger correlations of manufacturing output. Third, offshoring reduces the appreciation of the real exchange rate that follows an aggregate productivity improvement in the parent country, and thus dampens the Harrod-Balassa-Samuelson effect that occurs in the framework with firm entry and endogenously traded varieties.

There are a number of possible extensions to the model considered in this paper. First, the framework is useful to analyze the impact of offshore production on employment in the parent and the host countries. Second, a possible extension with rich policy implications would involve the study of interactions between offshore production and labor migration in a unified framework, in which both offshoring and labor mobility are driven by fluctuations in the relative wage across countries. Third, while this paper studies the fluctuations of offshoring over the business cycle, further research should address the long-run developments in offshore production and its implications for U.S. manufacturing.

References


A Appendix

A.1 Offshoring with Financial Autarky

Table A.1. Model Summary

<table>
<thead>
<tr>
<th>Category</th>
<th>Formula</th>
</tr>
</thead>
<tbody>
<tr>
<td>Euler equation, bonds</td>
<td>$C_t^{-\gamma} = \beta (1 + r_{t+1}) E_t \left[ C_{t+1}^{-\gamma} \right]$</td>
</tr>
<tr>
<td></td>
<td>$C_t^{<em>\gamma} = \beta (1 + r_{t+1}^</em>) E_t \left[ C_{t+1}^{*\gamma} \right]$</td>
</tr>
<tr>
<td>Euler equation, stocks</td>
<td>$\tilde{v}<em>t = \beta(1-\delta)E_t \left( C</em>{t+1}^{<em>\gamma} \right)^{-\gamma} (d_{t+1}^</em> + \tilde{v}_{t+1}^*)$</td>
</tr>
<tr>
<td></td>
<td>$\tilde{v}<em>t^* = \beta*(1-\delta*)E_t \left( C</em>{t+1}^{<em>\gamma} \right)^{-\gamma} (d_{t+1}^</em> + \tilde{v}_{t+1}^*)$</td>
</tr>
<tr>
<td>Free entry</td>
<td>$\tilde{v}<em>t = \frac{f</em>{E,t}}{Z_t}$</td>
</tr>
<tr>
<td></td>
<td>$\tilde{v}<em>t^* = \frac{f</em>{E,t}^<em>}{Z_t^</em>}$</td>
</tr>
<tr>
<td>Rule of motion, total number of firms</td>
<td>$N_{t+1} = (1-\delta)(N_t + N_{E,t})$</td>
</tr>
<tr>
<td></td>
<td>$N_{D,t+1}^* = (1-\delta)(N_{D,t}^* + N_{E,t}^*)$</td>
</tr>
<tr>
<td>Aggregate accounting</td>
<td>$C_t + N_{E,t} \tilde{v}_t = w_t L + N_t \tilde{d}_t$</td>
</tr>
<tr>
<td></td>
<td>$C_t^* + N_{E,t}^* \tilde{v}<em>t^* = w_t^* L^* + N</em>{D,t}^* \tilde{d}_t^*$</td>
</tr>
<tr>
<td>Consumption price index</td>
<td>$1 = N_{D,t}^* (\tilde{\rho}<em>{D,t}^*)^{1-\theta} + N</em>{V,t} (\tilde{\rho}<em>{V,t})^{1-\theta} + N</em>{H,t}^* (\tilde{\rho}_{H,t}^*)^{1-\theta}$</td>
</tr>
<tr>
<td></td>
<td>$1 = N_{D,t}^* (\tilde{\rho}<em>{D,t})^{1-\theta} + N</em>{H,t}^* (\tilde{\rho}_{H,t})^{1-\theta}$</td>
</tr>
<tr>
<td>Total profits</td>
<td>$N_t \tilde{d}<em>t = N</em>{D,t}^* \tilde{d}<em>{D,t}^* + N</em>{V,t} \tilde{d}<em>{V,t}^* + N</em>{H,t} \tilde{d}_{H,t}$</td>
</tr>
<tr>
<td></td>
<td>$N_{D,t}^* \tilde{d}<em>t^* = N</em>{D,t}^* \tilde{d}<em>{D,t}^* + N</em>{H,t}^* \tilde{d}_{H,t}$</td>
</tr>
<tr>
<td>Total number of firms (North)</td>
<td>$N_t = N_{D,t} + N_{V,t}$</td>
</tr>
<tr>
<td>Offshoring profits link (North)</td>
<td>$\tilde{d}<em>{V,t} = \frac{k}{k-(\theta-1)} \left( \frac{\tilde{v}</em>{V,t}}{Z_{D,t}} \right)^{\theta-1} \tilde{d}<em>{D,t} + \frac{\theta-1}{k-(\theta-1)} f</em>{H,t} w_t^* Q_t$</td>
</tr>
<tr>
<td>Export profits link</td>
<td>$\tilde{d}<em>{H,t} = \frac{\theta-1}{k-(\theta-1)} f</em>{H,t} w_t^* Z_t$</td>
</tr>
<tr>
<td></td>
<td>$\tilde{d}<em>{H,t}^* = \frac{\theta-1}{k-(\theta-1)} f</em>{H,t}^* w_t^* Z_t^*$</td>
</tr>
<tr>
<td>Avrg. prod. of domestic producers (North)</td>
<td>$\tilde{z}<em>{D,t} = \nu z</em>{min} \tilde{z}<em>{V,t} \left[ \frac{k-(\theta-1)}{z</em>{V,t} - k-(\theta-1) \tilde{z}<em>{V,t} - x</em>{min}} \right]^{1/k}$</td>
</tr>
<tr>
<td>Avrg. prod. of offshore producers (North)</td>
<td>$\tilde{z}<em>{V,t} = \nu z</em>{min} \left( \frac{N_t}{N_{V,t}} \right)^{1/k}$</td>
</tr>
<tr>
<td>Avrg. productivity of exporters</td>
<td>$\tilde{z}<em>{H,t} = \nu z</em>{min} \left( \frac{N_t}{N_{H,t}} \right)^{1/k}$</td>
</tr>
<tr>
<td></td>
<td>$\tilde{z}<em>{H,t}^* = \nu z</em>{min} \left( \frac{N_{D,t}^<em>}{N_{H,t}^</em>} \right)^{1/k}$</td>
</tr>
<tr>
<td>Balanced trade</td>
<td>$N_{H,t} (\tilde{\rho}<em>{H,t})^{1-\theta} C_t^* Q_t + N</em>{V,t} \tilde{d}<em>{V,t} = \frac{N</em>{V,t} (\tilde{\rho}<em>{V,t})^{1-\theta} C_t + N</em>{H,t}^* (\tilde{\rho}_{H,t}^*)^{1-\theta} C_t}{1}$</td>
</tr>
</tbody>
</table>
A.2 Offshoring with International Bond Trading

I introduce international bond trading in the model with offshoring. International asset markets are incomplete, as the representative household in each economy holds risk-free, country-specific bonds from both the North and the South. Each type of bonds provides a real return denominated in units of the issuing country’s consumption basket. Quadratic costs of adjustment for bond holdings ensure stationarity for the net foreign assets in the presence of temporary shocks.

The representative household in the North maximizes inter-temporal utility subject to:

\[(d_t + \tilde{u}_t)N_t x_t + w_t L + (1 + r_t) B_{h,t} + (1 + r_t^*) Q_t B_{f,t} + T_t \geq C_t + \tilde{v}_t (N_t + N_{E,t}) x_{t+1} + B_{h,t+1} + \frac{\pi}{2} (B_{h,t+1})^2 + Q_t B_{f,t+1} + \frac{\pi}{2} Q_t (B_{f,t+1})^2,\]

where \(r_t\) and \(r_t^*\) are the rates of return of the North and South-specific bonds; \((1+r_t)B_{h,t}\) and \((1+r_t^*)Q_t B_{f,t}\) denote the principal and interest income from each type of bonds; \(\frac{\pi}{2} (B_{h,t+1})^2\) and \(\frac{\pi}{2} Q_t (B_{f,t+1})^2\) are the adjustment costs for each type of bond holdings; \(T_t\) is the fee rebate. Setting \(\pi = 0.005\), I add the two Euler equations for bonds to the baseline model:

\[1 + \pi B_{h,t+1} = \beta(1 + r_{t+1})E_t \left( \frac{C_{t+1}}{C_t} \right)^{-\gamma},\]

\[1 + \pi B_{f,t+1} = \beta(1 + r_{t+1}^*)E_t \frac{Q_{t+1}}{Q_t} \left( \frac{C_{t+1}^*}{C_t^*} \right)^{-\gamma} .\]

For the Southern representative household, the Euler equations for bonds are:

\[1 + \pi B_{h,t+1}^* = \beta^*(1 + r_{t+1})E_t \frac{Q_{t+1}}{Q_t} \left( \frac{C_{t+1}^*}{C_t^*} \right)^{-\gamma} ,\]

\[1 + \pi B_{f,t+1}^* = \beta^*(1 + r_{t+1}^*)E_t \left( \frac{C_{t+1}^*}{C_t^*} \right)^{-\gamma} .\]

The market clearing conditions for bonds are:

\[B_{h,t+1} + B_{h,t+1}^* = 0,\]

\[B_{f,t+1} + B_{f,t+1}^* = 0.\]
Thus, financial integration through trade in bonds adds four new variables \((B_{h,t}, B_{f,t}, B_{h,t}^*, B_{f,t}^*)\) and six new equations (27, 28, 29, 30, 31 and 32) while removing the original two Euler equations from the baseline model with financial autarky. Also, the new expressions for aggregate accounting in the North and the South are:

\[
C_t + N_{E,t} n_t + B_{h,t+1} + Q_t B_{f,t+1} = w_t L + N_t d_t + (1 + r_t) B_{h,t} + (1 + r^*_t) Q_t B_{f,t},
\]

\[
C_t^* + N_{E,t}^* n_t^* + Q_t^{-1} B_{h,t+1}^* + B_{f,t+1}^* = w_t^* L^* + N_{D,t}^* d_t^* + (1 + r_t) Q_t^{-1} B_{h,t}^* + (1 + r^*_t) B_{f,t}^*.
\]

Finally, I replace the balanced current account condition from financial autarky with the expression for the balance of international payments:

\[
TB_t + \underbrace{N_{V,t} d_{V,t}}_{\text{Repatriated profits}} + r_t B_{h,t} + r_t^* Q_t B_{f,t} = (B_{h,t+1} - B_{h,t}) + Q_t (B_{f,t+1} - B_{f,t})
\]

which shows that the current account balance (trade balance plus repatriated profits plus investment income) must equal the negative of the financial account balance (the change in bond holdings).

### A.3 Average Firm-Specific Productivity Levels

**Firms producing offshore** I obtain the average productivity of the Northern firms that produce offshore by integrating over the upper range of the support interval, above the offshoring productivity cutoff:

\[
\tilde{z}_{V,t} = \left[ \frac{1}{1 - G(z_{V,t})} \int_{z_{V,t}}^{\infty} z^{\theta-1} g(z) dz \right]^\frac{1}{\theta-1} = \left[ \left( \frac{z_{V,t}}{z_{\min}} \right)^k \int_{z_{V,t}}^{\infty} z^{\theta-1-k} dz \right]^\frac{1}{\theta-1} =
\]

\[
= \left( \frac{k}{k - (\theta - 1)} \right) z_{V,t} - \frac{k}{k - (\theta - 1)} \frac{z_{V,t}^{\theta-1-k}}{\theta-1} = \nu z_{V,t},
\]

\[
\text{where } \nu \equiv \left[ \frac{k}{k - (\theta - 1)} \right]^{\frac{1}{\theta-1}}.
\]
Firms producing domestically  The average productivity of the Northern firms that produce domestically is:

\[
\tilde{z}_{D,t} = \left[ \frac{1}{G(z_{V,t})} \int_{z_{min}}^{z_{V,t}} z^\theta - 1 g(z) dz \right] \frac{1}{\theta - 1} = \frac{z_{k}^k z_{V,t}^{k-1 - k}}{z_{V,t}^{\theta (1-k)} z_{min}^k} \left[ \frac{z_{V,t}^{\theta - 1 - k}}{z_{min}^{\theta - 1 - k}} - \frac{z_{V,t}^{\theta - 1}}{z_{min}^{\theta - 1}} \right] \frac{1}{\theta - 1} = \\
\nu z_{V,t} \left[ \frac{z_{V,t}^{\theta - 1} z_{min}^{\theta - 1}}{z_{V,t}^{\theta - 1} z_{min}^{\theta - 1}} - \frac{z_{V,t}^{\theta - 1}}{z_{min}^{\theta - 1}} \right] \frac{1}{\theta - 1}.
\]

A.4 Average Profits from Domestic and Offshore Production

The average profit of the Northern firms producing domestically is:

\[
\bar{d}_{D,t} = d_{D,t}(\tilde{z}_{D,t}) = \frac{1}{\theta} \left[ \frac{\theta}{\theta - 1} \frac{w_t}{Z_t^{D,t}} \right] \left[ \frac{\theta}{\theta - 1} \frac{w_t}{Z_t^{D,t}} \right]^{1-\theta} C_t = \frac{1}{\theta} \left[ \frac{\theta}{\theta - 1} \frac{w_t}{Z_t^{D,t}} \right]^{1-\theta} C_t \tilde{z}_{D,t}^{\theta - 1} = \\
= \frac{1}{\theta} \left[ \frac{\theta}{\theta - 1} \frac{w_t}{Z_t^{D,t}} \right]^{1-\theta} C_t (\nu z_{min} z_{V,t})^{\theta - 1} \left[ \frac{z_{k}^{\theta - (1-k)} z_{min}^{\theta - 1}}{z_{V,t}^{\theta - (1-k)} z_{min}^{\theta - 1}} - \frac{z_{k}^{\theta - 1} z_{min}^{\theta - 1}}{z_{V,t}^{\theta - 1} z_{min}^{\theta - 1}} \right] \frac{1}{\theta - 1} \left[ \frac{\theta}{\theta - 1} \frac{w_t}{Z_t^{D,t}} \right]^{1-\theta} C_t z_{V,t}^{\theta - 1} = \\
= \frac{1}{\theta} \left[ \frac{\theta}{\theta - 1} \frac{w_t}{Z_t^{D,t}} \right]^{1-\theta} C_t (\nu z_{min} z_{V,t})^{\theta - 1} \left[ \frac{z_{k}^{\theta - (1-k)} z_{min}^{\theta - 1}}{z_{V,t}^{\theta - (1-k)} z_{min}^{\theta - 1}} - \frac{z_{k}^{\theta - 1} z_{min}^{\theta - 1}}{z_{V,t}^{\theta - 1} z_{min}^{\theta - 1}} \right] ^{1-\theta} C_t z_{V,t}^{\theta - 1} = \\
d_{D,t}(z_{V,t}) \left[ \frac{z_{k}^{\theta - (1-k)} z_{min}^{\theta - 1}}{z_{V,t}^{\theta - (1-k)} z_{min}^{\theta - 1}} - \frac{z_{k}^{\theta - 1} z_{min}^{\theta - 1}}{z_{V,t}^{\theta - 1} z_{min}^{\theta - 1}} \right] \left[ \frac{\theta}{\theta - 1} \frac{w_t}{Z_t^{D,t}} \right]^{1-\theta} C_t z_{V,t}^{\theta - 1}.
\]
The average profit of the Northern firms producing offshore is:

\[
\tilde{d}_{V,t} = d_{V,t}(\tilde{z}_{V,t}) = \frac{1}{\theta} \left[ \frac{\theta}{\theta - 1} w^*_t Q_t \right]^{1-\theta} C_t - f_V \frac{w^*_t Q_t}{Z^*_t} = \\
= \frac{1}{\theta} \left[ \frac{\theta}{\theta - 1} \frac{w^*_t Q_t}{Z^*_t} \right]^{1-\theta} C_t \tilde{z}_{V,t}^{\theta-1} - f_V \frac{w^*_t Q_t}{Z^*_t} = \\
= \left\{ \frac{1}{\theta} \left[ \frac{\theta}{\theta - 1} \frac{w^*_t Q_t}{Z^*_t} \right]^{1-\theta} C_t - f_V \frac{w^*_t Q_t}{Z^*_t} \right\} \nu^{\theta-1} + \\
\left. \right. \\
+ (\nu^{\theta-1} - 1) f_V \frac{w^*_t Q_t}{Z^*_t} = \\
= d_{V,t}(z_{V,t}) \nu^{\theta-1} + \frac{\theta - 1}{k - (\theta - 1)} f_V \frac{w^*_t Q_t}{Z^*_t}. \\
\tag{41}
\]

The Northern firm with productivity equal to the cutoff \(z_{V,t}\) is indifferent between producing domestically or offshore. I use the equality of profits at the productivity cutoff, \(d_{D,t}(z_{V,t}) = d_{V,t}(z_{V,t})\), and equations (40) and (41) to write the link between the two average profits as:

\[
\tilde{d}_{V,t} = \left( \frac{1}{\nu z_{\min}} \right)^{\theta-1} \left[ \frac{\tilde{z}_{V,t}^{k-(\theta-1)} - \tilde{z}_{V,t}^{k-(\theta-1)}}{z_{V,t}^{k} - z_{\min}^{k}} \right]^{1-\theta} \tilde{d}_{D,t} \nu^{\theta-1} + \frac{\theta - 1}{k - (\theta - 1)} f_V \frac{w^*_t Q_t}{Z^*_t} = \\
= \tilde{z}_{\min}^{1-\theta} \left[ \frac{\tilde{z}_{V,t}^{k-(\theta-1)} - \tilde{z}_{V,t}^{k-(\theta-1)}}{z_{V,t}^{k} - z_{\min}^{k}} \right]^{1-\theta} \tilde{d}_{D,t} + \frac{\theta - 1}{k - (\theta - 1)} f_V \frac{w^*_t Q_t}{Z^*_t} = \\
= \frac{k}{k - (\theta - 1)} \left( \frac{\tilde{z}_{V,t}^{\theta-1}}{\tilde{z}_{D,t}^{\theta-1}} \right) \tilde{d}_{D,t}^{\theta-1} + \frac{\theta - 1}{k - (\theta - 1)} f_V \frac{w^*_t Q_t}{Z^*_t}. \\
\tag{42}
\]

### A.5 The Real Exchange Rate

The CPI-based real exchange rate is \(Q^{1-\theta}_{\text{CPI},t} = \tilde{N}_t \left( \frac{P^*_{\text{CPI},t}}{P^*_{t}} \right)^{\frac{1}{\theta}}\), in which \(\tilde{N}_t \equiv N_{D,t} + N_{V,t} + N_{H,t}\) and \(\tilde{N}^*_t \equiv N^*_{D,t} + N^*_{H,t}\) are the total number of varieties available to consumers in the North and the South. For the price indexes \(P_t\) and \(P^*_t\), I use the average price formulas implied by the broader framework of offshoring described in footnotes 11 and 16 to obtain:
Using all of the above, I log-linearize the CPI-based real exchange rate:

\[
Q_{CPI,t}^{1-\theta} = \frac{\tilde{N}_t}{N_t} \frac{N_{D,t}^* \left( \tilde{\rho}_{D,t}^* P_t^* \varepsilon_t \right)^{1-\theta} + N_{H,t} \left( \tilde{\rho}_{H,t} P_t^* \varepsilon_t \right)^{1-\theta}}{N_{D,t}^* \left( \tilde{\rho}_{D,t} P_t \varepsilon_t \right)^{1-\theta} + N_{V,t} \left( \tilde{\rho}_{V,t} P_t \varepsilon_t \right)^{1-\theta} + N_{H,t}^* \left[ \tilde{\rho}_{H,t} P_t \varepsilon_t \right]^{1-\theta}} = \\
= \frac{\tilde{N}_t}{N_t} \frac{N_{D,t}^* \left( \frac{w_t^* P_t^* \varepsilon_t}{Z_t^D,t} \right)^{1-\theta} + N_{V,t} \left[ \left( \tau_{TOL,t} \right)^{1-\alpha} w_t P_t \right]^{1-\theta} + N_{H,t}^* \left[ \left( \tau_{TOL,t} \right)^{\alpha} w_t P_t \right]^{1-\theta}}{N_{D,t}^* \left( \frac{w_t P_t \varepsilon_t}{Z_t^D,t} \right)^{1-\theta} + N_{V,t} \left[ \left( \tau_{TOL,t} \right)^{1-\alpha} w_t P_t \right]^{1-\theta} + N_{H,t}^* \left[ \left( \tau_{TOL,t} \right)^{\alpha} w_t P_t \right]^{1-\theta}} = \\
= \frac{\tilde{N}_t}{N_t} \frac{N_{D,t}^* \left[ \frac{TOL_t}{Z_t^D,t} \right]^{1-\theta} + N_{V,t} \left[ \left( \tau_{TOL,t} \right)^{1-\alpha} \frac{w_t P_t}{Z_t^V,t} \right]^{1-\theta} + N_{H,t}^* \left[ \left( \tau_{TOL,t} \right)^{\alpha} \frac{w_t P_t}{Z_t^H,t} \right]^{1-\theta}}{N_{D,t}^* \left[ \frac{TOL_t}{Z_t^D,t} \right]^{1-\theta} + N_{V,t} \left[ \left( \tau_{TOL,t} \right)^{1-\alpha} \frac{w_t P_t}{Z_t^V,t} \right]^{1-\theta} + N_{H,t}^* \left[ \left( \tau_{TOL,t} \right)^{\alpha} \frac{w_t P_t}{Z_t^H,t} \right]^{1-\theta}}. \tag{43}
\]

In what follows I use the notation \( s_D \equiv N_D \left[ \frac{w}{Z^D} \right]^{1-\theta} + N_V \left[ \frac{w}{Z^V} \left( \tau TOL \right)^{1-\alpha} \right]^{1-\theta} \) to denote the steady-state share of spending in the North on varieties produced by the Northern firms both domestically and offshore. Expression \( s_V \equiv N_V \left[ \frac{w}{Z^V} \left( \tau TOL \right)^{1-\alpha} \right]^{1-\theta} \) denotes the steady-state share of spending in the North on goods produced by the Northern firms offshore only. (Note that \( s_D - s_V > 0 \)) Expression \( s_D^* \equiv N_D^* \left[ \frac{w^* Q}{Z^D} \right]^{1-\theta} \) denotes the steady-state share of spending in the South on goods produced by the Southern firms domestically. The average productivity of the Southern firms producing domestically \( z_D^* \) is constant over time.

Using all of the above, I log-linearize the CPI-based real exchange rate:

\[
(1-\theta) \hat{Q}_{CPI,t} = s_D^* \left[ \tilde{N}_{D,t}^* - \tilde{N}_t + (1-\theta) \tilde{TOL}_t \right] + \\
+ (1-s_D^*) \left[ \tilde{N}_{H,t}^* - \tilde{N}_t + (1-\theta) \left( \eta_{\tilde{T}_t}^* + (1-\eta) \tilde{TOL}_t - \tilde{z}_H,t \right) \right] - \\
- (s_D - s_V) \left[ \tilde{N}_{D,t} - \tilde{N}_t + (1-\theta) \tilde{z}_D,t \right] - \\
- s_V \left[ \tilde{N}_{V,t} - \tilde{N}_t + (1-\theta) \left( (1-\alpha) \tilde{TOL}_t - \tilde{z}_V,t \right) \right] - \\
- (1-s_D) \left[ \tilde{N}_{H,t}^* - \tilde{N}_t + (1-\theta) \left( \eta^* \tilde{T}_t + \tilde{TOL}_t - \tilde{z}_H,t \right) \right]. \tag{44}
\]

I set \( \eta = \eta^* = 1 \) so that my model of offshoring nests the framework with endogenous exports in Ghironi and Melitz (2005): In addition to the firms that produce domestically or offshore for their home market, a subset of firms from each economy also serve the foreign markets through exports. The log-linearized expression for the CPI-based real exchange rate
becomes:

\[
\hat{Q}_{CPI,t} = [s_D - (1 - \alpha)s_V + s_D^* - 1] TOL_t + \\
+ (s_D - s_V) \hat{z}_{D,t} + s_V \hat{z}_{V,t} - (1 - \alpha)s_V \hat{\tau}_t + \\
+ (1 - s_D) \left( \hat{z}_{H,t} - \hat{\tau}_t \right) - (1 - s_D^*) \left( \hat{z}_{H,t} - \hat{\tau}_t^* \right) + \\
+ \frac{1}{\theta - 1} \left( s_V - \frac{N_V}{N} \right) \left( \hat{N}_{V,t} - \hat{N}_{H,t}^* \right) + \\
+ \frac{1}{\theta - 1} \left[ \left( \frac{N_D^*}{N^*} - s_D^* \right) \left( \hat{N}_{D,t} - \hat{N}_{H,t} \right) - \left( \frac{N_D}{N} - (s_D - s_V) \right) \left( \hat{N}_{D,t} - \hat{N}_{H,t}^* \right) \right].
\] (45)

A.6 Asymmetric Steady State

In this section I provide the steady state solution for the model of offshoring in the presence of a cross-country asymmetry in the cost of effective labor (i.e. the terms of labor \( TOL < 1 \)). To this end, I use the broad framework described in footnotes 11 and 16 of the paper that nests both the baseline model of offshoring (for \( \alpha = 0, \eta = 1 \)) and the model with exports only in Ghironi and Melitz (2005) (for \( \alpha = 1, \eta = 1 \)).

I obtain a numerical solution for the unique steady state using a non-linear system of 12 equations in 12 unknowns. The equations are described by 46-57 below. The unknowns are the steady state values of \( z_V \) (the offshoring productivity cutoff in the North), \( z_H \) (the exporting productivity cutoff in the North), \( TOL \) (the terms of labor), \( \frac{C}{C^*Q} \) (the real consumption ratio in units of the same consumption basket), \( Q \) (the real exchange rate), \( \frac{\bar{\rho}_D}{w}, \frac{\bar{\rho}_V}{w}, \frac{\bar{\rho}_H}{w} \) (the real profits from domestic and offshore production for the domestic market, as well as the profit from production for the export market, each divided by the real wage in the North), \( z_H^* \) (the exporting productivity cutoff in the South), \( \bar{\rho}_H \) (the average price of Northern exports), \( \bar{\rho}_H^* \) (the average price of Southern exports), and \( \frac{N}{N_D} \) (the ratio of the number of firms in the North and the South). Subsequently, I use the numerical solutions for these 12 variables to compute the steady state values for the remaining variables.

The following price and profit formulas (in which the aggregate productivity is \( Z = Z^* = 1 \)) are useful in computing the steady state solution:
### Table A.2. Average Prices and Profits

#### Average Prices

| 1. Domestic production, North | $\tilde{\rho}_D = \frac{\theta}{\theta - 1} \frac{w_D}{z_D}$ |
| 2. Domestic production, South | $\tilde{\rho}^*_D = \frac{\theta}{\theta - 1} \frac{w^*_D}{z^*_D}$ |
| 3. Offshore production ($\alpha = 0$) | $\tilde{\rho}_V = \frac{\theta}{\theta - 1} \frac{w^*_V}{z^*_V} (\tau_{TOL})^{1-\alpha}$ |
| 4. Exports ($\eta = 1$) or horiz. FDI ($\eta = 0$), North | $\tilde{\rho}_H = \frac{\theta}{\theta - 1} \frac{\tau^*_V w TOL^{1-\eta}}{\tau_{TOL}}$ |
| 5. Exports ($\eta^* = 1$) or horiz. FDI ($\eta^* = 0$), South | $\tilde{\rho}^*_H = \frac{\theta}{\theta - 1} \frac{\tau_{TOL}}{z^*_H} (\frac{1}{TOL})^{1-\eta^*}$ |

#### Average Profits

| 1. Domestic production, North | $\tilde{d}_{D,t} = \frac{1}{\theta} (\tilde{\rho}_{D,t})^{1-\theta} C_t$ |
| 2. Domestic production, South | $\tilde{d}^*_{D,t} = \frac{1}{\theta} (\tilde{\rho}^*_{D,t})^{1-\theta} C^*_t$ |
| 3. Offshore production ($\alpha = 0$) | $\tilde{d}_{V,t} = \frac{1}{\theta} (\tilde{\rho}_{V,t})^{1-\theta} C^*_t - f_{V} w TOL^{1-\alpha}$ |
| 4. Exports ($\eta = 1$) or horiz. FDI ($\eta = 0$), North | $\tilde{d}_{H,t} = \frac{1}{\theta} (\tilde{\rho}_{H,t})^{1-\theta} C^*_t Q_t - f_{H} w TOL^{1-\eta}$ |
| 5. Exports ($\eta^* = 1$) or horiz. FDI ($\eta^* = 0$), South | $\tilde{d}^*_{H,t} = \frac{1}{\theta} (\tilde{\rho}^*_{H,t})^{1-\theta} C^*_t Q_t^{1-\eta^*}$ $- f^*_H w^* \left( \frac{w^*_D}{z^*_D} \right) \left( \frac{1}{TOL} \right)^{1-\eta^*}$ |

Introducing $v = \frac{\beta(1-\delta)}{1-\beta(1-\delta)} d$, $N_E = \frac{\delta}{1-\delta} N$, and $v = f_{w} w$ in the expression for the total profits in the North (see Table A.1), the first equation of the system is:

$$\frac{1 - \beta(1-\delta)}{\beta(1-\delta)} f_E = \frac{N_D}{N} \tilde{d}_D + \frac{N_V}{N} \tilde{d}_V + \frac{N_H}{N} \tilde{d}_H,$$  \hspace{1cm} (46)

where $\frac{N_H}{N} = \left( \frac{1}{z_H} \right)^k$, $\frac{N_D}{N} = 1 - \left( \frac{1}{z_H} \right)^k$, $\frac{N_V}{N} = \left( \frac{1}{z_V} \right)^k$. 

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Next, the profit formulas for the Northern economy (see Table A.3) imply:

\[
\begin{align*}
\tilde{d}_D &= \frac{k}{k - (\theta - 1)} f_H TOL^{\theta(1 - \eta)} \frac{C}{C^* Q} Q^{1 - \theta} \tau^{*(\theta - 1)\eta} \left( \frac{z_V}{z_H} \right)^{\theta - 1} \frac{\tau^{*(\theta - 1)\eta}}{\tau^{(1 - \alpha)(\theta - 1)}} - f_V TOL^{1 - \alpha}, \\
\tilde{d}_V &= \frac{k}{k - (\theta - 1)} f_H TOL^{1 - \alpha + \theta(\alpha - \eta)} \frac{C}{C^* Q} Q^{1 - \theta} \left( \frac{z_V}{z_H} \right)^{\theta - 1} \tau^{*(\theta - 1)\eta} f_V TOL^{1 - \alpha}, \\
\tilde{d}_H &= \frac{(\theta - 1)}{k - (\theta - 1)} f_H w TOL^{1 - \eta}, \\
\tilde{d}_V &= \frac{z^k_V - 1}{z^k_V - 1} \tilde{d}_D + \frac{(\theta - 1)}{k - (\theta - 1)} f_V w TOL^{1 - \alpha}.
\end{align*}
\]

Using the expression for the total profits in the South (see Table A.1), it follows that:

\[
\begin{align*}
1 - \beta^*(1 - \delta^*) f_E^* &= \frac{k}{k - (\theta - 1)} f_H^* TOL^{\theta(\eta^* - 1)} \tau^{*(\eta^* - 1)\eta} Q^{\theta - 1} z_H^* \left( \frac{1}{z_H^*} \right)^k f_H^* TOL^{\eta^* - 1}, \\
&+ \frac{(\theta - 1)}{k - (\theta - 1)} \left( \frac{1}{z_H^*} \right)^k f_H^* TOL^{\eta^* - 1}.
\end{align*}
\]

The consumption ratio in units of the same consumption basket is:

\[
\frac{C}{C^* Q} = \frac{f_H^*}{f_H} TOL^{\theta(\eta^* - 1)} Q^{\theta - 1} \left( \frac{z_H^* \tau^{*(\eta^* - 1)}}{z_H^* \tau^{\eta^*}} \right)^{\theta - 1}.
\]

From the balanced current account condition, I obtain:

\[
(1 - \alpha) z_V^{-k} TOL^{-\alpha} \left[ \frac{(\theta - 1) k}{k - (\theta - 1)} f_H^* \left( \frac{z_V}{z_H^*} \right)^{\theta - 1} TOL^{\theta(\alpha + \eta^*) - 1} \tau^{(1 - \eta^*)} \right] + f_V
\]

\[
= \Lambda \left( \eta + \frac{1 - \eta^*}{\theta} \right) \frac{k}{k - (\theta - 1)} + (1 - \eta) \] and \( \Lambda^* = \left( \eta^* + \frac{1 - \eta^*}{\theta} \right) \frac{k}{k - (\theta - 1)} - (1 - \eta^*). \)

The expression for the real exchange rate in steady state is:

\[
Q^{1 - \theta} = \frac{TOL^{1 - \theta} + (\tau^{\eta*} TOL^{1 - \eta})^{1 - \theta} z_H^{k - 1 - k} \frac{N}{N^*} \left( \tau TOL \right)^{\theta - 1 - \eta} \left( \frac{1}{z_V^{k - 1 - 1}} \right) \frac{N}{N^*} \left( \tau TOL \right)^{\eta^* - 1}}{(1 - z_V^{k}) z_V^{\theta - k} \frac{N}{N^*} \left( \tau TOL \right)^{1 - \theta} + \frac{N}{N^*} \left( \tau TOL \right)^{\eta - 1}}.
\]
Appendix

The remaining equations are:

\[ \frac{\theta k}{k - (\theta - 1)} f_H \frac{\tilde{\rho}_H^{-\theta - 1}}{TOL} = 1 + \frac{1 - \beta^*}{\beta^*(1 - \delta^*)} f_{E} \frac{\tilde{\rho}_H^{-\theta - 1}}{\Xi_t}, \quad (55) \]

\[ \frac{\theta k}{k - (\theta - 1)} f_H TOL^{\gamma - \tilde{\rho}_H^{-\theta - 1}} = 1 + \frac{1 - \beta}{\beta(1 - \delta)} f_{E} \frac{\tilde{\rho}_H^{\theta - 1}}{\Omega_t}, \quad (56) \]

\[ \frac{N}{N^*_D} \left( \frac{\tilde{\rho}_H}{\rho^*_H} \right)^{\theta - 1} = \Xi_t, \quad (57) \]

with:

\[ \Xi_t = \left[ \frac{1}{z_H} \left( \frac{\tau^*}{TOL} \right) \right]^{\theta - 1} + z_{-k}^{-k} \frac{N}{N^*_D}, \]

\[ \Omega_t = (1 - z_V^{-k}) \left( \frac{z_V}{z_H} \right)^{\theta - 1} \frac{z_{-k}^{-k(\theta - 1)} - 1}{z_V^{-k} - 1} (\tau TOL)^{\eta(\theta - 1)} \]

\[ + z_{-k}^{-k} \left[ \frac{z_V}{z_H} (\tau TOL)^{\eta + \alpha - 1} \right]^{\theta - 1} + z_{-k}^{-k} \left( \frac{N}{N^*_D} \right)^{-1}. \]

### A.7 Empirical Impulse Responses of Offshoring to Mexico

I estimate the empirical impulse responses of offshoring to Mexico’s maquiladora sector (total value added, number of plants, and the value added per plant) to permanent technology shocks in U.S. manufacturing. To this end, I use a structural VAR model with five variables: (i) labor productivity in U.S. manufacturing, (ii) labor productivity in Mexico’s maquiladora, (iii) value added per plant and (iv) the number of plants in Mexico’s maquiladora, as well as (v) hours worked in U.S. manufacturing. The estimation details are discussed in Zlate (2009). With the exception of the intensive margin, all variables have a unit root and therefore enter the VAR model in first differences. My identification strategy assumes that long-run labor productivity in U.S. manufacturing responds exclusively to U.S. technology shocks. Conversely, long-run labor productivity in Mexico’s maquiladora sector - which uses production machinery received on loan from U.S. firms - responds to both the U.S. and Mexico-specific permanent technology shocks.

In Figure A.1, I plot the estimated impulse responses of Mexico’s maquiladora variables, together with the +/- 2 standard error confidence intervals. Following a positive, one standard deviation, permanent technology shock to U.S. manufacturing, the number of
maquiladora plants (the extensive margin) does not react on impact, but increases gradually over time. The value added per maquiladora plant (the intensive margin) exhibits an immediate jump, followed by an additional increase until it reaches a peak two quarters after the shock. The intensive margin then declines below its initial level, but returns to it over time.

The predictions of the theoretical model of offshoring (illustrated in Figure 4) are consistent with the empirical impulse responses. In the model, following a positive transitory shock to aggregate productivity in the North, the extensive margin of offshoring increases gradually over time, despite the initial drop. The intensive margin jumps on impact, then declines below its initial steady state and returns to it in the medium run, as in the data.

Figure A.1. Empirical impulse responses of offshore production to a permanent U.S. technology shock.