Appendices to: Oil Efficiency, Demand, and Prices: a Tale of Ups and Downs

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Contents

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A Appendix: Model Solution

A.1 Equilibrium Conditions

This appendix summarizes the equilibrium conditions of the model both in nonlinear and linearized form. The stochastic processes for the exogenous variables are presented in Table 1, in the main body of the paper.
$X^*_{i,t}$ refers to the value of variable X_i at time t along the balanced growth path. $X^*_{i,0}$ is the initial value of the variable at time 0. $\hat{x}_{i,t}$ denotes the log-deviation of variable X_i at time t from its value along the balanced growth path. $x_{i,t}$ denotes the absolute deviation of variable X_i at time t from its value along the balanced growth path.

A.1.1 Country-Specific Relationships

1. First order condition for $C_{1,t}$:

$$\left(Z^c_{1,t}C_{1,t} - \kappa_1 C_{1,t-1}\right)^{-1} = \lambda^q_{1,t}\frac{P^C_{1,t}}{P^d_{1,t}}, \tag{1}$$

where $\lambda^q_{1,t} = \lambda^c_{1,t}P^d_{1,t}$ and $\lambda^c_{1,t}$ is the Lagrange multiplier on the households budget constraint. The linearized equation is given by:

$$\frac{-1}{\left(1-\frac{\kappa_1}{\mu_z}\right)}\left\{\hat{c}_{1,t} + \hat{z}^c_{1,t} - \frac{\kappa_1}{\mu_z}\hat{c}_{1,t-1}\right\} = \hat{\lambda}^q_{1,t} + \widehat{\left[\frac{P^c}{P^d}\right]}_{1,t}. \tag{2}$$

2. First order condition for $L_{1,t}$:

$$-\chi_{0,1}\left(1-L_{1,t}\right)^{\chi_1} = \lambda^q_{1,t}w^f_{1,t}. \tag{3}$$

The linearized equation is given by:

$$\hat{w}^f_{1,t} = \frac{L^*_{1,0}}{1-L^*_{1,0}}\chi_1\hat{l}_{1,t} - \hat{\lambda}^q_{1,t}. \tag{4}$$

Note, that $w^f_{1,t}$ is the desired real wage expressed in terms of $P^d_{1,t}$.

3. Relationship between desired and actual wage:

   (a) If wages are flexible:

$$w_{1,t} = w^f_{1,t} + \frac{\theta^{w*}_{1,0}}{1+\theta^{w*}_{1,0}}\hat{\theta}^w_{1,t} \tag{5}$$

(b) If wages are sticky:

$$E_t \sum_{j=0}^{\infty} \Xi^w_{1,t+j} \left[\frac{1+\tau^w_1}{\theta^w_{1,t+j}} W^r_{1,t} - \frac{W^f_{1,t+j}}{P^d_{1,t+j}} \frac{P^d_{1,t+j}}{W_{1,t+j}} \left(\frac{W_{1,t}\omega^l_{1,t,j}}{W_{1,t+j}} \right)^{-1} \frac{1+\theta^w_{1,t+j}}{\theta^w_{1,t+j}} \right] = 0$$

$$\Xi^w_{1,t+j} = (\xi^w_1)^j \psi_{1,t,t+j} L_{1,t+j}(h) \omega^l_{1,t,j}. \tag{6}$$

The relative wage is defined as $W^r_{1,t} = \frac{W_{1,t}(h)}{W_{1,t}}$. The linearized equation is given by:

$$\frac{1}{\pi^*_1}\left(\omega_{1,t} - \iota^w_1 \omega_{1,t-1}\right) - \frac{\beta_1}{\pi^*_1}\left(\omega_{1,t+1} - \iota^w_1 \omega_{1,t}\right)$$
$$= \frac{\left(1-\xi^w_1\beta_1\right)\left(1-\xi^w_1\right)}{\xi^w_1} \left(\hat{w}^f_{1,t+j} - \hat{w}_{1,t+j} + \frac{\theta^w_1}{1+\theta^w_1}\hat{\theta}^w_{1,t} \right) \tag{7}$$

As customary, in the numerical implementation we using the approximation $\frac{1}{\pi^*_1}\left(\omega_{1,t} - \iota^w_1 \omega_{1,t-1}\right) = \omega_{1,t} - \iota^w_1 \omega_{1,t-1}$. See for example **?**).

4. Wage inflation $\omega_{1,t}$:

$$\omega_{1,t} = log\left(\frac{W_{1,t}}{W_{1,t-1}}\right). \tag{8}$$

The linearized equation is given by:

$$\omega_{1,t} = \hat{w}_{1,t} - \hat{w}_{1,t-1} + \pi^d_{1,t}. \tag{9}$$

5. First order condition for $I_{1,t}$:

$$\begin{aligned} 0 \;=\; & 1 - q_{1,t} Z^i_{1,t}\left(1 - \frac{\psi^i_1}{2}\left(\frac{I_{1,t}}{I_{1,t-1}} - \mu_z\right)^2\right) + q_{1,t} Z^i_{1,t} I_{1,t} \psi^i_1 \left(\frac{I_{1,t}}{I_{1,t-1}} - \mu_z\right)\frac{1}{I_{1,t-1}} \\ & -\beta_1 \frac{P^i_{1,t+1}\lambda^c_{1,t+1}}{P^i_{1,t}\lambda^c_{1,t}} q_{1,t+1} Z^i_{1,t+1} I_{1,t+1} \psi^i_1 \left(\frac{I_{1,t+1}}{I_{1,t}} - \mu_z\right)\frac{I_{1,t+1}}{I^2_{1,t}}, \end{aligned} \tag{10}$$

where $q_{1,t} = \frac{Q_{1,t}}{P^i_{1,t}\lambda^c_{1,t}}$ and $Q_{1,t}$ is the Lagrange multiplier on the capital accumulation equation. The linearized equation is given by:

$$\hat{q}_{1,t} + \hat{z}^i_{1,t} = \psi^i_1 \mu^2_z \left(\hat{\imath}_{1,t} - \hat{\imath}_{1,t-1}\right) - \beta_1 \psi^i_1 \mu^2_z \left(\hat{\imath}_{1,t+1} - \hat{\imath}_{1,t}\right). \tag{11}$$

6. First order condition for $K_{1,t}$:

$$q_{1,t} = \beta_1 \frac{P^d_{1,t}}{P^i_{1,t}} \frac{\lambda^q_{1,t+1}}{\lambda^q_{1,t}} \frac{R^k_{1,t+1}}{P^d_{1,t+1}} + (1-\delta_1)\beta_1 q_{1,t+1} \frac{P^i_{1,t+1}}{P^d_{1,t+1}} \frac{P^d_{1,t}}{P^i_{1,t}} \frac{\lambda^q_{1,t+1}}{\lambda^q_{1,t}}. \tag{12}$$

The linearized equation is given by:

$$\hat{q}_{1,t} = \hat{\lambda}^q_{1,t+1} - \hat{\lambda}^q_{1,t} + \left(1 - \frac{\beta_1}{\mu_z}(1-\delta_1)\right)\left(\hat{r}^k_{1,t+1} - \widehat{\left[\frac{P^i_1}{P^d_1}\right]_t}\right) + \frac{\beta_1}{\mu_z}(1-\delta_1)\left(\hat{q}_{1,t+1} + \widehat{\left[\frac{P^i_1}{P^d_1}\right]_{t+1}} - \widehat{\left[\frac{P^i_1}{P^d_1}\right]_t}\right). \tag{13}$$

7. Capital accumulation:

$$K_{1,t} = (1-\delta_1) K_{1,t-1} + Z^i_{1,t} I_{1,t}\left(1 - \frac{\psi^i_1}{2}\left(\frac{I_{1,t}}{I_{1,t-1}} - \mu_z\right)^2\right). \tag{14}$$

The linearized equation is given by:

$$\hat{k}_{1,t} = \frac{1}{\mu_z}(1-\delta_1)\hat{k}_{1,t-1} + \left(1 - \frac{1}{\mu_z}(1-\delta_1)\right)\left(\hat{z}^i_{1,t} + \hat{\imath}_{1,t}\right). \tag{15}$$

8. Consumption basket:

$$C_{1,t} = \left((\omega^{cc}_1)^{\frac{\rho^o_1}{1+\rho^o_1}}\left(C^{ne}_{1,t}\right)^{\frac{1}{1+\rho^o_1}} + (\omega^{oc}_1)^{\frac{\rho^o_1}{1+\rho^o_1}}\left(\mu^t_{zo} Z^o_{1,t} O^c_{1,t}\right)^{\frac{1}{1+\rho^o_1}}\right)^{1+\rho^o_1}. \tag{16}$$

The linearized equation is given by:

$$\hat{c}_{1,t} = \omega^{cc}_1 \hat{c}^{ne}_{1,t} + \omega^{oc}_1\left(o^c_{1,t} + \hat{z}^o_{1,t}\right). \tag{17}$$

9. Nonoil consumption aggregate:

$$C^{ne}_{1,t} = \left((\omega^c_1)^{\frac{\rho^c_1}{1+\rho^c_1}}\left(C^d_{1,t}\right)^{\frac{1}{1+\rho^c_1}} + (\omega^{mc}_1)^{\frac{\rho^c_1}{1+\rho^c_1}}\left(Z^m_{1,t} M^c_{1,t}\right)^{\frac{1}{1+\rho^c_1}}\right)^{1+\rho^c_1}. \tag{18}$$

The linearized equation is given by:

$$\hat{c}^{ne}_{1,t} = \omega^c_1 \hat{c}^d_{1,t} + \omega^{mc}_1\left[\hat{m}^c_{1,t} + \hat{z}^m_{1,t}\right]. \tag{19}$$

10. First order condition for $C^d_{1,t}$ in consumption basket:

$$\frac{P^c_{1,t}}{P^d_{1,t}}\left(\frac{\omega^{cc}_1 C_{1,t}}{C^{ne}_{1,t}}\right)^{\frac{\rho^o_1}{1+\rho^o_1}}\left(\frac{\omega^c_1 C^{ne}_{1,t}}{C^d_{1,t}}\right)^{\frac{\rho^c_1}{1+\rho^c_1}} = 1. \tag{20}$$

The linearized equation is given by:

$$-\widehat{\left[\frac{P^c_1}{P^d_1}\right]_t} - \frac{\rho^o_1}{1+\rho^o_1}\left(\hat{c}_{1,t} - \hat{c}^{ne}_{1,t}\right) - \frac{\rho^c_1}{1+\rho^c_1}\left(\hat{c}^{ne}_{1,t} - \hat{c}^d_{1,t}\right) = 0. \tag{21}$$

11. First order condition for $M^c_{1,t}$ in consumption basket:

$$\frac{P^m_{1,t}}{P^d_{1,t}} = \frac{P^c_{1,t}}{P^d_{1,t}} \left(\omega^{cc}_1 \frac{C_{1,t}}{C^{ne}_{1,t}}\right)^{\frac{\rho^o_1}{1+\rho^o_1}} \left(\frac{\omega^{mc}_1 C^{ne}_{1,t}}{Z^m_{1,t} M^c_{1,t}}\right)^{\frac{\rho^c_1}{1+\rho^c_1}} Z^m_{1,t}. \tag{22}$$

The linearized equation is given by:

$$\widehat{\left[\frac{P^m_1}{P^d_1}\right]}_t = \widehat{\left[\frac{P^c_1}{P^d_1}\right]}_t + \frac{\rho^o_1}{1+\rho^o_1}\left(\hat{c}_{1,t} - \hat{c}^{ne}_{1,t}\right) + \frac{\rho^c_1}{1+\rho^c_1}\left(\hat{c}^{ne}_{1,t} - \hat{m}^c_{1,t} - \hat{z}^m_{1,t}\right) + \hat{z}^m_{1,t}. \tag{23}$$

12. First order condition for $O^c_{1,t}$ in consumption basket:

$$\frac{P^o_{1,t}}{P^d_{1,t}} = \frac{P^c_{1,t}}{P^d_{1,t}} \left(\frac{\omega^{oc}_1 C_{1,t}}{\mu^t_{zo} Z^c_{1,t} O^c_{1,t}}\right)^{\frac{\rho^o_1}{1+\rho^o_1}} \mu^t_{zo} Z^c_{1,t}. \tag{24}$$

The linearized equation is given by:

$$\widehat{\left[\frac{P^o_1}{P^d_1}\right]}_t = \widehat{\left[\frac{P^c_1}{P^d_1}\right]}_t + \frac{\rho^o_1}{1+\rho^o_1}\left(\hat{c}_{1,t} - \hat{o}^c_{1,t} - \hat{z}^o_{1,t}\right) + \hat{z}^o_{1,t}. \tag{25}$$

13. Investment basket:

$$I_{1,t} = \left(\left(\omega^i_1\right)^{\frac{\rho^c_i}{1+\rho^c_i}} \left(I^d_{1,t}\right)^{\frac{1}{1+\rho^c_i}} + \left(\omega^{mi}_1\right)^{\frac{\rho^c_i}{1+\rho^c_i}} \left(Z^i_{1,t} M^i_{1,t}\right)^{\frac{1}{1+\rho^c_i}}\right)^{1+\rho^c_i}. \tag{26}$$

The linearized equation is given by:

$$\hat{\imath}_{1,t} = \omega^i_1 \hat{\imath}^d_{1,t} + \omega^{mi}_1 \left[m^i_{1,t} + \hat{z}^m_{1,t}\right]. \tag{27}$$

14. First order condition for I^d_t in investment basket:

$$\frac{P^i_{1,t}}{P^d_{1,t}} \left(\frac{\omega^i_1 I_{1,t}}{I^d_{1,t}}\right)^{\frac{\rho^c_1}{1+\rho^c_1}} = 1. \tag{28}$$

The linearized equation is given by:

$$\widehat{\left[\frac{P^i_1}{P^d_1}\right]}_t + \frac{\rho^c_1}{1+\rho^c_1}\left(\hat{\imath}_{1,t} - \hat{\imath}^d_{1,t}\right) = 0. \tag{29}$$

15. First order condition for $M^i_{12,t}$ in investment basket:

$$\frac{P^m_{1,t}}{P^d_{1,t}} = \frac{P^i_{1,t}}{P^d_{1,t}} \left(\frac{\omega^{mi}_1 I_{1,t}}{Z^m_{1,t} M^i_{1,t}} \right)^{\frac{\rho^c_1}{1+\rho^c_1}} Z^m_{1,t}. \tag{30}$$

The linearized equation is given by:

$$\widehat{\left[\frac{P^m_1}{P^d_1}\right]}_t = \widehat{\left[\frac{P^i_1}{P^d_1}\right]}_t + \frac{\rho^c_1}{1+\rho^c_1} \left(\hat{\imath}_{1,t} - m^i_{12,t} - \hat{z}^m_{1,t} \right) + \hat{z}^m_{1,t}. \tag{31}$$

16. Value added aggregator:

$$V_{1,t} = \left(\left(\omega^k_1\right)^{\frac{\rho^v_1}{1+\rho^v_1}} \left(K_{1,t-1}\right)^{\frac{1}{1+\rho^v_1}} + \left(\omega^l_1\right)^{\frac{\rho^v_1}{1+\rho^v_1}} \left(\mu^t_z Z_{1,t} L_{1,t}\right)^{\frac{1}{1+\rho^v_1}} \right)^{1+\rho^v_1}. \tag{32}$$

The linearized equation is given by:

$$\hat{v}_{1,t} = \phi^k_1 \hat{k}_{1,t-1} + \phi^l_1 \left(\hat{z}_{1,t} + \hat{l}_{1,t} \right), \tag{33}$$

with

$$\phi^k_1 = \omega^k_1 \left(\frac{K^*_{1,0}}{\omega^k_1 \mu_z V^*_{1,0}} \right)^{\frac{1}{1+\rho^v_1}}, \tag{34}$$

$$\phi^l_1 = \omega^l_1 \left(\frac{L^*_{1,0}}{\omega^l_1 V^*_{1,0}} \right)^{\frac{1}{1+\rho^v_1}}, \tag{35}$$

and $\phi^k_1 + \phi^l_1 = 1$.

17. Output aggregator:

$$Y_{1,t} = \left(\left(\omega^{vy}_1\right)^{\frac{\rho^o_1}{1+\rho^o_1}} \left(V_{1,t}\right)^{\frac{1}{1+\rho^o_1}} + \left(\omega^{oy}_1\right)^{\frac{\rho^o_1}{1+\rho^o_1}} \left(\mu^t_{zo} Z^o_{1,t} O^y_{1,t}\right)^{\frac{1}{1+\rho^o_1}} \right)^{1+\rho^o_1}. \tag{36}$$

The linearized equation is given by:

$$\hat{y}_{1,t} = \omega^{vy}_1 \hat{v}_{1,t} + \omega^{oy}_1 \left(\hat{o}^y_{1,t} + \hat{z}^o_{1,t} \right). \tag{37}$$

18. First order condition for $K_{1,t-1}$ in output aggregator:

$$\frac{R^k_{1,t}}{P^d_{1,t}} = \frac{MC_{1,t}}{P^d_{1,t}} \left(\omega^{vy}_1 \frac{Y_{1,t}}{V_{1,t}} \right)^{\frac{\rho^o_1}{1+\rho^o_1}} \left(\frac{\omega^k_1 V_{1,t}}{K_{1,t-1}} \right)^{\frac{\rho^v_1}{1+\rho^v_1}}. \tag{38}$$

The linearized equation is given by:

$$\hat{r}^k_{1,t} = \widehat{mc}_{1,t} + \frac{\rho^o_1}{1+\rho^o_1} \left(\hat{y}_{1,t} - \hat{v}_{1,t} \right) + \frac{\rho^v_1}{1+\rho^v_1} \left(\hat{v}_{1,t} - \hat{k}_{1,t-1} \right). \tag{39}$$

19. First order condition for $L_{1,t}$ in output aggregator:

$$\frac{1}{\mu_z^t Z_{1,t}} \frac{W_{1,t}}{P_{1,t}^d} = \frac{MC_{1,t}}{P_{1,t}^d} \left(\frac{\omega_1^{vy} Y_{1,t}}{V_{1,t}} \right)^{\frac{\rho_1^o}{1+\rho_1^o}} \left(\frac{\omega_1^l V_{1,t}}{\mu_z^t Z_{1,t} L_{1,t}} \right)^{\frac{\rho_1^v}{1+\rho_1^v}}. \tag{40}$$

The linearized equation is given by:

$$\hat{w}_{1,t} = \widehat{mc}_{1,t} + \frac{\rho_1^o}{1+\rho_1^o} \left(\hat{y}_{1,t} - \hat{v}_{1,t} \right) + \frac{\rho_1^v}{1+\rho_1^v} \left(\hat{v}_{1,t} - \hat{z}_{1,t} - \hat{l}_{1,t} \right) + \hat{z}_{1,t}. \tag{41}$$

20. First order condition for $O_{1,t}^y$ in output aggregator:

$$\frac{P_{1,t}^o}{P_{1,t}^d} = \frac{MC_{1,t}}{P_{1,t}^d} \left(\frac{\omega_1^{oy} Y_{1,t}}{\mu_{zo}^t Z_{1,t}^o O_{1,t}^y} \right)^{\frac{\rho_1^o}{1+\rho_1^o}} \mu_{zo}^t Z_{1,t}^o. \tag{42}$$

The linearized equation is given by:

$$\widehat{\left[\frac{P_1^o}{P_1^d} \right]_t} = \widehat{mc}_{1,t} + \frac{\rho_1^o}{1+\rho_1^o} \left(\hat{y}_{1,t} - \hat{o}_{1,t} - \hat{z}_{1,t}^o \right) + \hat{z}_{1,t}^o. \tag{43}$$

21. Evolution of marginal costs:

    (a) If prices are flexible:

$$\frac{MC_{1,t}}{P_{1,t}^d} = \frac{1+\tau_1^p}{1+\theta_{1,t}^p}. \tag{44}$$

    The linearized equation is given by:

$$\widehat{mc}_{1,t} = -\frac{1}{1+\theta_{1,0}^{p*}} \hat{\theta^p}_{1,t}. \tag{45}$$

    (b) If prices are sticky and export prices are set in the currency of the producer:

$$\begin{aligned} & P_{1,t}^r(i) \sum_{j=0}^{\infty} \left[(\xi_1^p)^j \psi_{1,t,t+j} \frac{1+\tau_1^p}{\theta_{1,t+j}^p} \pi_{1,t,j}^l Y_{1,t+j}^d(i) \right] \\ = & \left[\sum_{j=0}^{\infty} (\xi_1^p) \psi_{1,t,t+j} \frac{MC_{1,t+j}}{P_{1,t+j}^d} \frac{P_{1,t+j}^d}{P_{1,t}^d} Y_{1,t+j}^d(i) \frac{1+\theta_{1,t+j}^p}{\theta_{1,t+j}^p} \right], \end{aligned} \tag{46}$$

where

$$P^r_{1,t} = \frac{P^d_{1,t}(i)}{P^d_{1,t}} \tag{47}$$

$$\psi_{1,t,t+j} = \beta_1^j \frac{P^d_{1,t}}{P^d_{1,t+j}} \frac{\lambda^c_{1,t+j}}{\lambda^c_{1,t}} \tag{48}$$

$$\pi^l_{1,t,j} = \prod_{i=1}^{j} \left\{ \left(\pi^d_{1,t-1+i}\right)^{\iota} \left(\pi^*_1\right)^{1-\iota} \right\} \tag{49}$$

$$Y^d_{1,t+j}(i) = Y^d_{1,t+j} \left(\frac{P^d_{1,t}(i)}{P^d_{1,t}} \frac{P^d_{1,t}\pi^l_{1,t,j}}{P^d_{1,t+j}} \right)^{-\frac{1+\theta^p_{1,t+j}}{\theta^p_{1,t+j}}} \tag{50}$$

or after simplifying

$$\sum_{j=0}^{\infty} \Xi_{1,t+j} \left[\frac{1+\tau^p_1}{\theta^p_{1,t+j}} P^r_{1,t} - \frac{MC_{1,t+j}}{P^d_{1,t+1}} \left(\frac{P^d_{1,t}\pi^l_{1,t,j}}{P^d_{1,t+j}} \right)^{-1} \frac{1+\theta^p_{1,t+j}}{\theta^p_{1,t+j}} \right] = 0$$

$$\Xi_{1,t+j} = \left(\xi^p_1 \beta_1\right)^j \frac{\lambda^c_{1,t+j}}{\lambda^c_{1,t}} \left(\frac{P^d_{1,t}\pi^l_{1,t,j}}{P^d_{1,t+j}} \right)^{-\frac{1}{\theta^p_{1,t+j}}} \left(P^r_{1,t}\right)^{-\frac{1+\theta^p_{1,t+j}}{\theta^p_{1,t+j}}} Y^d_{1,t+j}. \tag{51}$$

The linearized equation is given by

$$\frac{1}{\pi^*_1}\left(\pi^d_{1,t} - \iota^p_1 \pi^d_{t-1}\right) = \frac{\beta_1}{\pi^*_1}\left(\pi^d_{t+1} - \iota^p_1 \pi^d_t\right) + \frac{\left(1-\xi^p_1\beta_1\right)\left(1-\xi^p_1\right)}{\xi^p_1} \left(\widehat{mc}_{1,t} + \frac{\theta^{p*}_{1,0}}{1+\theta^{p*}_{1,0}} \hat{\theta}^p_{1,t} \right) \tag{52}$$

In the numerical implementation we use the approximation typically used in the literature $\frac{1}{\pi^*_1}\left(\pi^d_{1,t} - \iota^p_1 \pi^d_{t-1}\right) = \pi^d_{1,t} - \iota^p_1 \pi^d_{t-1}$. See, for example, **?**).

22. Government spending $G^d_{1,t}$:

$$G^d_{1,t} = g_1 Z^g_{1,t} Y^d_{1,t}. \tag{53}$$

The linearized equation is given by:

$$\hat{g}^d_{1,t} = \hat{y}^d_{1,t} + \hat{z}^g_{1,t}. \tag{54}$$

23. Market clearing condition for $Y^d_{1,t}$:

$$Y^d_{1,t} = I^d_{1,t} + C^d_{1,t} + G^d_{1,t} + X_{1,t}. \tag{55}$$

The linearized equation is given by:

$$\hat{y}^d_{1,t} = \frac{I^{d*}_{1,0}}{Y^{d*}_{1,0}} \hat{\imath}^d_{1,t} + \frac{C^{d*}_{1,0}}{Y^{d*}_{1,0}} \hat{c}^d_{1,t} + \frac{G^{d*}_{1,0}}{Y^{d*}_{1,0}} \hat{g}^d_{1,t} + \frac{X^{d*}_{1,0}}{Y^{d*}_{1,0}} \hat{x}_{1,t}. \tag{56}$$

24. Oil demand $O_{1,t}$:

$$O_{1,t} = O^{c}_{1,t} + O^{y}_{1,t}. \tag{57}$$

The linearized equation is given by:

$$\hat{o}_{1,t} = \frac{O^{c*}_{1,0}}{O^{*}_{1,0}}\hat{o}^{c}_{1,t} + \frac{O^{y*}_{1,0}}{O^{*}_{1,0}}\hat{o}^{y}_{1,t}. \tag{58}$$

25. Nominal interest rate $R^{s}_{1,t}$ and real interest rate $R^{rs}_{1,t}$:

$$\frac{1}{1+R^{s}_{1,t}} = \beta\frac{\lambda^{q}_{1,t+1}}{\lambda^{q}_{1,t}}\frac{P^{d}_{1,t}}{P^{d}_{1,t+1}}. \tag{59}$$

The linearized equation is given by:

$$r^{rs}_{1,t} = r^{s}_{1,t} - \pi^{d}_{1,t+1} = -\left(\hat{\lambda}^{q}_{1,t+1} - \hat{\lambda}^{q}_{1,t}\right), \tag{60}$$

assuming that β_1/μ_z is close to 1. $r^{s}_{1,t}$ and $r^{rs}_{1,t}$ are measured in absolute deviation from their values along the balanced growth path.

26. Monetary policy reaction function $i_{1,t} = R^{s}_{1,t} - 1$:

$$i_{1,t} = \bar{i}_1 + \gamma^{i}_1(i_{1,t-1} - \bar{i}_1) + (1-\gamma^{i}_1)(\pi^{core}_{1,t} + \gamma^{\pi}_1(\pi^{core}_{1,t} - \bar{\pi}^{core}_{1,t}) + \gamma^{y}_1 y^{gap}_{1,t}). \tag{61}$$

The linearized equation is given by:

$$r^{s}_{1,t} = \gamma^{i}_1 r^{s}_{1,t-1} + (1-\gamma^{i}_1)(\pi^{core}_{1,t} + \gamma^{\pi}_1(\pi^{core}_{1,t} - \bar{\pi}^{core}_{1,t}) + \gamma^{y}_1 y^{gap}_{1,t}). \tag{62}$$

27. Core price level $P^{ne}_{1,t}$:

$$P^{ne}_{1,t} = P^{c}_{1,t}\left(\frac{\omega^{cc}_1 C_{1,t}}{C^{ne}_{1,t}}\right)^{\frac{\rho^{o}_1}{1+\rho^{o}_1}}. \tag{63}$$

The linearized equation is given by:

$$\begin{aligned}\widehat{\left[\frac{P^{ne}_1}{P^{d}_1}\right]_t} &= \widehat{\left[\frac{P^{c}_1}{P^{d}_1}\right]_t} + \frac{\rho^{o}_1}{1+\rho^{o}_1}\left(\hat{c}_{1,t} - \hat{c}^{ne}_{1,t}\right) \\ &= \frac{1}{\omega^{cc}_1}\widehat{\left[\frac{P^{c}_1}{P^{d}_1}\right]_t} - \frac{\omega^{oc}_1}{\omega^{cc}_1}\left(\widehat{\left[\frac{P^{o}_1}{P^{d}_1}\right]_t} - \hat{z}^{o}_{1,t}\right).\end{aligned} \tag{64}$$

In constructing the core price index the shock to oil efficiency enters, as this shock changes the share of oil in the headline price index $P^{c}_{1,t}$.

28. Inflation of domestic prices $\pi^d_{1,t}$:

$$\pi^d_{1,t} = log\left(\frac{P^d_{1,t}}{P^d_{1,t-1}}\right). \tag{65}$$

29. Inflation of core prices:

$$\pi^{core}_{1,t} = log\left(\frac{P^{ne}_{1,t}}{P^{ne}_{1,t-1}}\right). \tag{66}$$

The linearized equation is given by:

$$\pi^{core}_{1,t} = \widehat{\left[\frac{P^{ne}_1}{P^d_1}\right]}_t - \widehat{\left[\frac{P^{ne}_1}{P^d_1}\right]}_{t-1} + \pi^d_{1,t}. \tag{67}$$

30. Inflation of headline prices:

$$\pi^{head}_{1,t} = log\left(\frac{P^c_{1,t}}{P^c_{1,t-1}}\right). \tag{68}$$

The linearized equation is given by:

$$\pi^{head}_{1,t} = \widehat{\left[\frac{P^c_1}{P^d_1}\right]}_t - \widehat{\left[\frac{P^c_1}{P^d_1}\right]}_{t-1} + \pi^d_{1,t}. \tag{69}$$

31. Aggregate imports $M_{1,t}$:

$$M_{1,t} = \frac{P^m_{1,t}}{P^d_{1,t}} M^c_{1,t} + \frac{P^m_{1,t}}{P^d_{1,t}} M^i_{1,t}. \tag{70}$$

The linearized equation is given by:

$$\hat{m}_{1,t} = \frac{M^{c*}_{1,0}}{M^*_{1,0}}\left(\widehat{\left[\frac{P^m_1}{P^d_1}\right]}_t + \hat{m}^c_{1,t}\right) + \frac{M^{i*}_{1,0}}{M^*_{1,0}}\left(\widehat{\left[\frac{P^m_1}{P^d_1}\right]}_t + \hat{m}^i_{1,t}\right), \tag{71}$$

as relative prices are calibrated to be 1 along the balanced growth path.

32. Aggregate exports $X_{1,t}$:

$$X_{1,t} = \frac{1}{\zeta_1}\left(M^c_{2,t} + M^i_{2,t}\right), \tag{72}$$

as country 1's real per capita exports $X_{1,t}$ and country 2's real per capita imports $M^c_{2,t} + M^i_{2,t}$ are related through ζ_1. The linearized equation is given by:

$$\hat{x}_{1,t} = \frac{M^{c*}_{2,0}}{M^{c*}_{2,0} + M^{i*}_{2,0}}\hat{m}^c_{2,t} + \frac{M^{i*}_{2,0}}{M^{c*}_{2,0} + M^{i*}_{2,0}}\hat{m}^i_{2,t}. \tag{73}$$

33. Trade balanced to gross output ratio:

$$\frac{NT^{bal}_{1,t}}{P^d_{1,t}Y^d_{1,t}} = \frac{X_{1,t} - M_{1,t} + \frac{P^o_{1,t}}{P^d_{1,t}}\left(Y^o_{1,t} - O_{1,t}\right)}{Y^d_{1,t}}. \tag{74}$$

The linearized equation is given by:

$$t^{bal}_{1,t} = \frac{X^*_{1,0}}{Y^{d*}_{1,0}}\hat{x}_{2,t} - \frac{M^*_{1,0}}{Y^{d*}_{1,0}}\hat{m}_{1,t} + \frac{P^{o*}_{1,0}Y^{o*}_{1,0}}{P^{d*}_{1,0}Y^{d*}_{1,0}}\hat{y}^o_{1,t} - \frac{P^{o*}_{1,0}O^*_{1,0}}{P^{d*}_{1,0}Y^{d*}_{1,0}}\hat{o}_{1,t} - \frac{P^{o*}_{1,0}\left(O^*_{1,0} - Y^{o*}_{1,0}\right)}{P^{d*}_{1,0}Y^{d*}_{1,0}}\widehat{\left[\frac{P^o_1}{P^d_1}\right]_t}. \tag{75}$$

34. Nonoil trade balance to gross output ratio:

$$\frac{NG^{bal}_{1,t}}{P^d_{1,t}Y^d_{1,t}} = \frac{X_{1,t} - M_{1,t}}{Y^d_{1,t}}. \tag{76}$$

The linearized equation is given by:

$$g^{bal}_{1,t} = \frac{X^*_{1,0}}{Y^{d*}_{1,0}}\hat{x}_{1,t} - \frac{M^*_{1,0}}{Y^{d*}_{1,0}}\hat{m}_{1,t} - \left(\frac{X^*_{1,0}}{Y^{d*}_{1,0}} - \frac{M^*_{1,0}}{Y^{d*}_{1,0}}\right)\hat{y}^d_{1,t}. \tag{77}$$

The conditions for country 2 are analogous.

A.1.2 Bilateral Relationships

35. Relative import prices under producer currency pricing:

(a) for country 1

$$\frac{P^m_{1,t}}{P^d_{1,t}} = \frac{e_{1,t}P^c_{2,t}}{P^c_{1,t}}\frac{P^d_{2,t}}{P^c_{2,t}}\frac{P^c_{1,t}}{P^d_{1,t}}, \tag{78}$$

where $e_{1,t}$ is the nominal exchange rate and $rer_{1,t} = \frac{e_{1,t}P^c_{2,t}}{P^c_{1,t}}$ the consumption real exchange rate. The linearized equation is given by:

$$\widehat{\left[\frac{P^m_{1,t}}{P^d_{1,t}}\right]} = \widehat{rer}_{1,t} - \widehat{\left[\frac{P^c_2}{P^d_2}\right]_t} + \widehat{\left[\frac{P^c_1}{P^d_1}\right]_t}. \tag{79}$$

(b) for country 2

$$\widehat{\left[\frac{P^m_{2,t}}{P^d_{2,t}}\right]} = -\widehat{rer}_{1,t} + \widehat{\left[\frac{P^c_2}{P^d_2}\right]_t} - \widehat{\left[\frac{P^c_1}{P^d_1}\right]_t}. \tag{80}$$

36. Uncovered interest rate parity condition:

$$\frac{\lambda^q_{2,t+1}}{\lambda^q_{2,t}}\frac{P^d_{2,t}}{P^c_{2,t}}\frac{P^c_{2,t+1}}{P^d_{2,t+1}} = \phi^b_{1,t}\frac{rer_{1,t+1}}{rer_{1,t}}\frac{\lambda^q_{1,t+1}}{\lambda^q_{1,t}}\frac{P^d_{1,t}}{P^c_{1,t}}\frac{P^c_{1,t+1}}{P^d_{1,t+1}}. \tag{81}$$

The linearized equation is given by:

$$\left(\hat{\lambda}^q_{2,t+1} - \hat{\lambda}^q_{2,t}\right) = \left(\hat{\lambda}^q_{1,t+1} - \hat{\lambda}^q_{1,t}\right) + \phi^b_1 b_{1,t} + \widehat{rer}_{1,t+1} - \widehat{rer}_{1,t} \tag{82}$$

$$-\widehat{\left[\frac{P^c_1}{P^d_1}\right]}_t + \widehat{\left[\frac{P^c_1}{P^d_1}\right]}_{t+1} + \widehat{\left[\frac{P^c_2}{P^d_2}\right]}_t - \widehat{\left[\frac{P^c_2}{P^d_2}\right]}_{t+1}. \tag{83}$$

37. Net foreign asset condition:

$$\frac{e_{1,t}P^b_{2,t}B_{1,t}}{\phi^b_{1,t}} = e_{1,t}B_{1,t-1} + NT^{bal}_{1,t}. \tag{84}$$

The linearized equation is given by:

$$\beta b_{1,t} = b_{1,t-1} + t^{bal}_{1,t}. \tag{85}$$

where $b_{1,t-1}$ is the absolute deviation of $\frac{e_{1,t}B_{1,t-1}}{P^d_{1,t}Y^d_{1,t}}$ from 0. $t^{bal}_{1,t}$ is the deviation of the trade balance to gross output ratio from its value along the balanced growth path.

38. Oil market clearing condition:

$$\zeta_1 Y^o_{1,t} + Y^o_{2,t} = \zeta_1 O_{1,t} + O_{2,t}. \tag{86}$$

The linearized equation is given by:

$$\frac{\zeta_1 Y^{o*}_{1,0}}{\zeta_1 Y^{o*}_{1,0} + Y^{o*}_{2,0}}\hat{y}^o_{1,t} + \frac{Y^{o*}_{2,0}}{\zeta_1 Y^{o*}_{1,0} + Y^{o*}_{2,0}}\hat{y}^o_{2,t} = \frac{\zeta_1 O^*_{1,0}}{\zeta_1 O^*_{1,0} + O^*_{2,0}}\hat{o}_{1,t} + \frac{O^*_{2,0}}{\zeta_1 O^*_{1,0} + O^*_{2,0}}\hat{o}_{2,t}. \tag{87}$$

39. Law of one price for oil:

$$\frac{P^o_{1,t}}{P^d_{1,t}} = rer_{1,t}\frac{P^c_{1,t}}{P^d_{1,t}}\frac{P^d_{2,t}}{P^c_{2,t}}\frac{P^o_{2,t}}{P^d_{2,t}}. \tag{88}$$

The linearized equation is given by:

$$\widehat{\left[\frac{P^o_1}{P^d_1}\right]}_t = \widehat{rer}_{1,t} + \widehat{\left[\frac{P^c_1}{P^d_1}\right]}_t - \widehat{\left[\frac{P^c_2}{P^d_2}\right]}_t + \widehat{\left[\frac{P^o_2}{P^d_2}\right]}_t. \tag{89}$$

A.1.3 Important Definitions

40. Definition of $GDP_{1,t}$ using the Laspeyres index:

$$GDP_{1,t} = GDP_{1,t-1} \frac{P^d_{1,t-1} Y_{1,t} - P^o_{1,t-1} O^y_{1,t} + P^o_{1,t-1} Y^o_{1,t}}{P^d_{1,t-1} Y_{1,t-1} - P^o_{1,t-1} O^y_{1,t-1} + P^o_{1,t-1} Y^o_{1,t-1}}. \tag{90}$$

The linearized equation is given by:

$$\begin{aligned} & \left(1 - \frac{P^{o*}_{1,0} O^{y*}_{1,0}}{P^{d*}_{1,0} Y^{d*}_{1,0}} + \frac{P^{o*}_{1,0} Y^{o*}_{1,0}}{P^{d*}_{1,0} Y^{d*}_{1,0}}\right) \left(\widehat{gdp}_{1,t} - \widehat{gdp}_{1,t-1}\right) - \left(\frac{\mu_z}{\mu_{gdp,1}} \hat{y}_{1,t} - \hat{y}_{1,t-1}\right) \\ = & -\frac{P^{o*}_{1,0} O^{y*}_{1,0}}{P^{d*}_{1,0} Y^{d*}_{1,0}} \left(\frac{\mu_o}{\mu_{gdp,1}} \hat{o}^y_t - \hat{o}^y_{t-1}\right) + \frac{P^{o*}_{1,0} Y^{o*}_{1,0}}{P^{d*}_{1,0} Y^{d*}_{1,0}} \left(\frac{\mu_o}{\mu_{gdp,1}} \hat{y}^o_{1,t} - \hat{y}^o_{1,t-1}\right) \\ & - \left[\frac{P^{o*}_{1,0} O^{y*}_{1,0}}{P^{d*}_{1,0} Y^{d*}_{1,0}} - \frac{P^{o*}_{1,0} Y^{o*}_{1,0}}{P^{d*}_{1,0} Y^{d*}_{1,0}}\right] \left(\frac{\mu_o}{\mu_{gdp,1}} - 1\right) \widehat{\left[\frac{P^o_1}{P^d_1}\right]}_{t-1}. \end{aligned} \tag{91}$$

Absent trend growth the linear approximation of the Laspeyres index for GDP is a constant price aggregate.

41. Ratio between nominal GDP and nominal gross output:

$$\frac{NGDP_{1,t}}{P^d_{1,t} Y_{1,t}} = 1 - \frac{P^o_{1,t} O^y_{1,t}}{P^d_{1,t} Y_{1,t}} + \frac{P^o_{1,t} Y^o_{1,t}}{P^d_{1,t} Y_{1,t}}. \tag{92}$$

The linearized equation is given by:

$$\frac{NGDP^*_{1,0}}{P^{d*}_{1,0} Y^{d*}_{1,0}} \widehat{\left[\frac{NGDP_1}{P^d_1 Y_1}\right]}_t = \left(\frac{P^{o*}_{1,0} O^{y*}_{1,0}}{P^{d*}_{1,0} Y^{d*}_{1,0}} - \frac{P^{o*}_{1,0} Y^{o*}_{1,0}}{P^{d*}_{1,0} Y^{d*}_{1,0}}\right) \left(\hat{y}_{1,t} - \widehat{\left[\frac{P^o_1}{P^d_1}\right]}_t\right) \tag{93}$$

$$- \frac{P^{o*}_{1,0} O^{y*}_{1,0}}{P^{d*}_{1,0} Y^{d*}_{1,0}} \hat{o}^y_{1,t} + \frac{P^{o*}_{1,0} Y^{o*}_{1,0}}{P^{d*}_{1,0} Y^{d*}_{1,0}} \hat{y}^o_{1,t}. \tag{94}$$

42. Oil price deflated by GDP deflator:

$$\frac{P^o_{1,t}}{P^{GDP}_{1,t}} \frac{P^{GDP}_{1,t-1}}{P^o_{1,t-1}} = \frac{P^o_{1,t}}{P^d_{1,t}} \frac{P^d_{1,t-1}}{P^o_{1,t-1}} \frac{\frac{NGDP_{1,t-1}}{P^d_{1,t-1} Y_{1,t-1}}}{\frac{NGDP_{1,t}}{P^d_{1,t} Y_{1,t}}} \frac{GDP_{1,t}}{GDP_{1,t-1}} \frac{Y_{1,t-1}}{Y_{1,t}}. \tag{95}$$

The linearized equation is given by:

$$\begin{aligned}\log\left(\left[\frac{P^o_{1,t}}{P^{GDP}_{1,t}}\right]^{obs}\right)-\log\left(\left[\frac{P^o_{1,t-1}}{P^{GDP}_{1,t-1}}\right]^{obs}\right) &= -\left[\left[\widehat{\frac{NGDP_1}{P^d_1Y_1}}\right]_t-\left[\widehat{\frac{NGDP_1}{P^d_1Y_1}}\right]_{t-1}\right]-\left[\hat{y}_{1,t}-\hat{y}_{1,t-1}\right]\\ &+\widehat{\left[\frac{P^o_1}{P^d_1}\right]_t}-\widehat{\left[\frac{P^o_1}{P^d_1}\right]_{t-1}}+\widehat{gdp}_{1,t}-\widehat{gdp}_{1,t-1}\\ &+\left(\frac{\mu_{zo}\mu_{gdp,1}}{\mu_z}-1\right). \end{aligned} \tag{96}$$

43. Trade balance to GDP ratio:

$$\frac{NT^{bal}_{1,t}}{NGDP_{1,t}}=\frac{NT^{bal}_{1,t}}{P^d_{1t}Y_{1,t}}\frac{P^d_{1,t}Y_{1,t}}{NGDP_{1,t}}. \tag{97}$$

The linearized equation is given by:

$$\left[\widehat{\frac{NT^{bal}_1}{NGDP_1}}\right]_t=\frac{P^{d*}_{1,0}Y^{d*}_{1,0}}{NGDP^*_{1,0}}t^{bal}_{1,t}. \tag{98}$$

44. Nonoil trade balance to GDP ratio:

$$\frac{NG^{bal}_{1,t}}{NGDP_{1,t}}=\frac{1}{\frac{NGDP_{1,t}}{P^d_{1,t}Y_{1,t}}}\left(\frac{X_{1,t}}{Y_{1,t}}-\frac{P^d_{1,t}}{P^d_{1,t}}\frac{M_{1,t}}{Y_{1,t}}\right). \tag{99}$$

The linearized equation is given by:

$$\left[\widehat{\frac{NG^{bal}_1}{NGDP_1}}\right]_t=-\frac{1}{\frac{NGDP^*_{1,0}}{P^{d*}_{1,0}Y^{d*}_{1,0}}}\left(\frac{X^*_{1,0}}{Y^{d*}_{1,0}}-\frac{P^{d*}_{1,0}M^*_{1,0}}{P^{d*}_{1,0}Y^{d*}_{1,0}}\right)\left[\widehat{\frac{NGDP_1}{P^d_1Y_1}}\right]_t+\frac{1}{\frac{NGDP^*_{1,0}}{P^{d*}_{1,0}Y^{d*}_{1,0}}}g^{bal}_{1,t} \tag{100}$$

A.1.4 Relationships Between Model Variables and Observed Data

The observed data carries the superscript "*obs*".

45. Observation equation for GDP:

$$\log\left(GDP^{obs}_{1,t}\right)-\log\left(GDP^{obs}_{1,t-1}\right)=\widehat{gdp}_{1,t}-\widehat{gdp}_{1,t-1}+\left(\mu_{gdp,1}-1\right). \tag{101}$$

46. Observation equation for oil production:

$$\log\left(Y^{o,obs}_{1,t}\right)-\log\left(Y^{o,obs}_{1,t-1}\right)=\hat{y}^o_{1,t}-\hat{y}^o_{1,t-1}+\left(\mu_o-1\right). \tag{102}$$

47. Observation equation for oil imports as share of GDP:

$$\frac{P_{1,t}^{o}\left(O_{1,t}-Y_{1,t}^{o}\right)}{NGDP_{1,t}}=\frac{P_{1t}^{d}Y_{1,t}}{NGDP_{1,t}}\frac{P_{1,t}^{o}}{P_{1,t}^{d}}\left(\frac{O_{1,t}}{Y_{1,t}}-\frac{Y_{1,t}^{o}}{Y_{1,t}}\right). \tag{103}$$

The linearized equation is given by:

$$\begin{aligned}\left[\frac{P_1^o\,\widehat{(O_1-Y_1^o)}}{NGDP_1}\right]_t &= \frac{P_{1,0}^{d*}Y_{1,0}^{d*}}{NGDP_{1,0}^{*}}\left(\frac{O_{1,0}^{*}}{Y_{1,0}^{d*}}-\frac{Y_{1,0}^{o*}}{Y_{1,0}^{*}}\right)\left[-\left[\widehat{\frac{NGDP_1}{P_1^dY_1}}\right]_t+\left[\widehat{\frac{P_1^o}{P_1^d}}\right]_t-\hat{y}_{1,t}\right]\\ &\quad+\frac{P_{1,0}^{d*}Y_{1,0}^{*}}{NGDP_{1,0}^{*}}\left(\frac{O_{1,0}^{*}}{Y_{1,0}^{d*}}\hat{o}_{1t}-\frac{Y_{1,0}^{o*}}{Y_{1,0}^{*}}\hat{y}_{1,t}^{o}\right). \end{aligned} \tag{104}$$

48. Observation equation for the price of oil:

$$\log\left(\left[\frac{P_{1,t}^{o}}{P_{1,t}^{GDP}}\right]^{obs}\right)-\log\left(\left[\frac{P_{1,t-1}^{o}}{P_{1,t-1}^{GDP}}\right]^{obs}\right)=\left[\widehat{\frac{P_1^o}{P_1^{GDP}}}\right]_t-\left[\widehat{\frac{P_1^o}{P_1^{GDP}}}\right]_{t-1}+\frac{\mu_{zo}\mu_{gdp,1}-\mu_z}{\mu_z}. \tag{105}$$

49. Observation of nonoil import share $\frac{P_{1,t}^dM_{1,t}}{NGDP_{1,t}}$:

$$\frac{P_{1,t}^{d}M_{1,t}}{NGDP_{1,t}}=\frac{P_{1,t}^{d}M_{1,t}}{P_{1,t}^{d}Y_{1,t}}\frac{P_{1,t}^{d}Y_{1,t}}{NGDP_{1,t}}. \tag{106}$$

The linearized equation is given by:

$$\left[\widehat{\frac{P_1^dM_1}{NGDP_1}}\right]_t=\frac{P_{1,0}^{d*}M_{1,0}^{*}}{P_{1,0}^{d*}Y_{1,0}^{d*}}\frac{NGDP_{1,0}^{*}}{P_{1,0}^{d*}Y_{1,0}^{d*}}\left(\hat{m}_{1,t}-\hat{y}_{1,t}-\left[\widehat{\frac{NGDP_1}{P_1^dY_1}}\right]_t\right). \tag{107}$$

50. Observation of nonoil export share $\frac{P_{1,t}^dX_{1,t}}{NGDP_{1,t}}$:

$$\frac{P_{1,t}^{d}X_{1,t}}{NGDP_{1,t}}=\frac{P_{1,t}^{d}X_{1,t}}{P_{1,t}^{d}Y_{1,t}}\frac{P_{1,t}^{d}Y_{1,t}}{NGDP_{1,t}}. \tag{108}$$

The linearized equation is given by:

$$\left[\widehat{\frac{P_1^dX_1}{NGDP_1}}\right]_t=\frac{P_{1,0}^{d*}X_{1,0}^{d*}}{P_{1,0}^{d*}Y_{1,0}^{d*}}\frac{NGDP_{1,0}^{*}}{P_{1,0}^{d*}Y_{1,0}^{d*}}\left(\hat{x}_{1,t}-\hat{y}_{1,t}-\left[\widehat{\frac{NGDP_1}{P_1^dY_1}}\right]_t\right). \tag{109}$$

51. Observation of the real exchange rate:

$$\widehat{rer}_{1,t}^{obs} = \widehat{rer}_{1,t}. \tag{110}$$

52. Observation of consumption share $\frac{P_{1,t}^c C_{1,t}}{NGDP_{1,t}}$:

$$\frac{P_{1,t}^c C_{1,t}}{NGDP_{1,t}} = \frac{P_{1,t}^c C_{1,t}}{P_{1,t}^d Y_{1,t}} \frac{P_{1,t}^d Y_{1,t}}{NGDP_{1,t}}. \tag{111}$$

The linearized equation is given by:

$$\left[\widehat{\frac{P_1^c C_1}{NGDP_1}}\right]_t = \frac{P_{1,0}^{c*} C_{1,0}^*}{P_{1,0}^{d*} Y_{1,0}^{d*}} \frac{NGDP_{1,0}^*}{P_{1,0}^{d*} Y_{1,0}^*} \left(\hat{c}_{1,t} + \left[\widehat{\frac{P_1^c}{P_1^d}}\right]_t - \hat{y}_{1,t} - \left[\widehat{\frac{NGDP_1}{P_1^d Y_1}}\right]_t \right). \tag{112}$$

53. Observation of (fixed) investment share in GDP $\frac{P_{1,t}^i I_{1,t}}{NGDP_{1,t}}$:

$$\frac{P_{1,t}^i I_{1,t}}{NGDP_{1,t}} = \frac{P_{1,t}^c I_{1,t}}{P_{1,t}^d Y_{1,t}} \frac{P_{1,t}^d Y_{1,t}}{NGDP_{1,t}}. \tag{113}$$

The linearized equation is given by:

$$\left[\widehat{\frac{P_1^i I_1}{NGDP_1}}\right]_t = \frac{P_{1,0}^{c*} I_{1,0}^*}{P_{1,0}^{d*} Y_{1,0}^{d*}} \frac{NGDP_{1,0}^*}{P_{1,0}^{d*} Y_{1,0}^{d*}} \left(\hat{\imath}_{1,t} + \left[\widehat{\frac{P_1^i}{P_1^d}}\right]_t - \hat{y}_{1,t} - \left[\widehat{\frac{NGDP_1}{P_1^d Y_1}}\right]_t \right). \tag{114}$$

54. Observation of core price inflation:

$$\pi_{1,t}^{core,obs} = \pi_{1,t}^{core} + (\pi_1^* - 1). \tag{115}$$

55. Observation of wage inflation:

$$\omega_{1,t}^{obs} = \omega_{1,t}. \tag{116}$$

56. Observation of the nominal interest rate:

$$r_{1,t}^{s,obs} = r_{1,t}^s. \tag{117}$$

57. Cost minimization of firms implies for ϕ_1^k and ϕ_1^l:

$$\phi_1^k = \omega_1^k \left(\frac{shareky_1}{\omega_1^k \mu_z sharevy_1} \right)^{\frac{1}{1+\rho_1^v}}, \tag{118}$$

and

$$\phi_1^l = \omega_1^l \left(\frac{sharely_1}{\omega_1^l sharevy_1} \right)^{\frac{1}{1+\rho_1^v}}, \tag{119}$$

where $\phi_1^k + \phi_1^l = 1$.

A.1.5 Decomposition of Marginal Costs

Define the marginal factor products as:

$$\widehat{mpo}_{1,t} = \frac{\rho_1^o}{1+\rho_1^o}\left(\hat{y}_{1,t} - \hat{o}_{1,t} - \hat{z}_{1,t}^o\right) + \hat{z}_{1,t}^o \tag{120}$$

$$\widehat{mpk}_{1,t} = \frac{\rho_1^o}{1+\rho_1^o}\left(\hat{y}_{1,t} - \hat{v}_{1,t}\right) + \frac{\rho_1^v}{1+\rho_1^v}\left(\hat{v}_{1,t} - \hat{k}_{1,t-1}\right) \tag{121}$$

$$\widehat{mpl}_{1,t} = \frac{\rho_1^o}{1+\rho_1^o}\left(\hat{y}_{1,t} - \hat{v}_{1,t}\right) + \frac{\rho_1^v}{1+\rho_1^v}\left(\hat{v}_{1,t} - \hat{z}_{1,t} - \hat{l}_{1,t}\right) + \hat{z}_{1,t}, \tag{122}$$

and notice that the first order conditions for firms can thus be rewritten as

$$\widehat{\left[\frac{P_1^o}{P_1^d}\right]_t} = \widehat{mc}_{1,t} + \widehat{mpo}_{1,t} \tag{123}$$

$$\hat{r}_{1,t}^k = \widehat{mc}_{1,t} + \widehat{mpk}_{1,t} \tag{124}$$

$$\hat{w}_{1,t} = \widehat{mc}_{1,t} + \widehat{mpl}_{1,t}. \tag{125}$$

Multiplying equation (123) by ω_1^{oy}, equation (124) by $\omega_1^{vy}\phi_1$, and equation (125) by $\omega_1^{vy}(1-\phi_1)$, and adding over these three equations, we obtain the desired expression:

$$\widehat{mc}_{1,t} = \omega_1^{oy}\left(\widehat{\left[\frac{P_1^o}{P_1^d}\right]_t} - \widehat{mpo}_{1,t}\right) + \omega_1^{vy}\phi_1\left(\hat{r}_{1,t}^k - \widehat{mpk}_{1,t}\right) + \omega_1^{vy}(1-\phi_1)\left(\hat{w}_{1,t} - \widehat{mpl}_{1,t}\right) \tag{126}$$

since $\omega_1^{oy} + \omega_1^{vy}\phi_1 + \omega_1^{vy}(1-\phi_1) = 1$.

A.2 Balanced Growth Path

Along the balanced growth path real quantities grow at the common rate μ_z, except for oil demand and supply, and hours worked. Prices (relative to the domestic good), including real marginal costs, are constant except for real wages and the real price of oil. With labor augmenting technological progress, hours worked are stationary and real wages need to grow with the common growth rate μ_z. Oil supply and oil demand grow at the rate $\mu_o < \mu_z$, while the price of oil grows at the rate $\mu_{zo} < \frac{\mu_z}{\mu_o}$. Nominal prices grow at the inflation rate π_1^*. Below, we define relationships that need to hold along the balanced growth path. Unless noted otherwise, an expression presented for the home country s identical to the one for the foreign country. The size of country 1 relative to that of country 2 is denoted by ζ_1. $X_{i,t}^*$ refers to the value of variable X_i at time t along the balanced growth path. $X_{i,0}^*$ is the initial value of the variable at time 0.

A.2.1 Calibrated Expressions

Some parameters in our model are not estimated, but are implicitly pinned down by assigning data means to the following expressions.

1. Nominal oil use as share of nominal gross output $shareoy_1$:

$$shareoy_1 = \frac{P^{o*}_{1,t} O^*_{1,t}}{P^{d*}_{1,t} Y^{d*}_{1,t}} = \frac{P^{o*}_{1,0} \pi^{*t}_1 O^*_{1,0} \mu^t_o}{P^{d*}_{1,0} \pi^{*t}_1 Y^{d*}_{1,0} \mu^t_z} \mu^t_{zo} = \frac{P^{o*}_{1,0} O^*_{1,0}}{P^{d*}_{1,0} Y^{d*}_{1,0}} = \frac{O^*_{1,0}}{Y^{d*}_{1,0}}, \tag{127}$$

as the real price of oil is assumed to be 1 in period 0.

2. Ratio of oil use in production and oil use in consumption $ratiooyoc_1$:

$$ratiooyoc_1 = \frac{P^{o*}_{1,t} O^{y*}_{1,t}}{P^{o*}_{1,t} O^{c*}_{1,t}} = \frac{O^{y*}_{1,0}}{O^{c*}_{1,0}}. \tag{128}$$

3. Overall investment as share of gross output $shareiy_1$:

$$shareiy_1 = \frac{I^*_{1,t}}{Y^{d*}_{1,t}} = \frac{I^*_{1,0}}{Y^{d*}_{1,0}}. \tag{129}$$

4. Overall government consumption as share of gross output $sharegy_1$:

$$sharegy_1 = \frac{G^*_{1,t}}{Y^{d*}_{1,t}} = \frac{G^*_{1,0}}{Y^{d*}_{1,0}}. \tag{130}$$

5. Ratio of oil production to oil consumption $ratioyoo_1$:

$$ratioyoo_1 = \frac{Y^{o*}_{1,t}}{O^*_{1,t}} = \frac{Y^{o*}_{1,0}}{O^*_{1,0}}. \tag{131}$$

6. Overall imports as share of gross output $sharemy_1$:

$$sharemy_1 = \frac{M^*_{1,t}}{Y^{d*}_{1,t}} = \frac{M^*_{1,0}}{Y^{d*}_{1,0}}. \tag{132}$$

7. Ratio of imports in investment relative to imports in consumption $ratiomimc_1$:

$$ratiomimc_1 = \frac{M^{i*}_{1,t}}{M^{c*}_{1,t}} = \frac{M^{i*}_{1,0}}{M^{c*}_{1,0}}. \tag{133}$$

8. Share of hours worked $labshare_1$:

$$labshare_1 = \frac{L^*_{1,t}}{1 - L^*_{1,t}} = \frac{L^*_{1,0}}{1 - L^*_{1,0}}. \tag{134}$$

9. Normalization of ω^l_1:

$$\omega^l_1 = 1. \tag{135}$$

A.2.2 Composite Parameters

Given the parameter choices and the expressions described above, the remaining parameters of the model can be computed as shown below.

1. From condition 20 define $shareoyy_1$ and compute ω_1^{oy}:

$$\omega_1^{oy} = \frac{\mu_{zo}^t O_{1,t}^{y*}}{Y_{1,t}^{d*}} = \frac{O_{1,0}^{y*}}{Y_{1,0}^{d*}} = shareoyy_1 \tag{136}$$

$$shareoyy_1 = shareoy_1 \left(1 - \frac{1}{1 + ratiooyoc_1}\right). \tag{137}$$

2. From condition 17 define $sharevy_1$ and compute ω_1^{vy}:

$$\omega_1^{vy} = 1 - \omega_1^{oy} = \frac{V_{1,t}^*}{Y_{1,t}^{*d}} = \frac{V_{1,0}^*}{Y_{1,0}^{*d}} = sharevy_1. \tag{138}$$

3. From condition 6 compute $r_{1,0}^{k*}$:

$$r_{1,0}^{k*} = \frac{\mu_z}{\beta_1} - 1 + \delta_1. \tag{139}$$

4. From condition 7 define $shareky_1$:

$$\frac{K_{1,t}^*}{Y_{1,t}^{d*}} = \frac{K_{1,0}^*}{Y_{1,0}^{d*}} = \frac{1}{1 - \frac{1-\delta_1}{\mu_z}} shareiy_1 = shareky_1. \tag{140}$$

5. From conditions 7 and 18 define $omegak_1$:

$$\omega_1^k = \frac{1}{1 - \frac{1-\delta_1}{\mu_z}} \left(\frac{\mu_z}{\beta_1} - 1 + \delta_1\right)^{\frac{1+\rho_1^v}{\rho_1^v}} \frac{1}{\mu_z} \frac{shareiy_1}{sharevy_1}. \tag{141}$$

6. From condition 16 define $sharely_1$:

$$\frac{\mu_z^t L_{1,t}^*}{Y_{1,t}^{d*}} = \frac{L_{1,0}^*}{Y_{1,0}^{d*}} = sharevy_1 \left(1 - \omega_1^k \left(r_{1,0}^{k*}\right)^{\frac{-1}{\rho_1^v}}\right)^{1+\rho_1^v} = sharely_1. \tag{142}$$

7. From consolidated budget constraint define $sharecy_1$:

$$\frac{P_{1,t}^{c*} C_{1,t}^*}{P_{1,t}^{d*} Y_{1,t}^{d*}} = 1 - \frac{P_{1,t}^{d*} I_{1,t}^*}{P_{1,t}^{d*} Y_{1,t}^{d*}} - \frac{P_{1,t}^{d*} G_{1,t}^*}{P_{1,t}^{d*} Y_{1,t}^{d*}} + \frac{P_{1,t}^{o*} Y_{1,t}^{o*}}{P_{1,t}^{d*} Y_{1,t}^{d*}} - \frac{P_{1,t}^{o*} O_{1,t}^{y*}}{P_{1,t}^{d*} Y_{1,t}^{d*}} = sharecy_1 \tag{143}$$

$$sharecy_1 = 1 - shareiy_1 - sharegy_1 + shareyoy_1 - shareoyy_1. \tag{144}$$

8. From condition 12 define $shareocc_1$ and compute ω_1^{oc}:

$$\omega_1^{oc} = \frac{\mu_{zo}^t O_{1,t}^{c*}}{C_{1,t}^*} = \frac{O_{1,0}^{c*}}{C_{1,0}^*} = shareocc_1, \tag{145}$$

where

$$shareocc_1 = \frac{shareoy_1 - shareoyy_1}{sharecy_1}. \tag{146}$$

9. From condition 8 compute ω_1^{cc}:

$$\omega_1^{cc} = 1 - \omega_1^{oc} = \frac{C_{1,t}^{ne*}}{C_{1,t}^*} = \frac{C_{1,0}^{ne*}}{C_{1,0}^*}. \tag{147}$$

10. From condition 10 compute ω_1^c:

$$\omega_1^c = 1 - \omega^{mc} = \frac{C_{1,t}^{d*}}{C_{1,t}^{ne*}} = \frac{C_{1,0}^{d*}}{C_{1,0}^{ne*}} = sharecdcn_1. \tag{148}$$

11. From condition 11 define $sharemccn_1$:

$$\omega_1^{mc} = \frac{M_{1,t}^{c*}}{{C^{ne*}}_{1,t}} = \frac{M_{1,0}^{c*}}{C_{1,0}^{ne*}} = sharemccn_1, \tag{149}$$

where $sharemccn_1$ is computed from

$$sharemccn_1 = \frac{sharemy_1}{sharecy_1 sharecnc_1}\frac{1}{1 + ratiomimc_1}. \tag{150}$$

12. From condition 14 compute ω_1^i:

$$\omega_1^i = 1 - \omega_1^{mi} = \frac{I_{1,t}^{d*}}{I_{1,t}^*} = \frac{I_{1,0}^{d*}}{I_{1,0}^*}. \tag{151}$$

13. From condition 15 define $sharemii_1$ and compute ω^{mi}:

$$\omega_1^{mi} = \frac{M_{1,t}^{i*}}{I_{1,t}^*} = \frac{M_{1,0}^{i*}}{I_{1,0}^*} = sharemii_1, \tag{152}$$

where

$$sharemii_1 = \frac{sharemy_1}{shareiy_1}\frac{ratiomimc_1}{1 + ratiomimc_1}. \tag{153}$$

14. Define exports relative of gross output country 1 $sharexy_1$:

$$\frac{P^{d*}_{1,t}X^*_{1,t}}{P^{d*}_{1,t}Y^d_{1,t}} = \frac{P^{d*}_{1,t}M^*_{1,t}}{P^{d*}_{1,t}Y^{d*}_{1,t}} + \frac{P^{o*}_{1,t}O^*_{1,t}}{P^{d*}_{1,t}Y^{d*}_{1,t}} - \frac{P^{o*}_{1,t}Y^{o*}_{1,t}}{P^{d*}_{1,t}Y^{d*}_{1,t}} = sharexy_1, \tag{154}$$

or

$$sharexy_1 = sharemy_1 + shareoy_1 - shareyoy_1. \tag{155}$$

15. Define exports relative of gross output country 2 $sharexy_2$:

$$\frac{X^*_{2,t}}{Y^d_{2,t}} = \frac{M^*_{2.t}}{Y^{d*}_{2,t}} - \left(\frac{O^{y*}_{1,t}}{Y^{d*}_{1,t}} + \frac{Y^{o*}_{1,t}}{Y^{d*}_{1,t}}\right)\frac{Y^{d*}_{1,t}}{Y^{d*}_{2,t}}\zeta_1, = sharexy_2, \tag{156}$$

or

$$sharexy_2 = sharemy_2 - (shareoy_1 - shareyoy_1)\frac{Y^{d*}_{1,0}}{Y^{d*}_{2,0}}\zeta_1. \tag{157}$$

16. If trade is balanced along the balanced growth path, we define $sharemy_2$:

$$\frac{P^{d*}_{1,t}M^*_{1,t}}{P^{d*}_{1,t}Y^{d*}_{1,t}} + \frac{P^{o*}_{1,t}\left(O^*_{1,t} - Y^{o*}_{1,t}\right)}{P^{d*}_{1,t}Y^{d*}_{1,t}} = \frac{e^*_{1,t}P^{d*}_{2,t}M^*_{2,t}}{P^{d*}_{2,t}Y^{d*}_{2,t}}\frac{P^{d*}_{2,t}Y^{d*}_{2,t}}{P^{d*}_{1,t}Y^{d*}_{1,t}}\frac{1}{\zeta_1} = sharemy_2, \tag{158}$$

or

$$sharemy_2 = (sharemy_1 + (shareoy_1 - shareyoy_1))\frac{Y^{d*}_{1,0}}{Y^{d*}_{2,0}}\zeta_1 \tag{159}$$

17. From condition 42 define $sharengdpny_1$:

$$\frac{NGDP^*_{1,t}}{P^{d*}_{1,t}Y^{d*}_{1,t}} = 1 - \frac{P^{o*}_{1,t}O^{y*}_{1,t}}{P^{d*}_{1,t}Y^{d*}_{1,t}} + \frac{P^{o*}_{1,t}Y^{o*}_{1,t}}{P^{d*}_{1,t}Y^{d*}_{1,t}} = 1 - \frac{P^{o*}_{1,0}O^{y*}_{1,0}}{P^{d*}_{1,0}Y^{d*}_{1,0}} + \frac{P^{o*}_{1,0}Y^{o*}_{1,0}}{P^{d*}_{1,0}Y^{d*}_{1,0}}, \tag{160}$$

or

$$sharengdpny_1 = 1 - shareoyy_1 + shareyoy_1. \tag{161}$$

18. From condition 34 define $mugdpss_1$:

$$\mu_{gdp,1} = \frac{\mu_z - (shareoyy_1 - shareyoy_1)\,\mu_o}{1 - (shareoyy_1 - shareyoy_1)}. \tag{162}$$

19. Gross output ratio $\frac{Y^{d*}_{1,t}}{Y^{d*}_{2,t}}$:

$$\frac{Y^{d*}_{1,t}}{Y^{d*}_{2,t}} = \frac{Y^{d*}_{1,0}}{Y^{d*}_{2,0}} = \frac{sharely_2}{sharely_1}\frac{L^*_{1,0}}{L^*_{2,0}}. \tag{163}$$

20. From condition 39 define the share in world oil production for country 2 $shareoprod_2$:

$$\frac{Y^{o*}_{2,t}}{\zeta_1 Y^{o*}_{1,t} + Y^{o*}_{2,t}} = \frac{\frac{Y^{o*}_{2,0}}{Y^{d*}_{2,0}}}{\zeta_1 \frac{Y^{o*}_{1,0}}{Y^{d*}_{1,0}} \frac{Y^{d*}_{1,0}}{Y^{d*}_{2,0}} + \frac{Y^{o*}_{2,0}}{Y^{d*}_{2,0}}} = shareoprod_2, \tag{164}$$

or

$$shareoprod_2 = \frac{shareyoy_2}{shareyoy_2 + shareyoy_1 \zeta_1 \frac{Y^{d*}_{1,0}}{Y^{d*}_{2,0}}}. \tag{165}$$

21. From condition 39 define the share in world oil consumption for country 2 $shareocon_2$:

$$\frac{O^{*}_{2,t}}{\zeta_1 O^{*}_{1,t} + Y^{o*}_{2,t}} = \frac{\frac{O^{*}_{2,0}}{Y^{d*}_{2,0}}}{\zeta_1 \frac{O^{*}_{1,0}}{Y^{d*}_{1,0}} \frac{Y^{d*}_{1,0}}{Y^{d*}_{2,0}} + \frac{O^{*}_{2,0}}{Y^{d*}_{2,0}}} = shareocon_2, \tag{166}$$

or

$$shareocon_2 = \frac{shareoy_2}{shareoy_2 + shareoy_1 \zeta_1 \frac{Y^{d*}_{1,0}}{Y^{d*}_{2,0}}}. \tag{167}$$

22. Overall oil production as share of gross output $shareyoy_1$:

$$\frac{Y^{o*}_{1,t}}{Y^{d*}_{1,t}} = \frac{Y^{o*}_{1,t}}{O^{*}_{1,t}} \frac{O^{*}_{1,t}}{Y^{d*}_{1,t}} = shareyoy_1, \tag{168}$$

or

$$shareyoy_1 = ratioyoo_1 shareoy_1. \tag{169}$$

B Appendix: Data

The model is estimated by the method of maximum likelihood. The data are quarterly and run between 1984 and the third quarter of 2008. A presample of 10 years from 1974 to 1984 is used to train the Kalman filter used to form the likelihood. The observed series are the following:

1. the log of U.S. real GDP, from NIPA Table 1.1.3 (line 1);

2. the log of trade-weighted foreign GDP. The series reflects GDP data from national sources for the 26 most important trading partners of the United States. A description of the export weights is in **?**). The countries included account for well over 90% of U.S. exports, as well as imports.

3. the log of the U.S. real dollar price of oil defined as the refiners' acquisition cost for imported crude from the U.S. Energy Information Administration ($http://www.eia.doe.gov/dnav/pet/pet\_pri\_rac2\_dcu\_nus\_m.htm$) normalized by the GDP deflator from NIPA Table 1.1.4 (line 1);

4. the log of U.S. crude oil production from Table 11.1b of the Monthly Energy Review of the U.S. Energy Information Administration ($http://www.eia.doe.gov/totalenergy/data/monthly/\#petroleum$);

5. the log of foreign oil production (calculated as world production net of U.S. production) from Table 11.b of the Monthly Energy Review of the U.S. Energy Information Administration ($http://www.eia.doe.gov/totalenergy/data/monthly/\#petroleum$);

6. the log of U.S. hours worked in the nonfarm business sector from the Labor Productivity and Cost Database of the U.S. Bureau of Labor Statistics, normalized by the U.S. civilian non-institutional population from the U.S. Bureau of Labor Statistics ($http://data.bls.gov/pdq/querytool.jsp?survey = pr$);

7. the log of the broad real dollar exchange rate from the U.S. Federal Reserve Board ($http://www.federalreserve.gov/releases/H10/summary/$, the weights are the same as the ones described for the measure of foreign GDP above);

8. U.S. personal consumption expenditures from NIPA Table 1.1.5 (line 2), expressed as a share of U.S GDP from NIPA Table 1.1.5 (line 1);

9. U.S. crude oil imports from the Energy Information Administration ($http://www.eia.doe.gov/dnav/pet/pet\_move\_impcus\_d\_NUS\_Z00\_mbbl\_m.htm$), expressed as a share of U.S. GDP using the refiners' acquisition cost for imported crude and GDP from NIPA Table 1.1.5 (line 1);

10. U.S. imports of non-petroleum goods from NIPA Table 4.2.5 (line 54), expressed as a share of GDP;

11. U.S. goods exports from NIPA Table 4.2.5 (line 2), expressed as a share of GDP;

12. U.S. fixed investment from NIPA Table 4.2.5 (line 8), expressed as a share of GDP;
13. U.S. core inflation measured as the log change in the deflator for personal consumption expenditures excluding food and energy prices from the Federal Reserve Bank of St. Louis Fred Database,
14. U.S. wage inflation (demeaned) measured using the log change in nominal compensation from the Labor Productivity and Cost Database of the U.S. Bureau of Labor Statistics ($http://data.bls.gov/pdq/querytool.jsp?survey = pr$);
15. and the U.S. effective federal funds rate (demeaned) from the Federal Reserve Board ($http://www.federalreserve.gov/releases/h15/data.htm$).

C Appendix: Plots of Data and In-Sample Forecast Errors

This appendix contains plots of all the observed data, the in-sample 1 step ahead forecasts, and the forecast errors.

Figure 1: Data and Forecast Errors

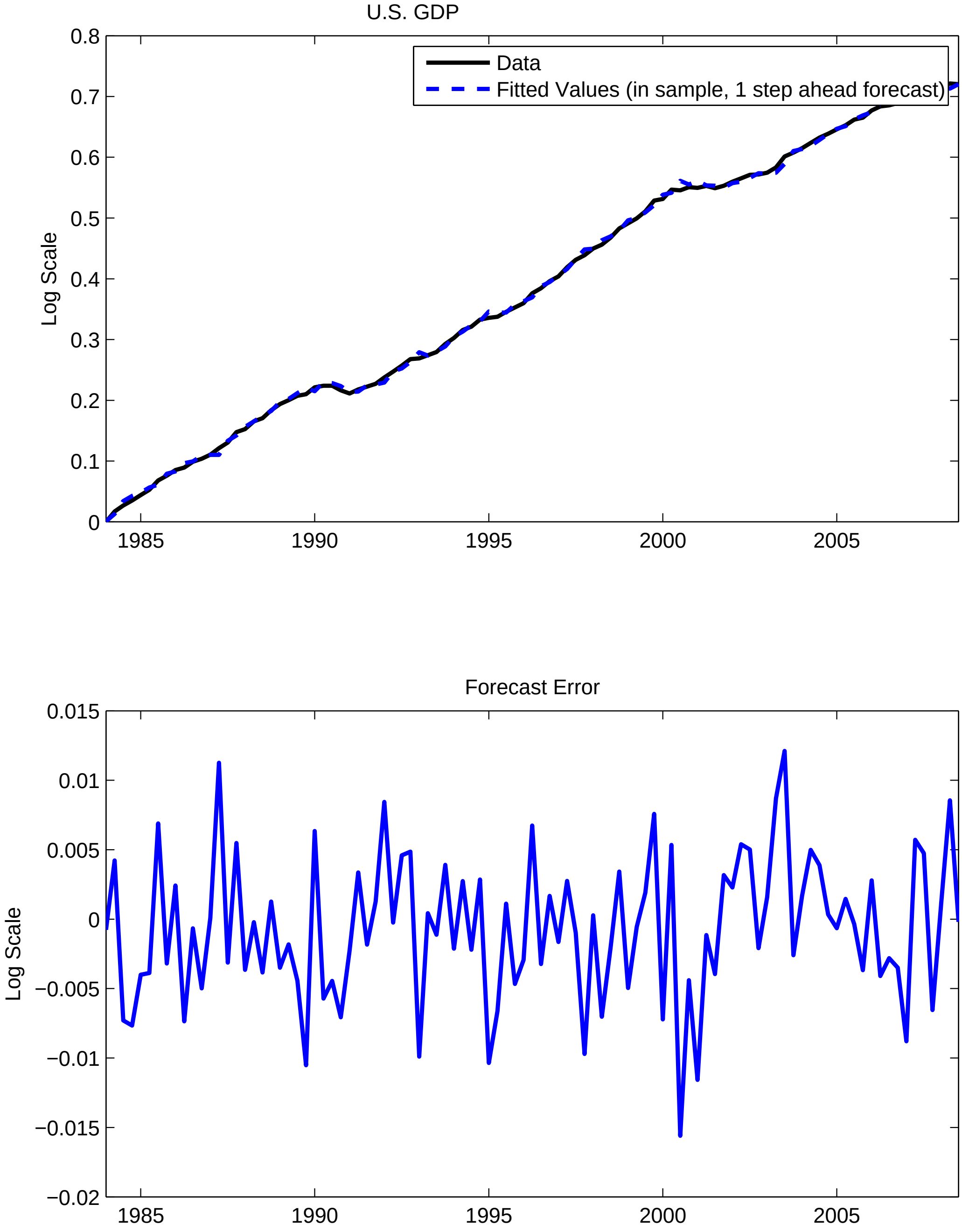
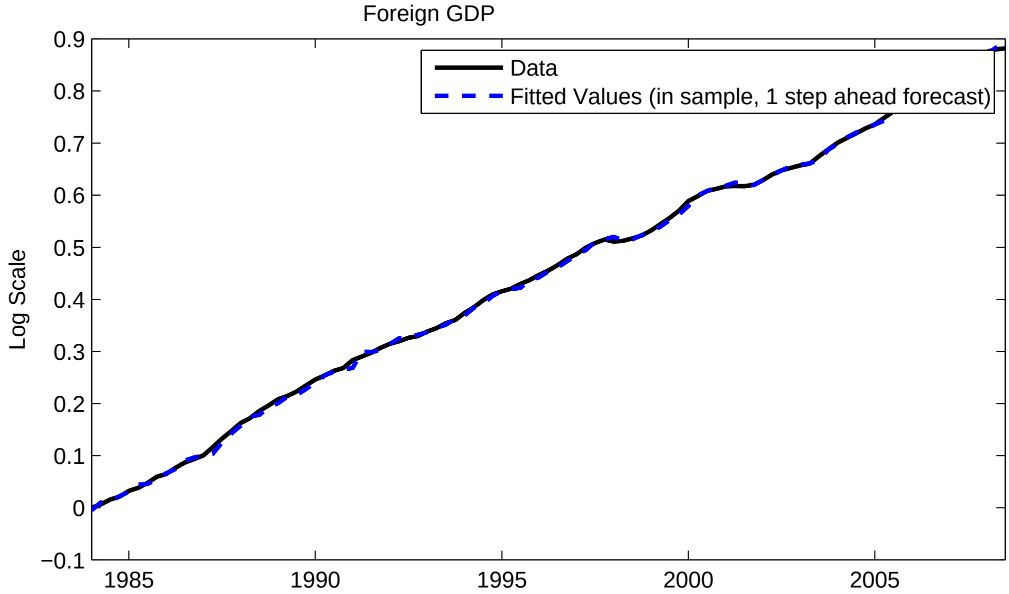


Figure 2: Data and Forecast Errors



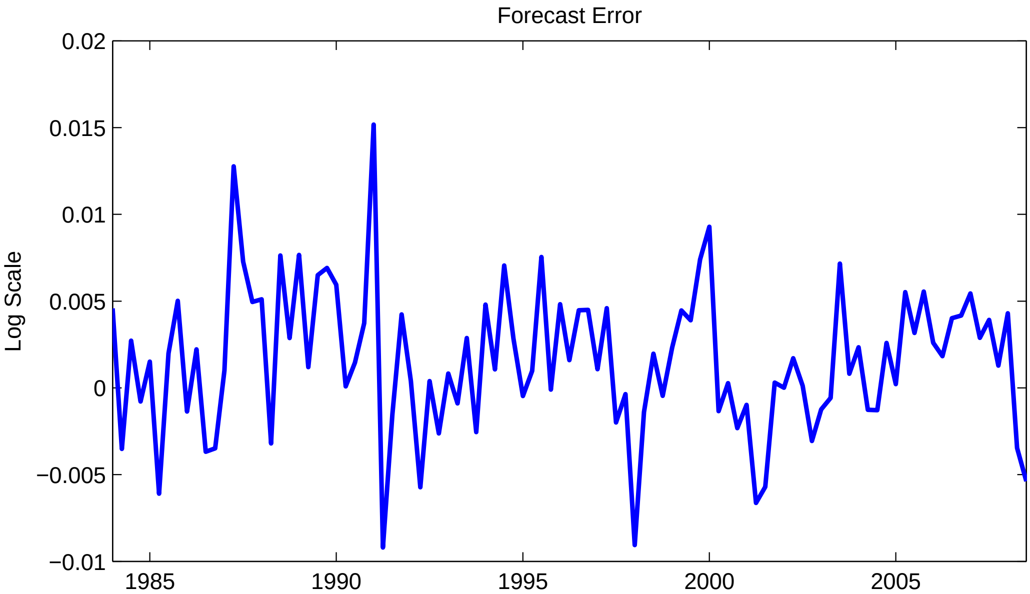
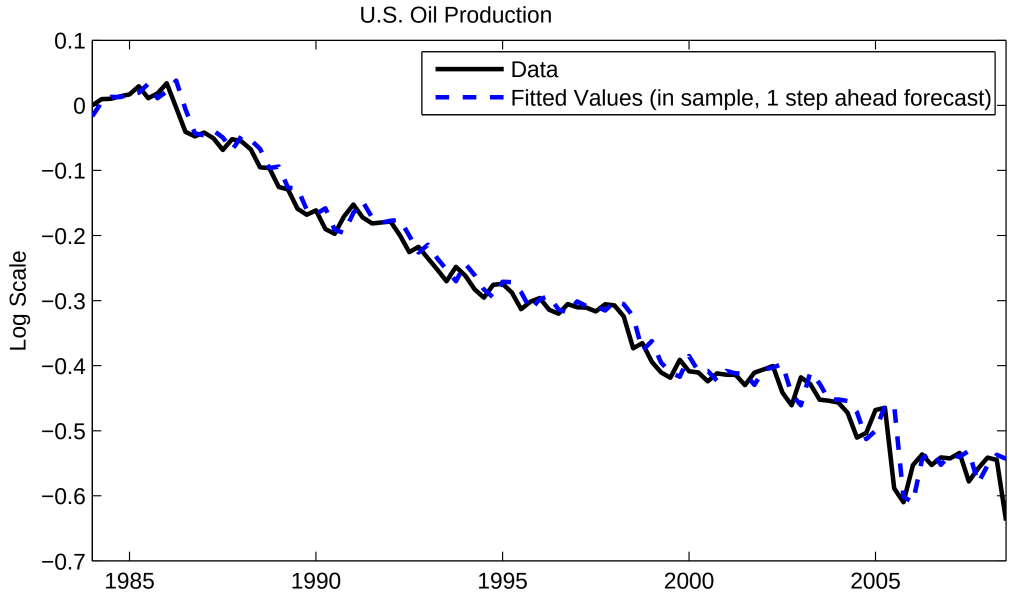


Figure 3: Data and Forecast Errors



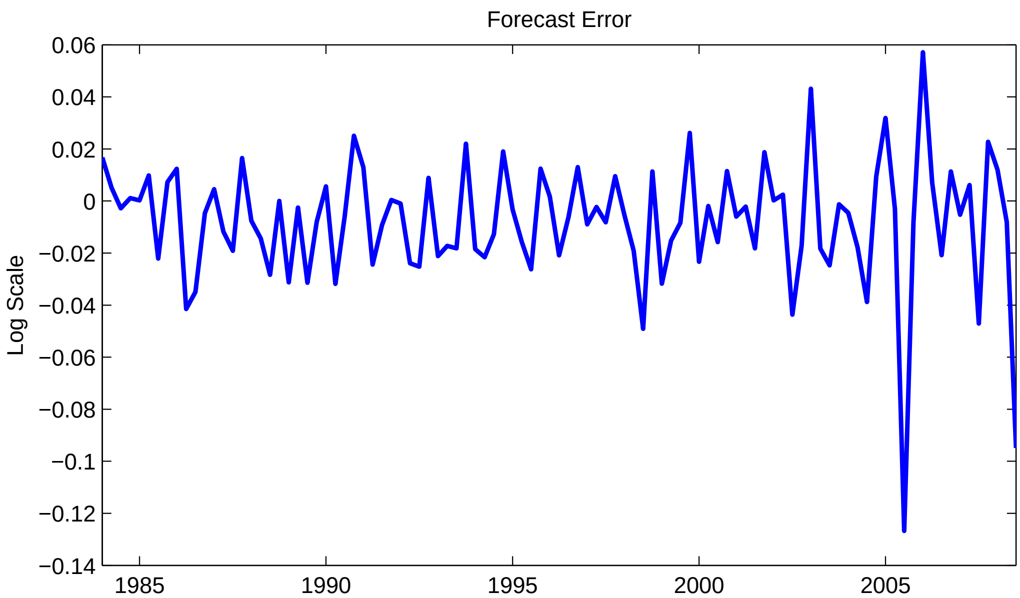


Figure 4: Data and Forecast Errors

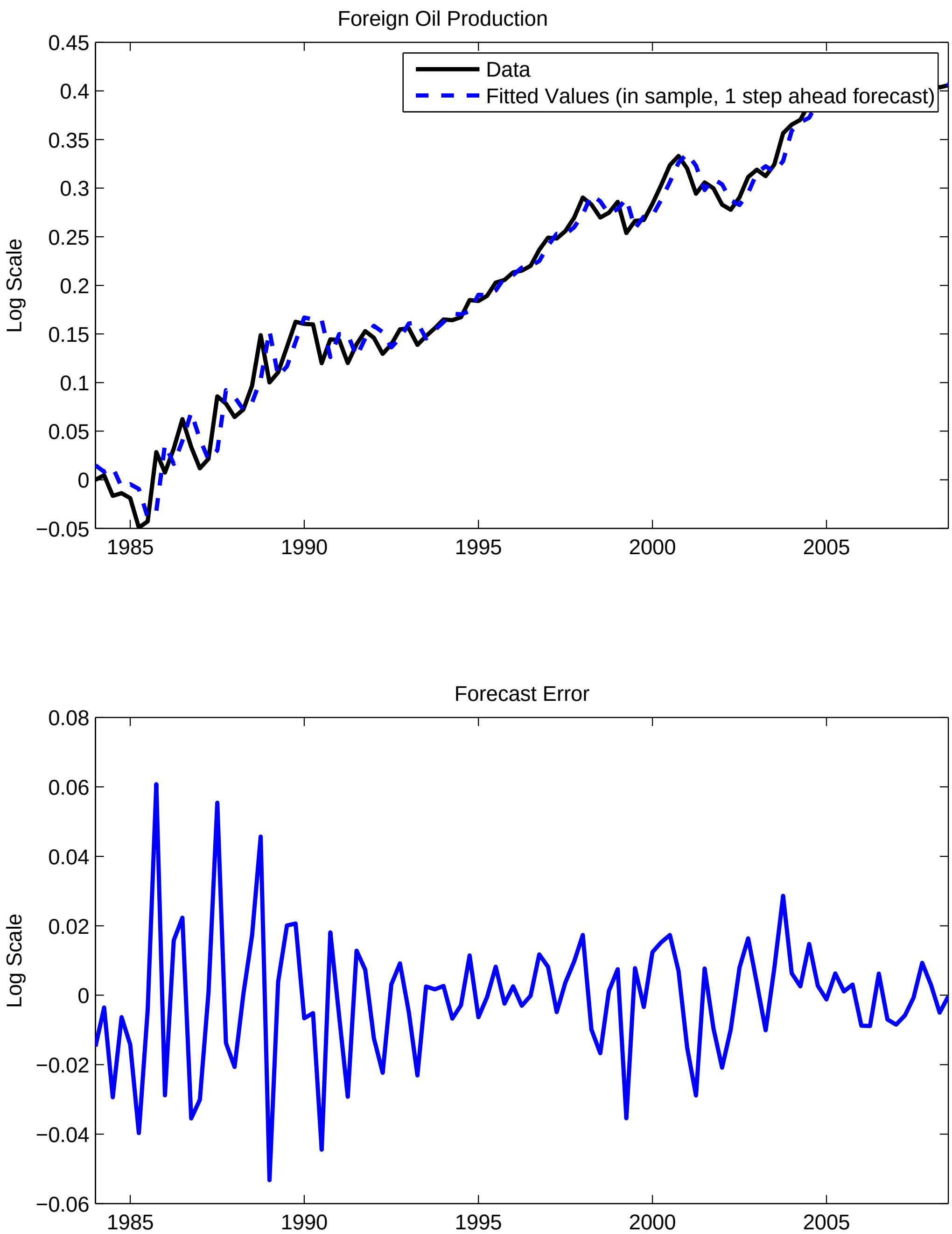


Figure 5: Data and Forecast Errors

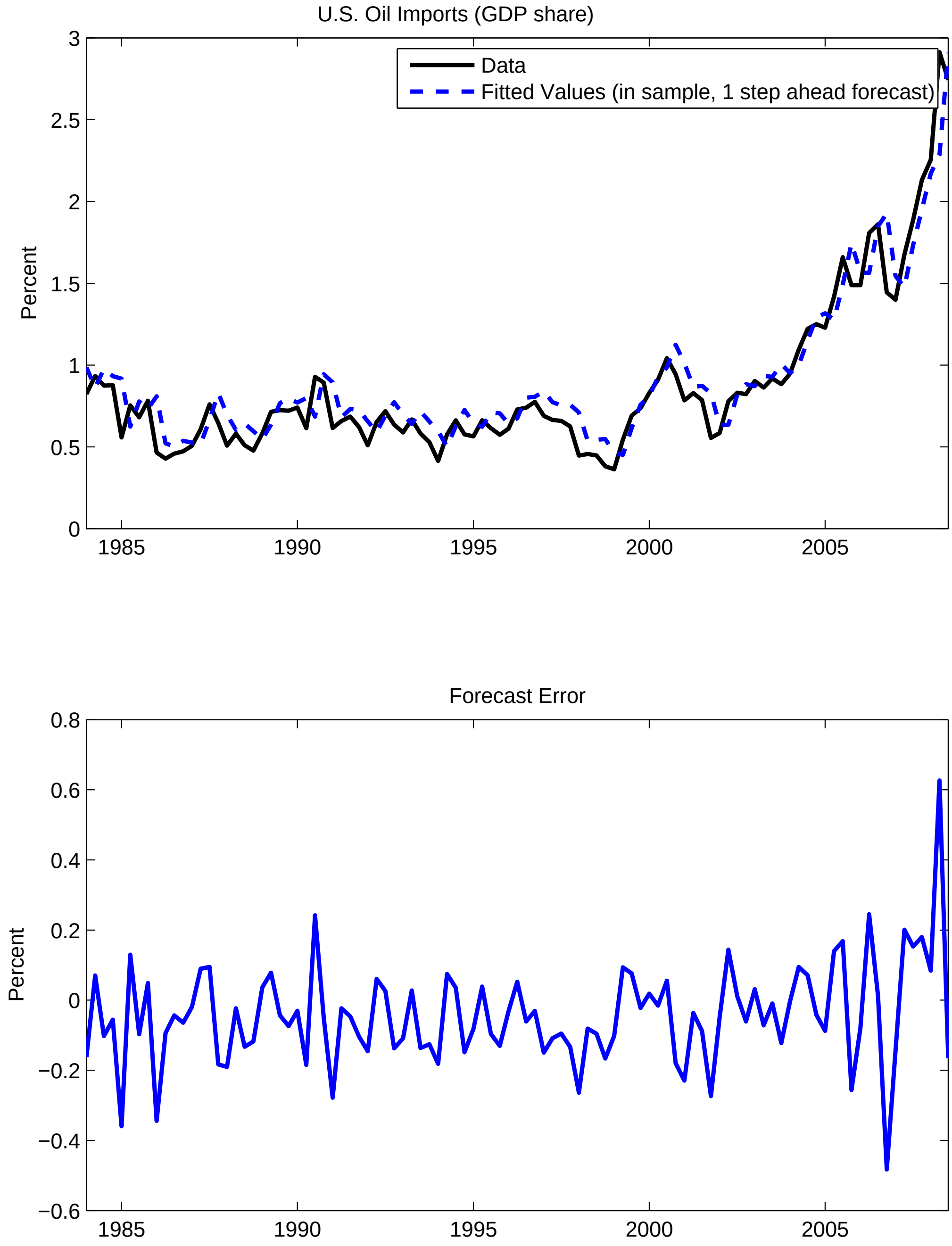


Figure 6: Data and Forecast Errors

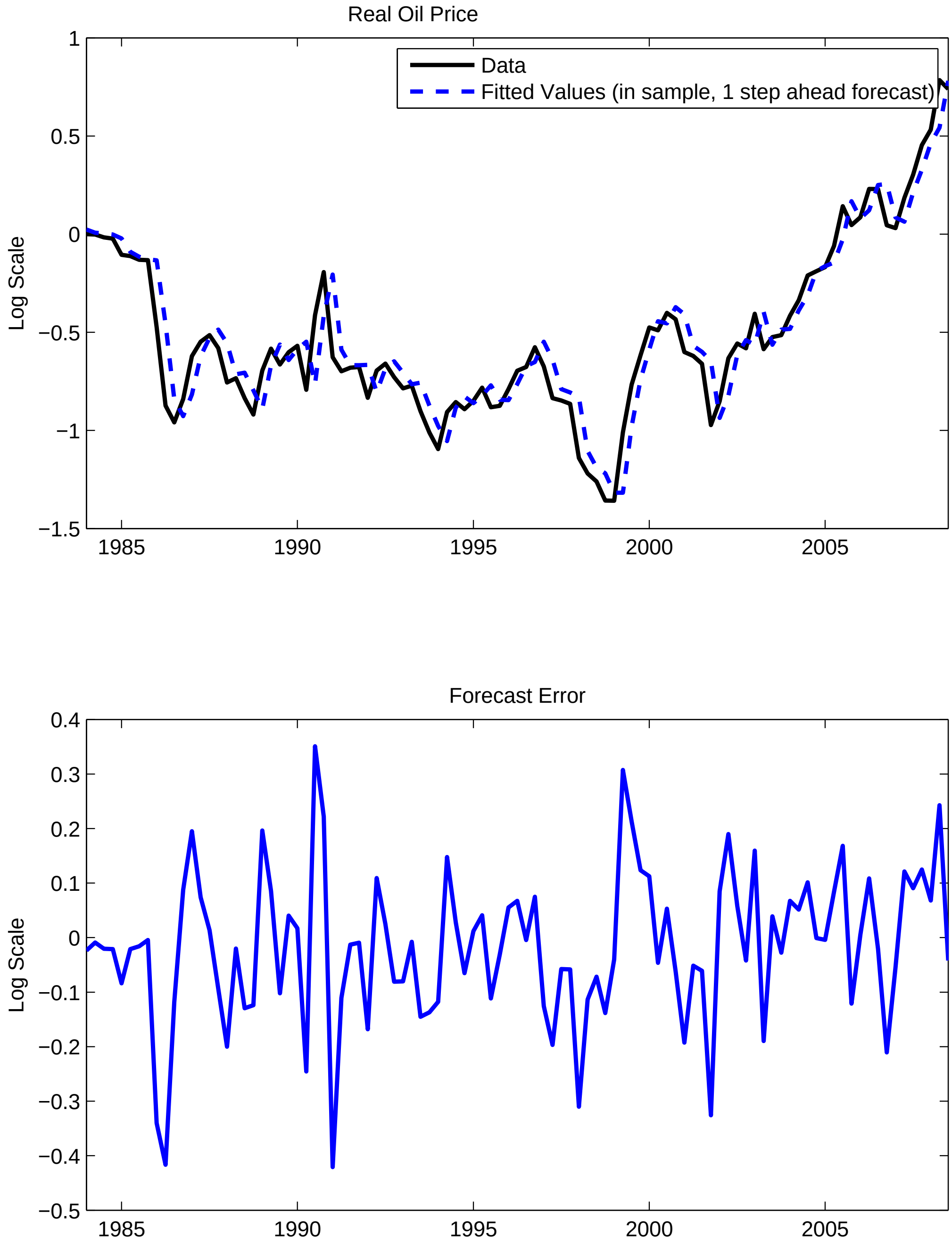


Figure 7: Data and Forecast Errors

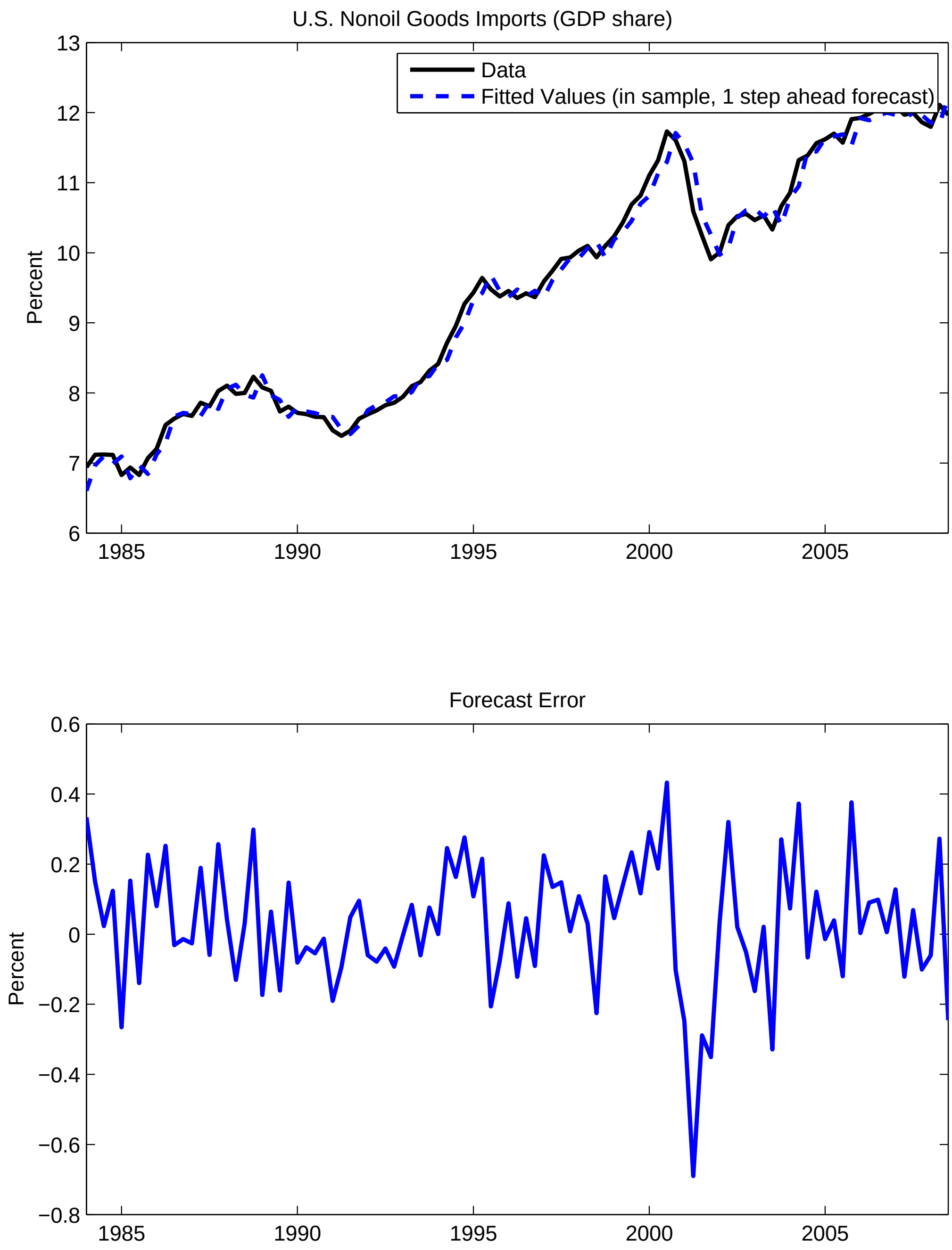
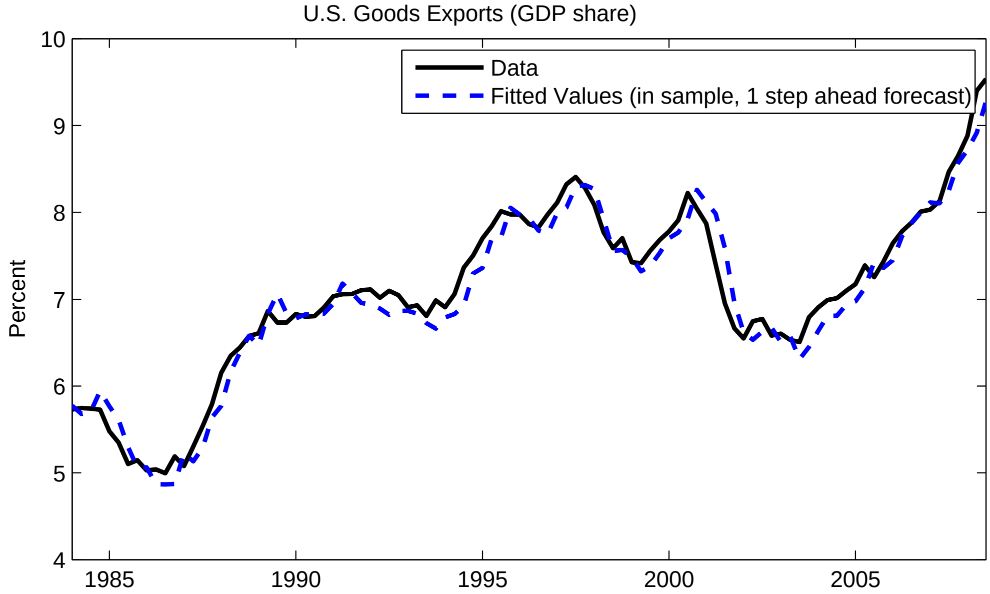


Figure 8: Data and Forecast Errors



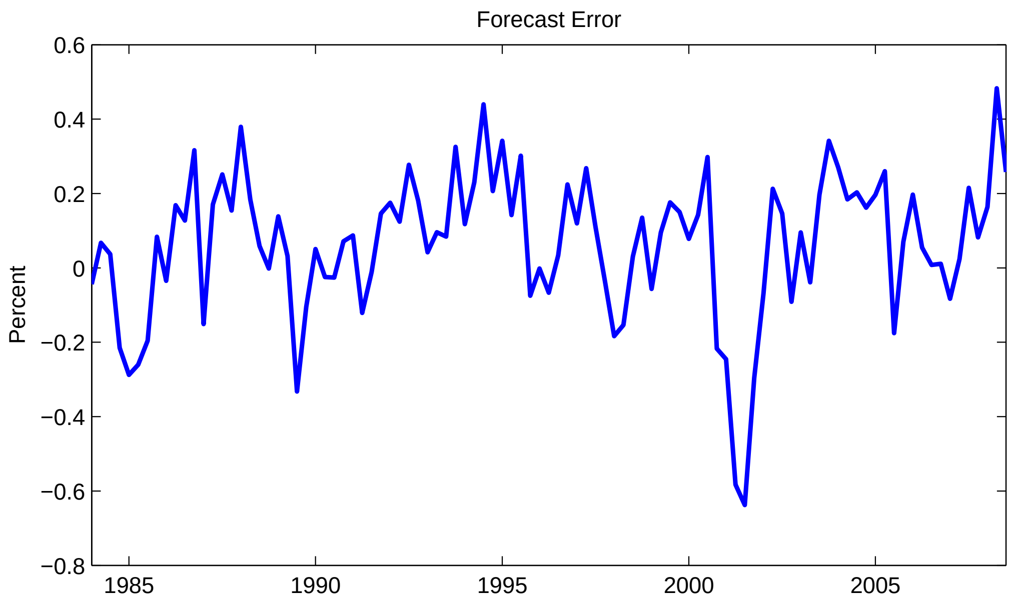


Figure 9: Data and Forecast Errors

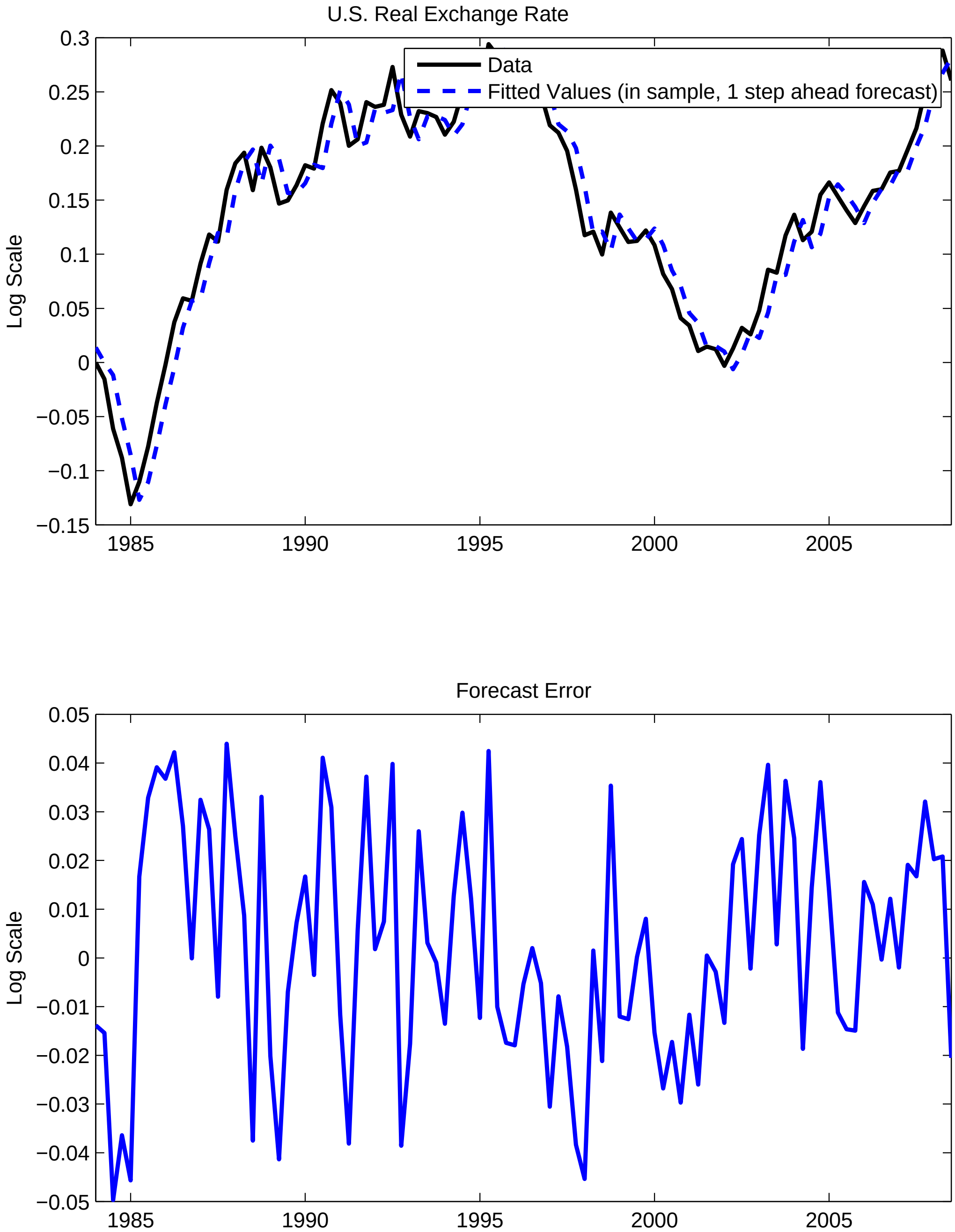


Figure 10: Data and Forecast Errors

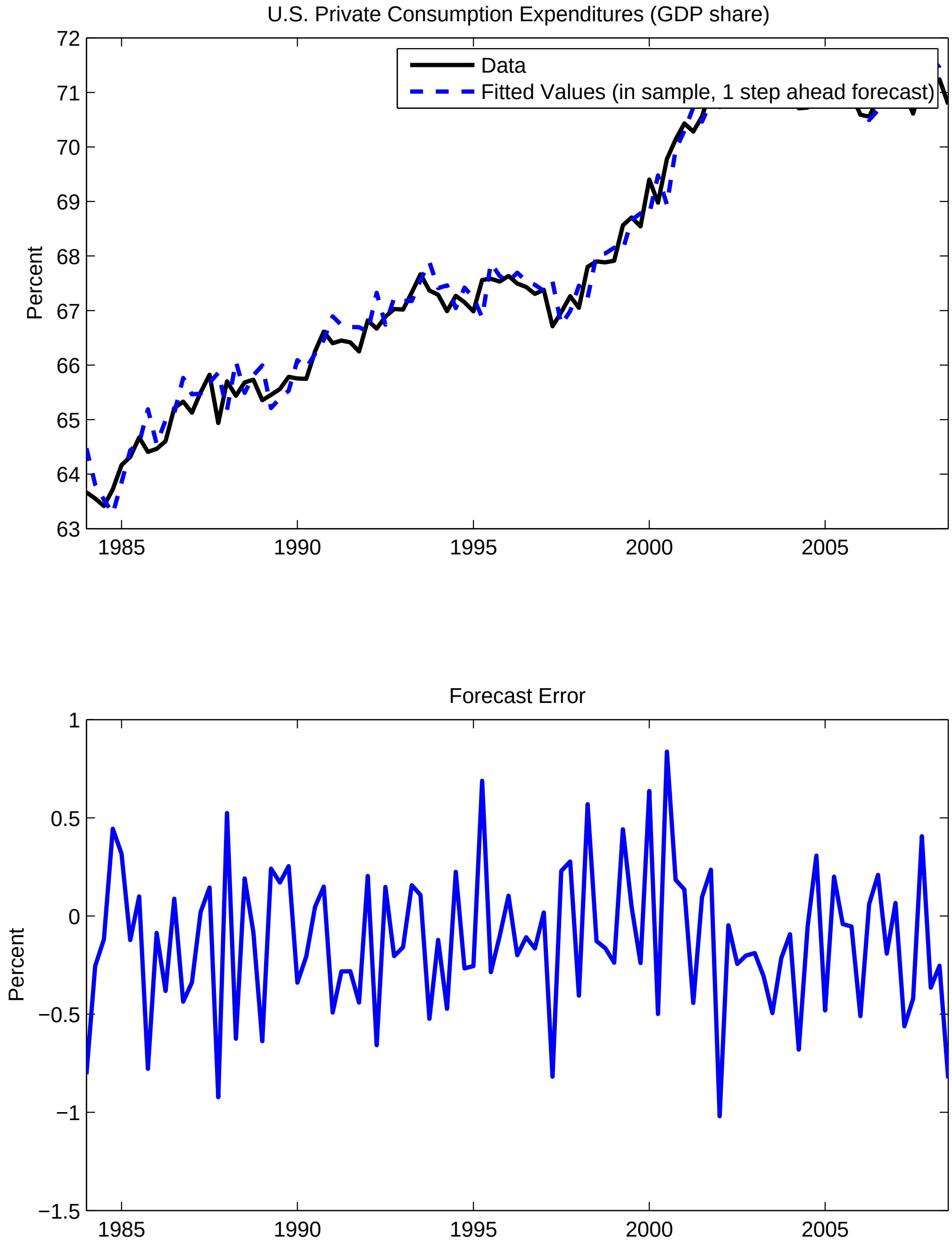
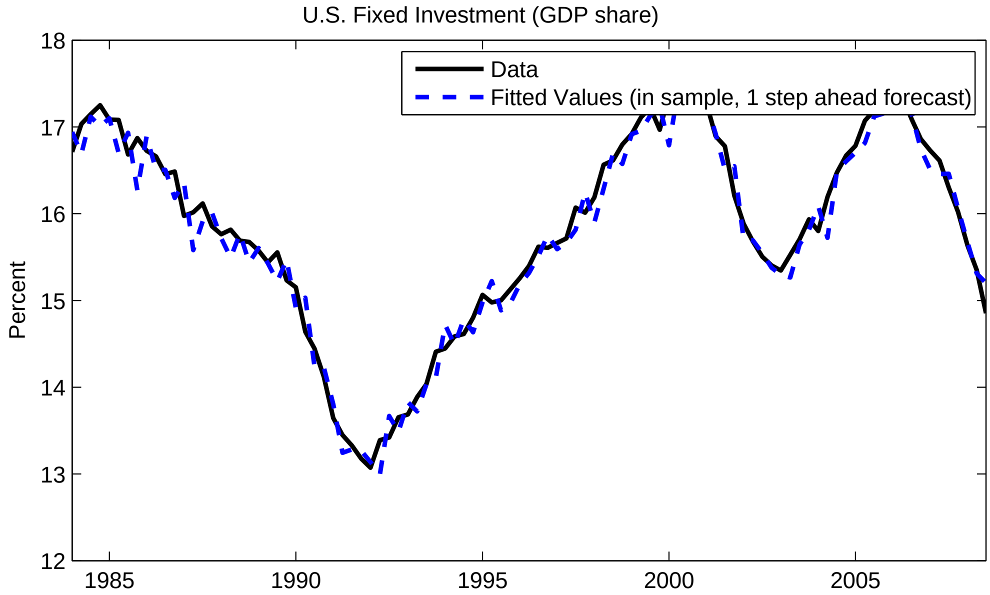


Figure 11: Data and Forecast Errors



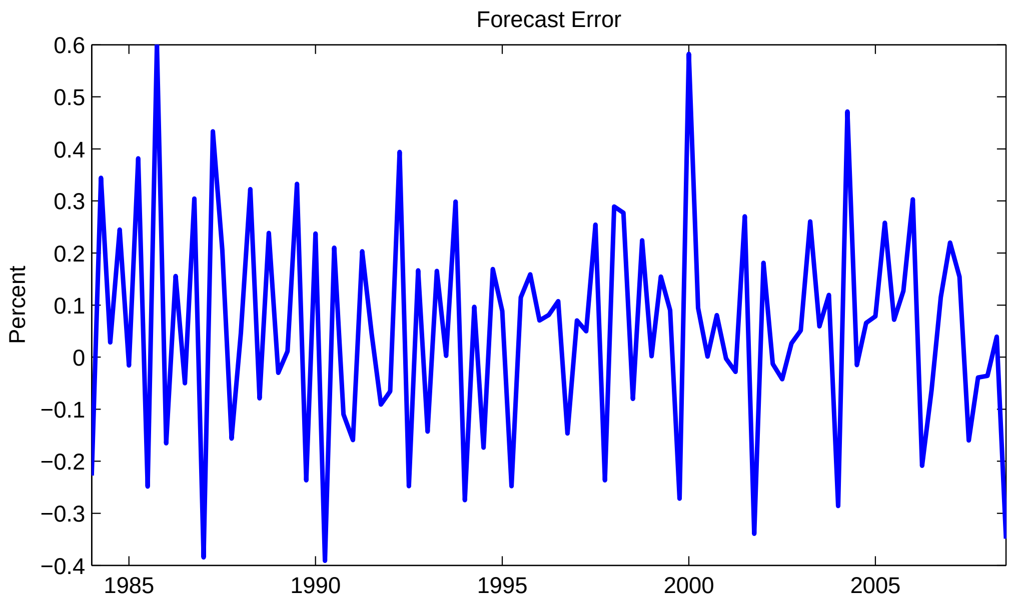


Figure 12: Data and Forecast Errors

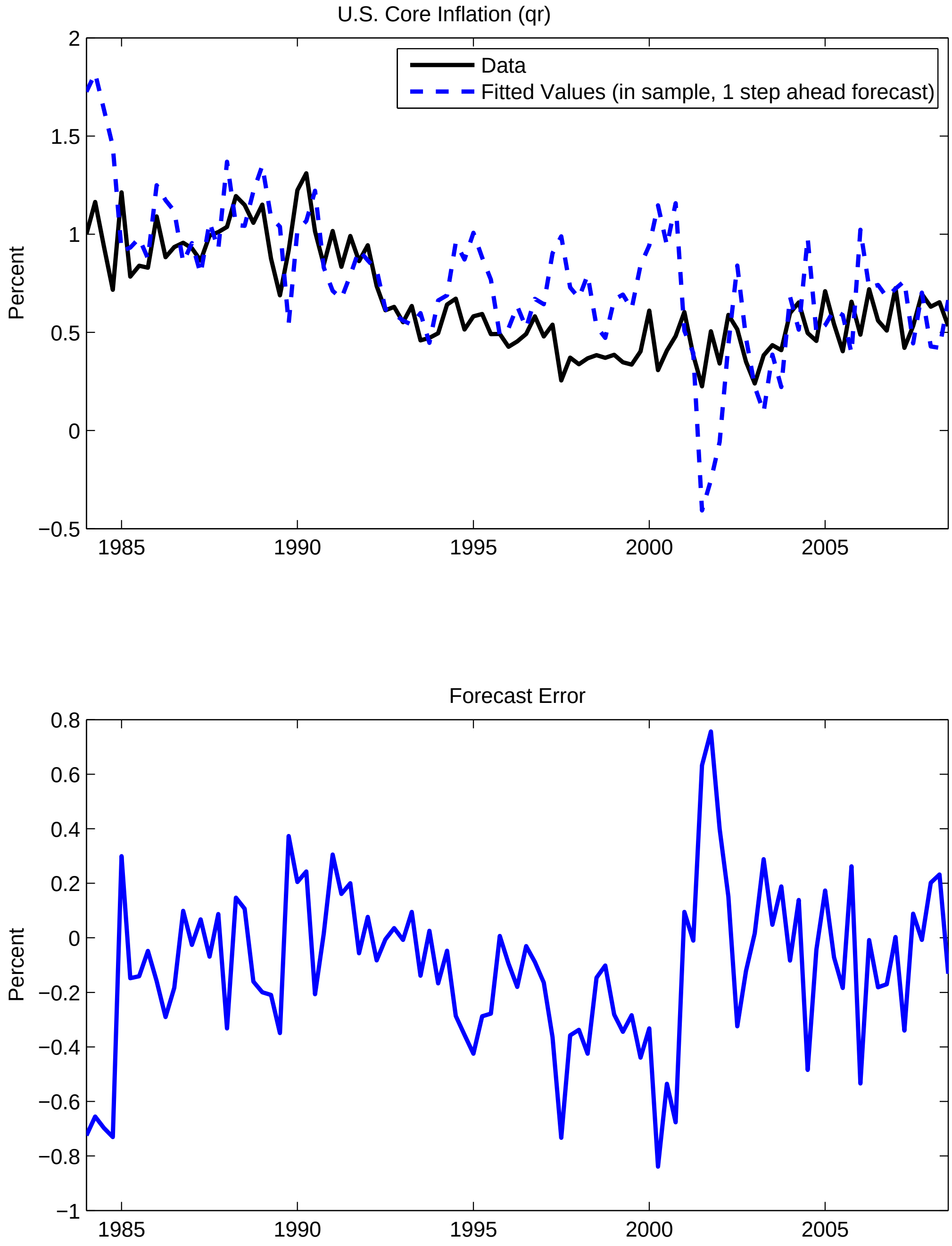


Figure 13: Data and Forecast Errors

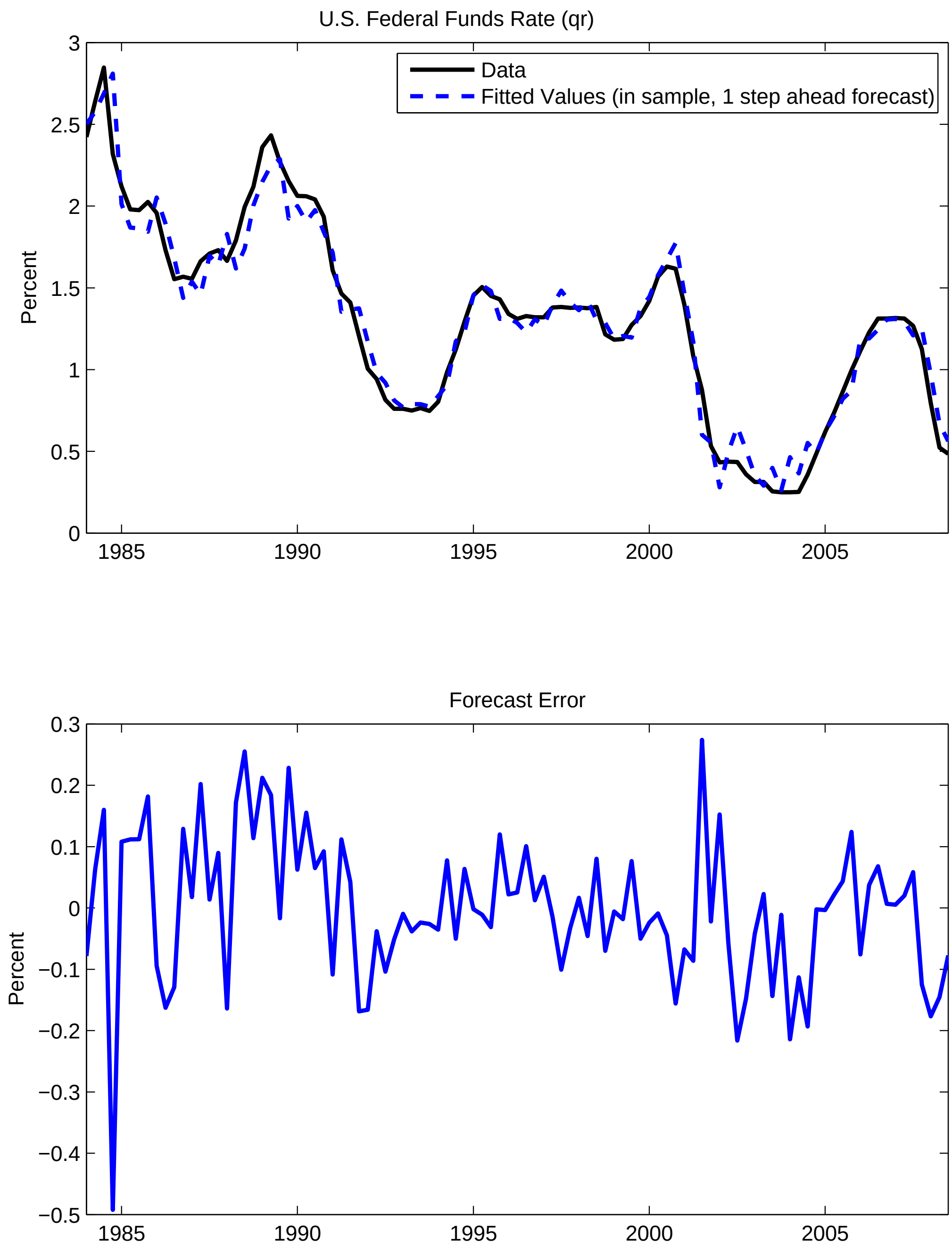
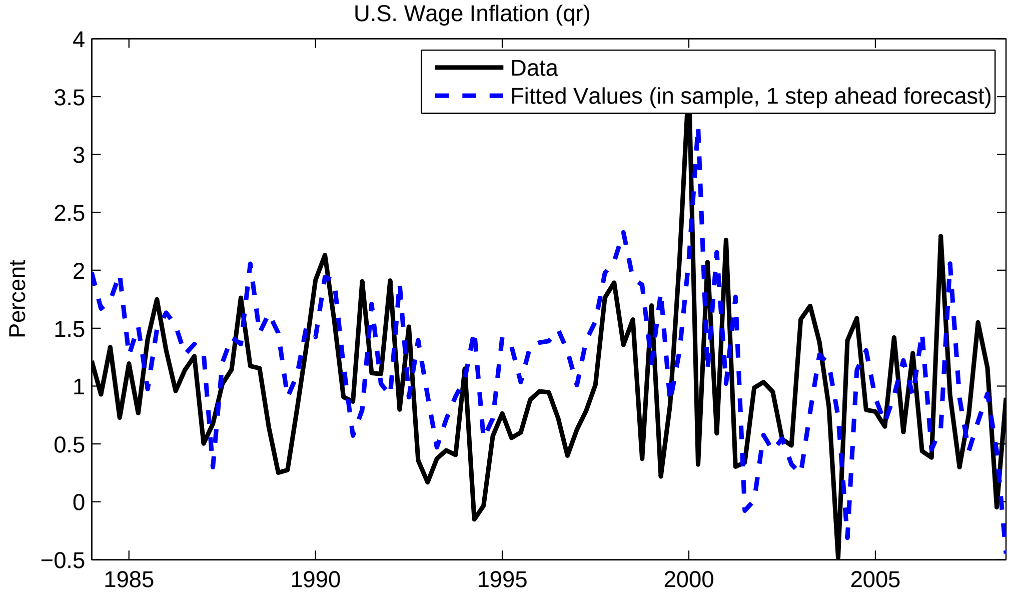


Figure 14: Data and Forecast Errors



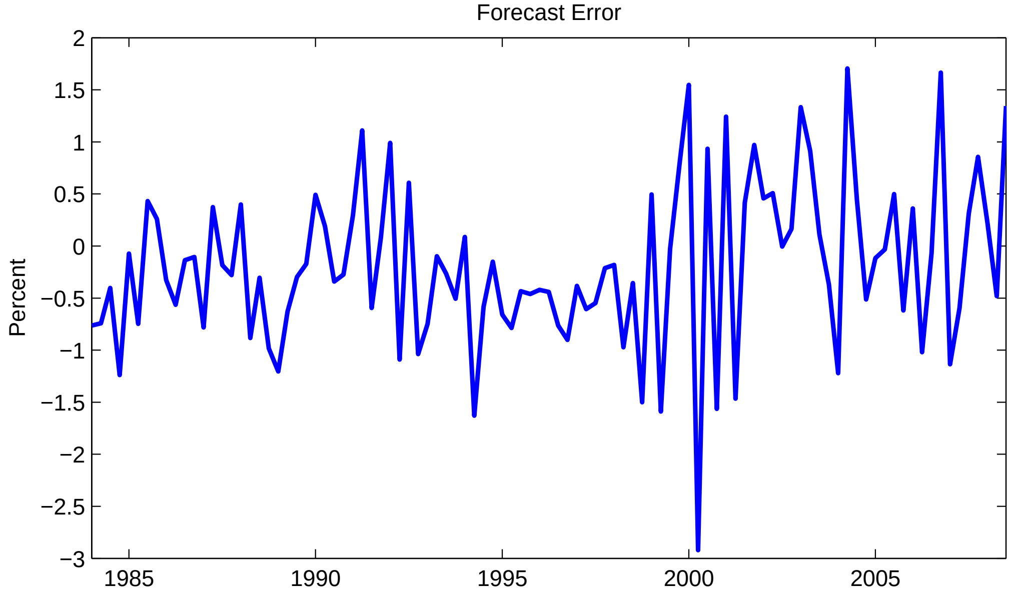
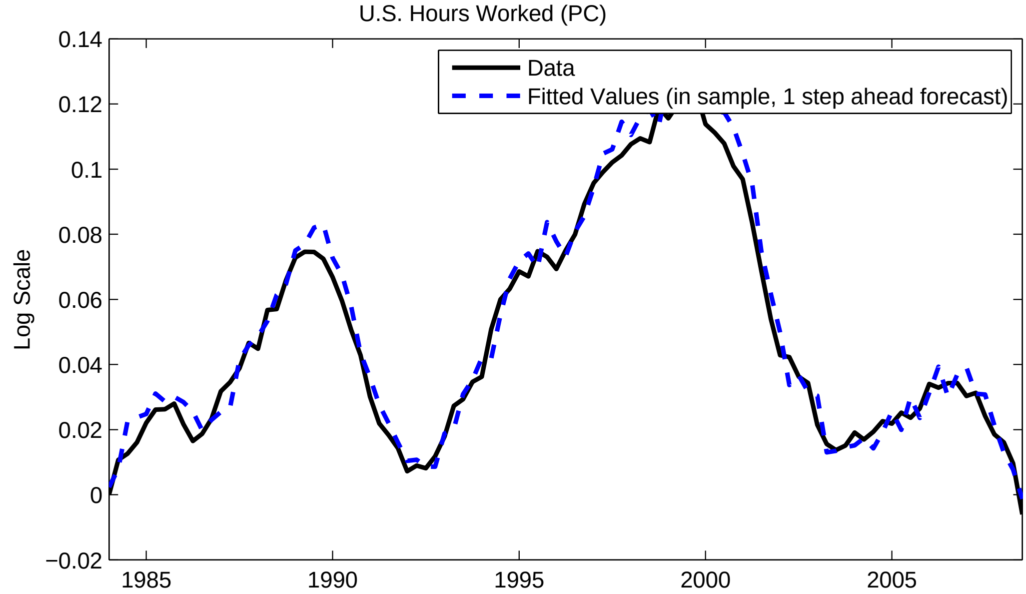
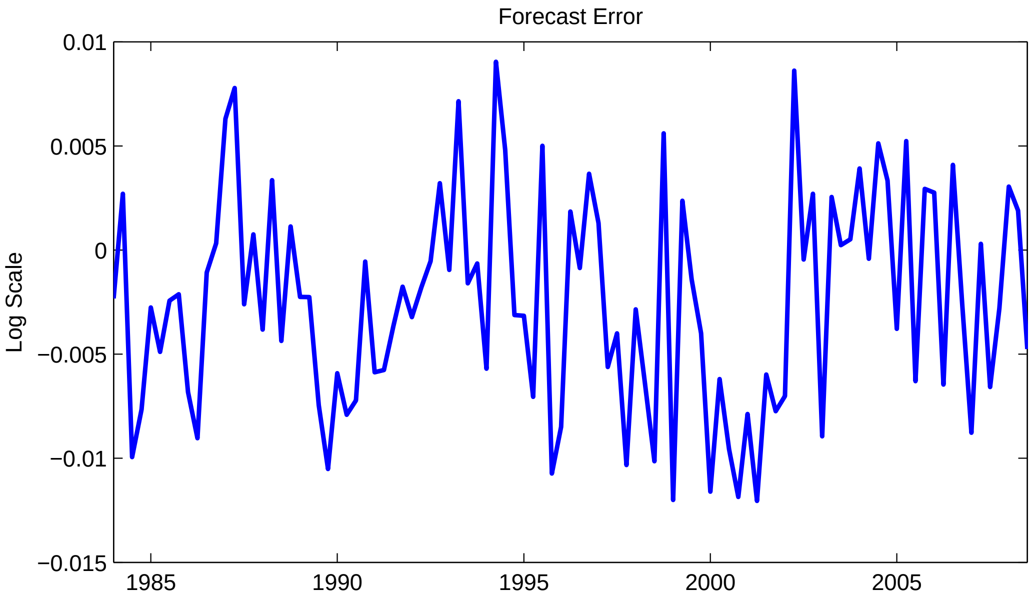


Figure 15: Data and Forecast Errors

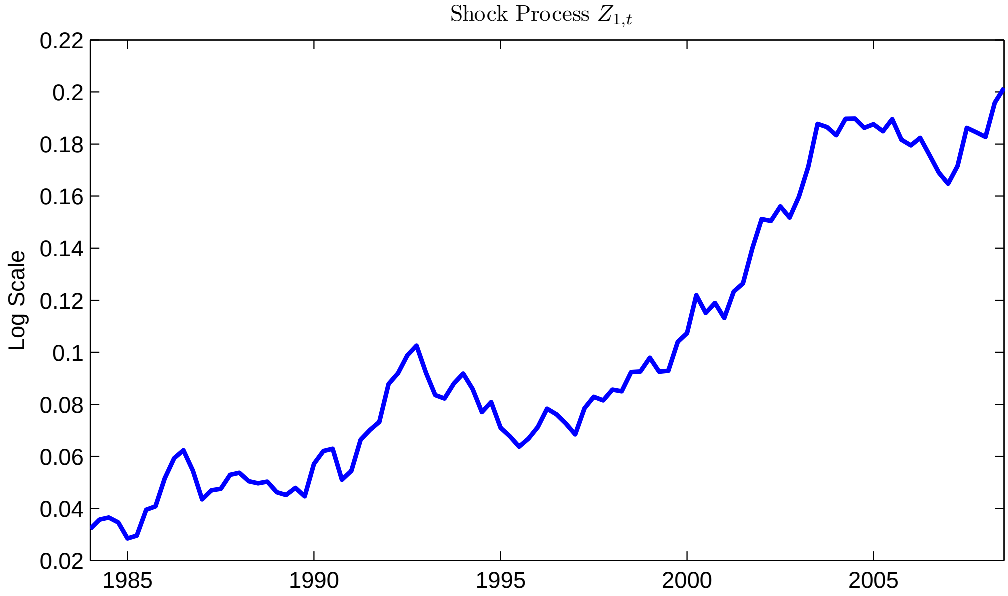




D Appendix: Plots of Smoothed Estimates of Shock Processes and Their Innovations

This appendix contains plots of all the shock processes and their innovations. A key to the symbols denoting each shock process is given in Table 1 in the main body of the paper.

Figure 16: Data and Forecast Errors



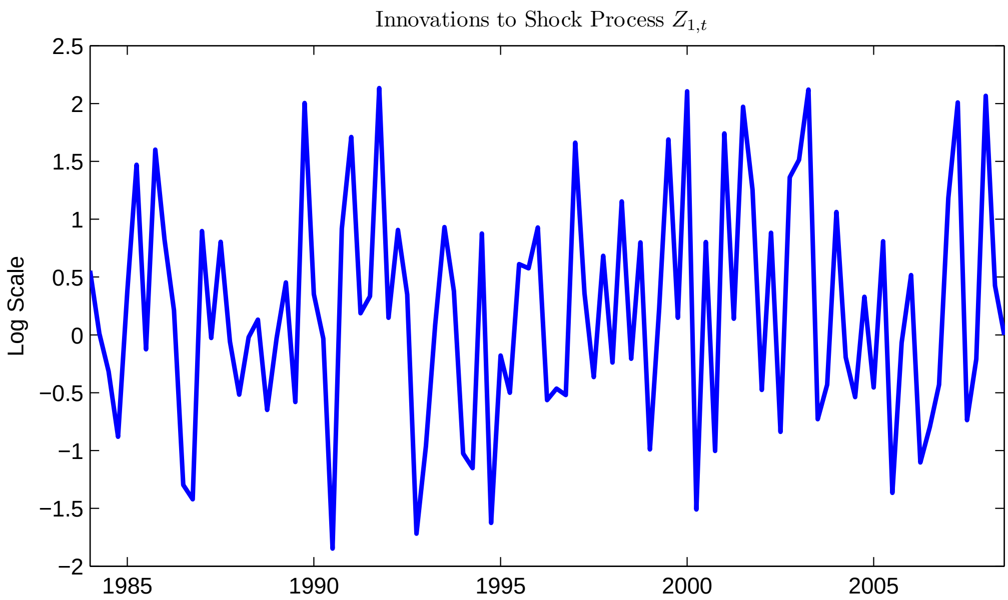
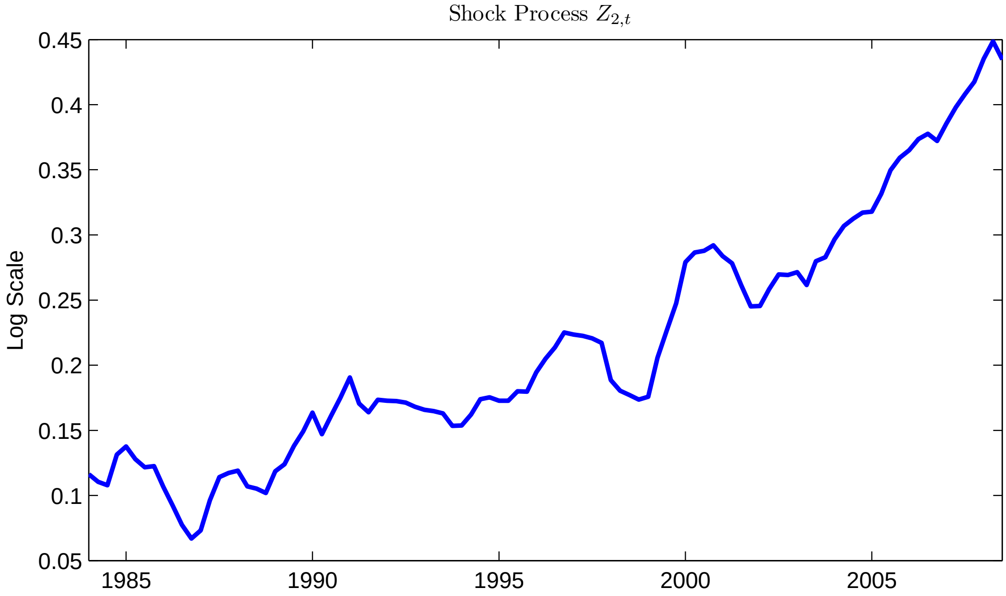


Figure 17: Data and Forecast Errors



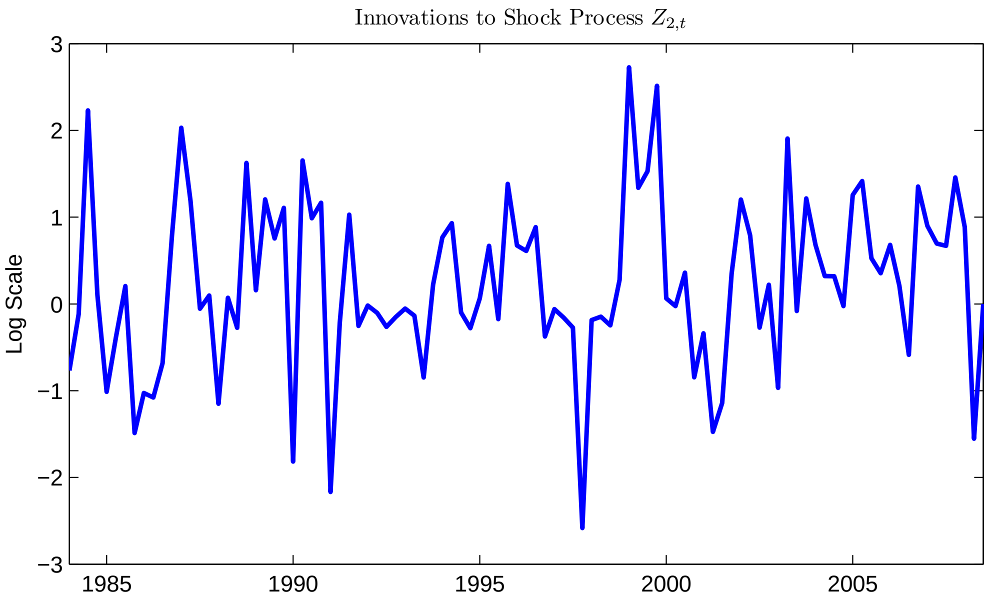
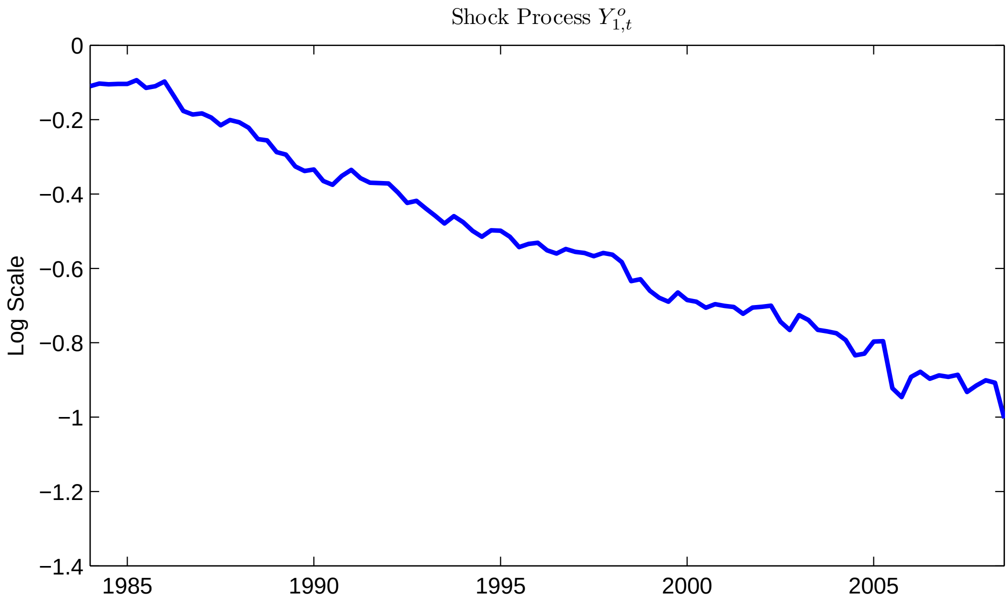


Figure 18: Data and Forecast Errors



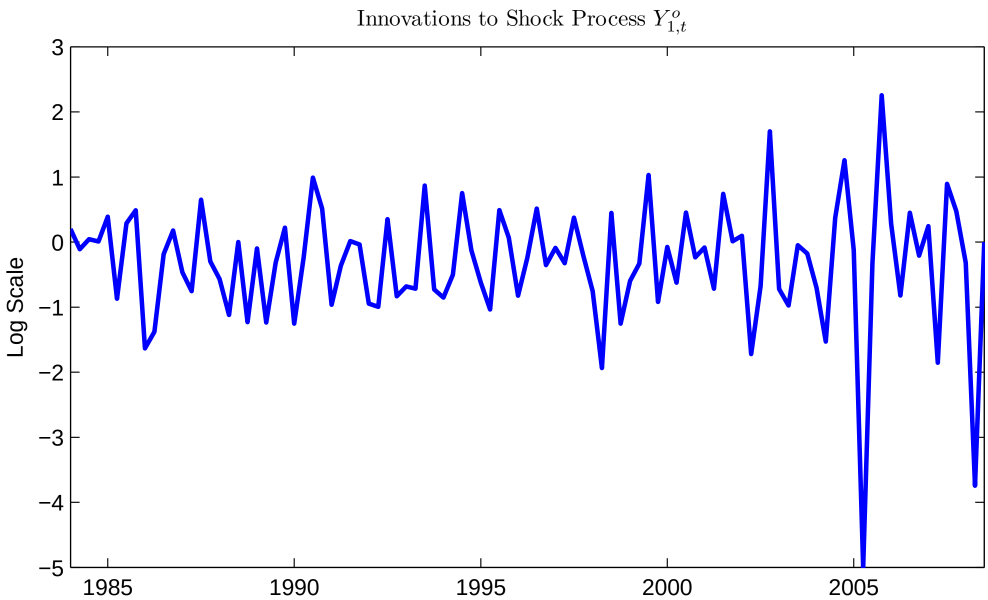


Figure 19: Data and Forecast Errors

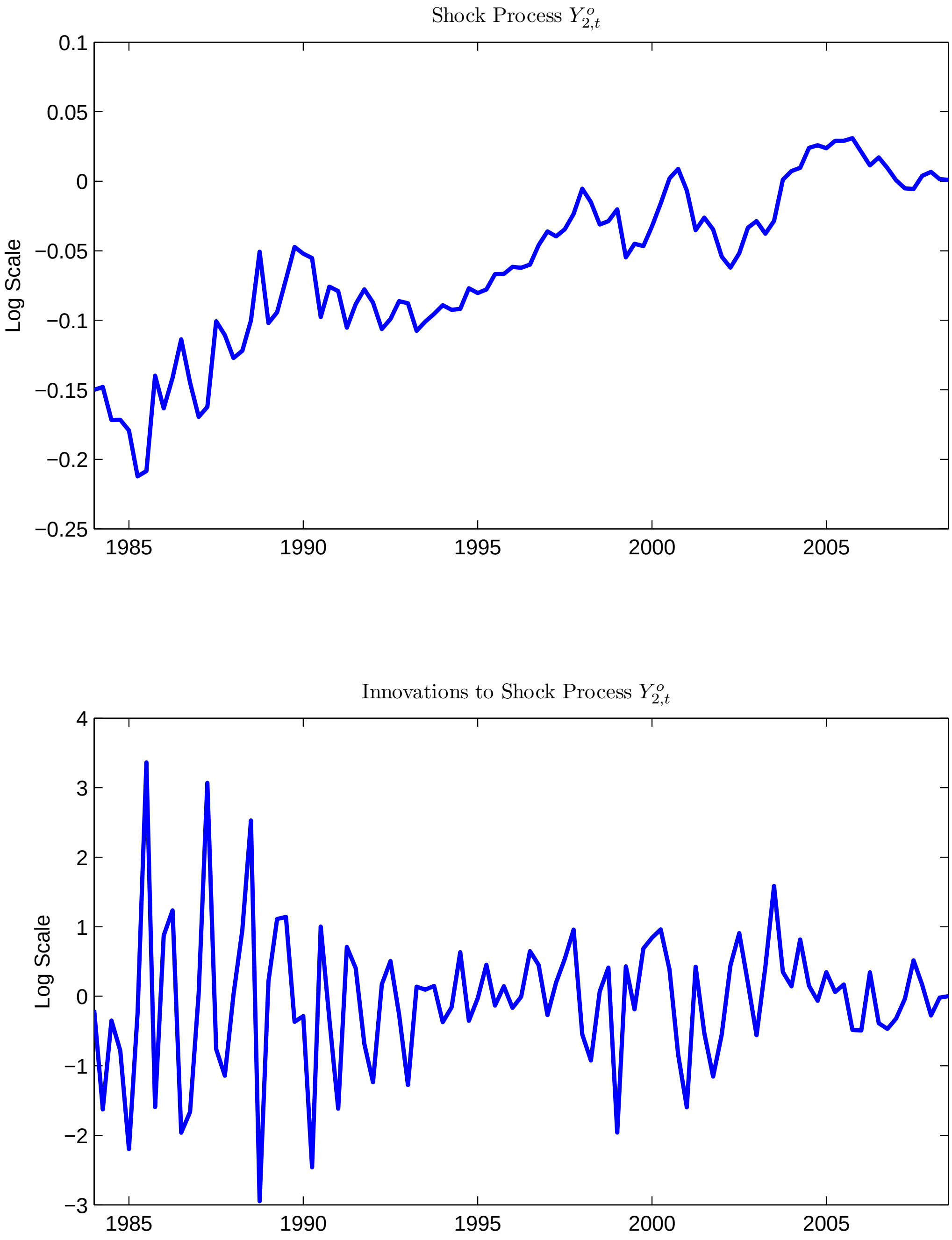


Figure 20: Data and Forecast Errors

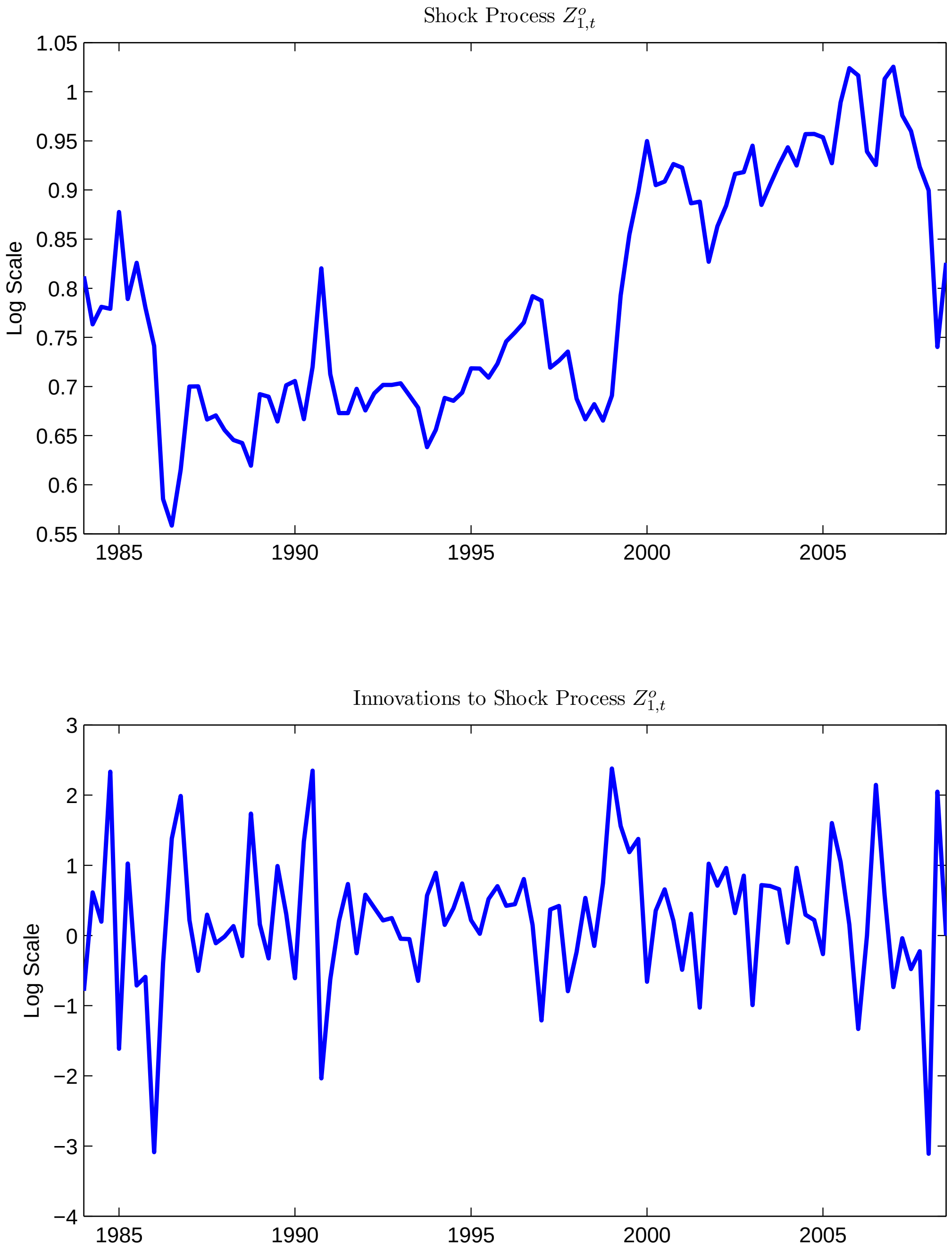
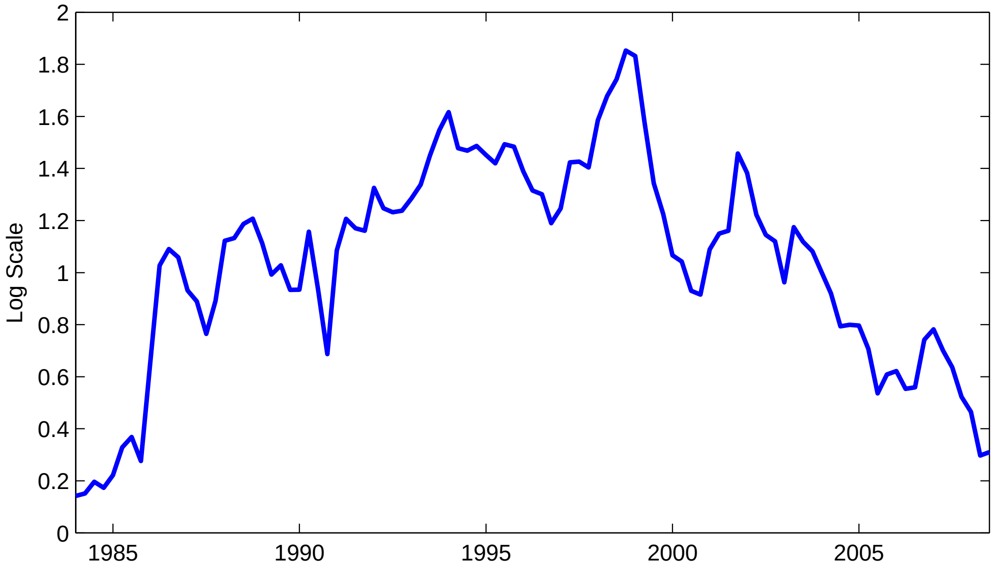


Figure 21: Data and Forecast Errors

Shock Process $Z^o_{2,t}$



Innovations to Shock Process $Z^o_{2,t}$

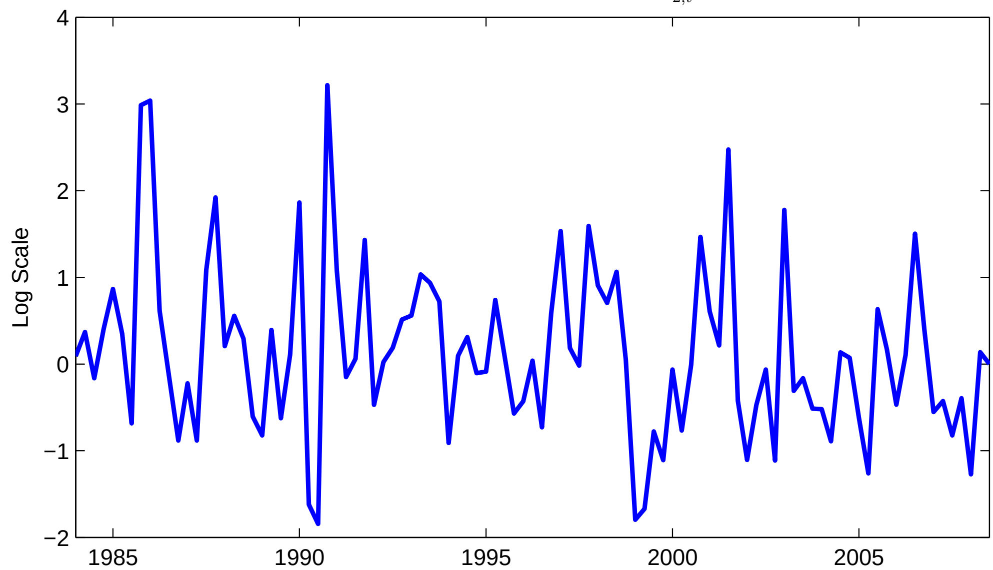
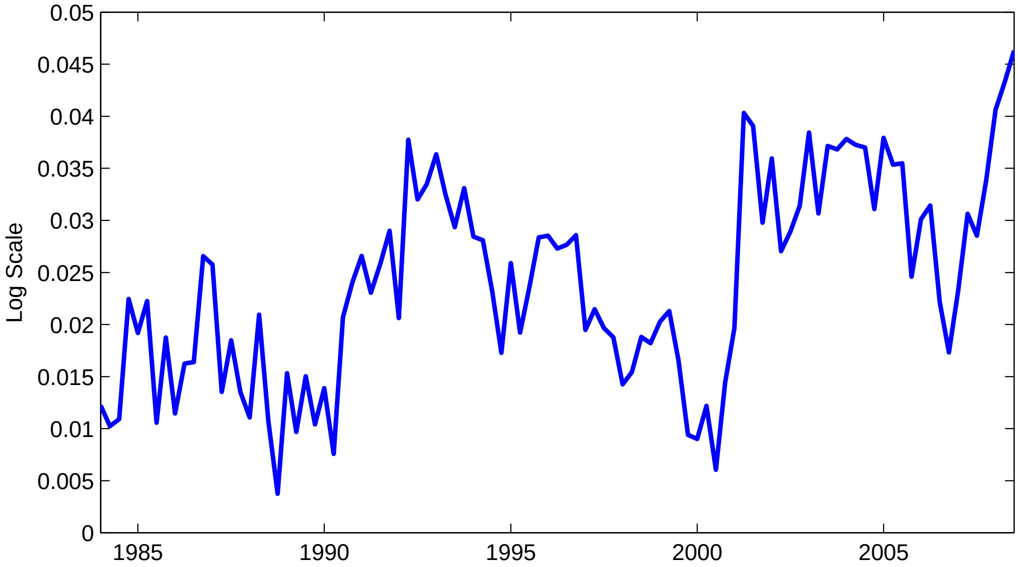


Figure 22: Data and Forecast Errors

Shock Process $Z^c_{1,t}$



Innovations to Shock Process $Z^c_{1,t}$

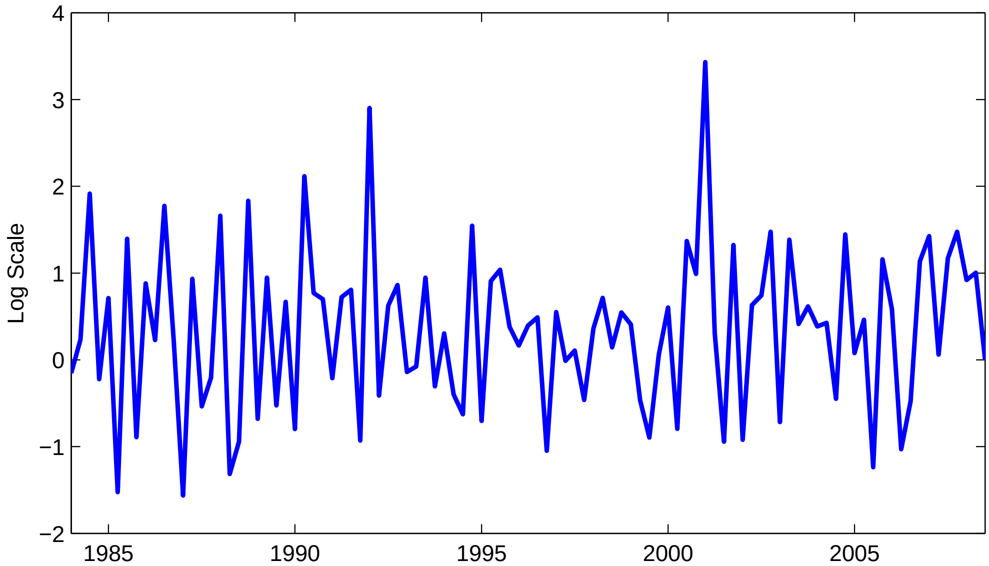
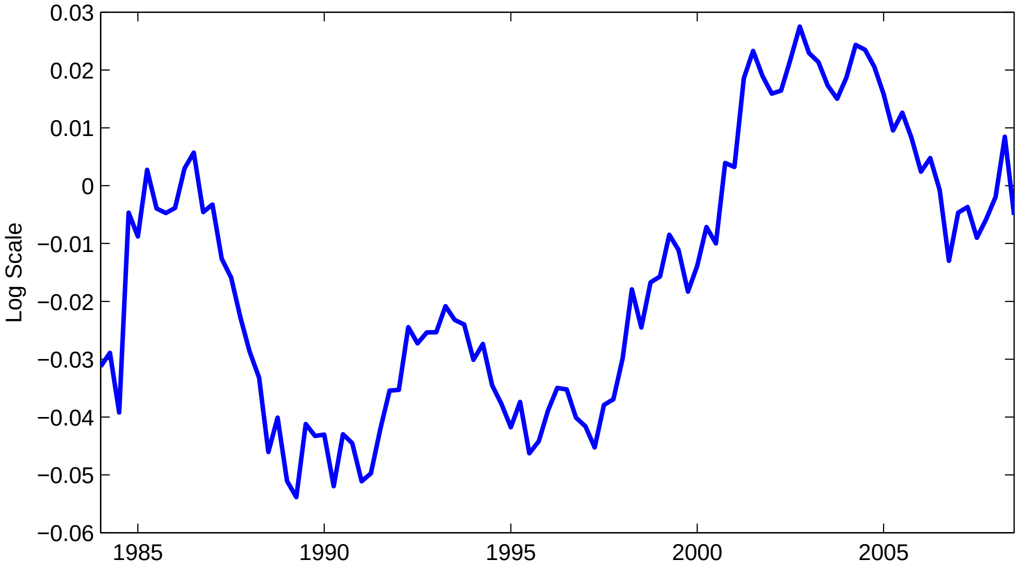


Figure 23: Data and Forecast Errors

Shock Process $Z^c_{2,t}$



Innovations to Shock Process $Z^c_{2,t}$

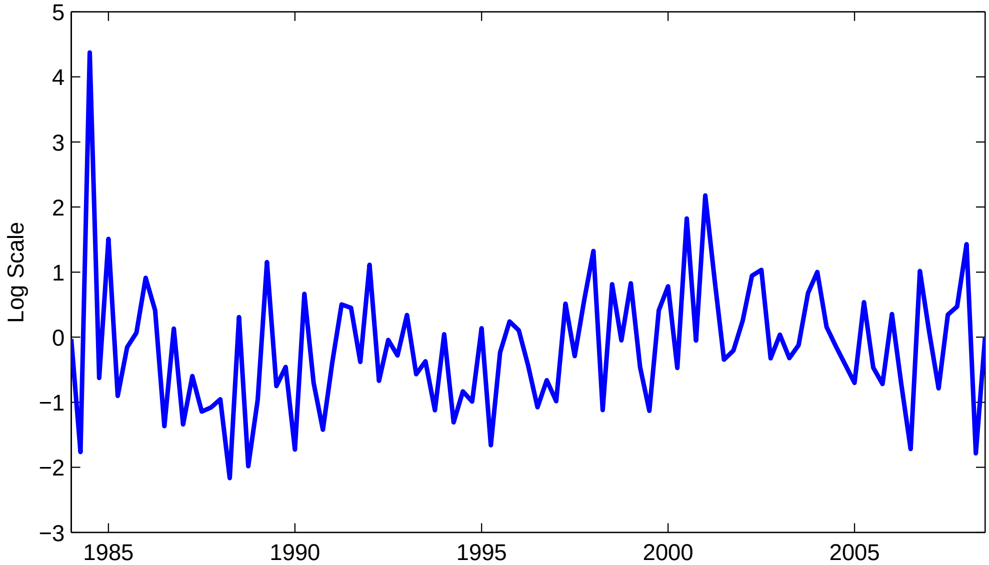
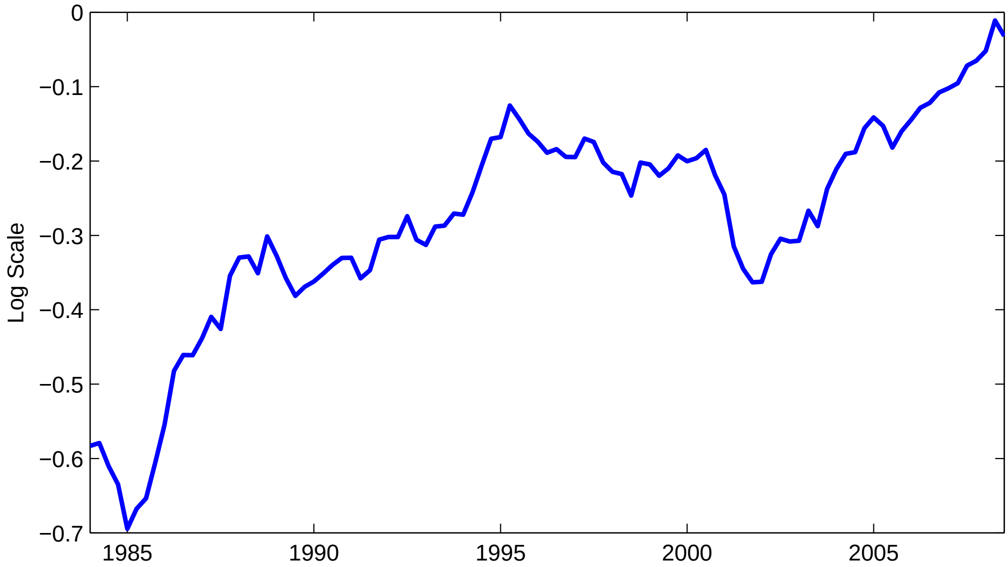


Figure 24: Data and Forecast Errors

Shock Process $Z^m_{1,t}$



Innovations to Shock Process $Z^m_{1,t}$

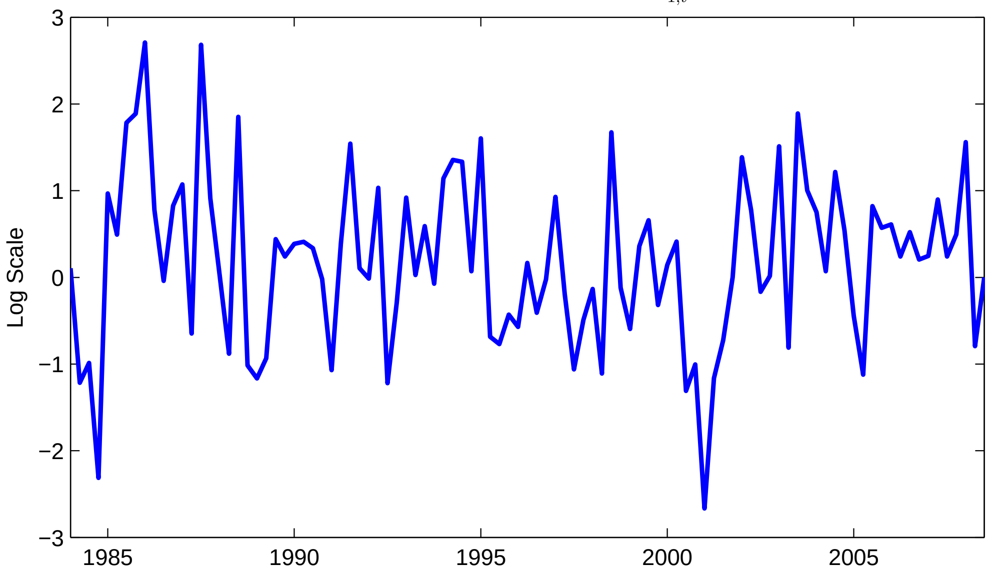
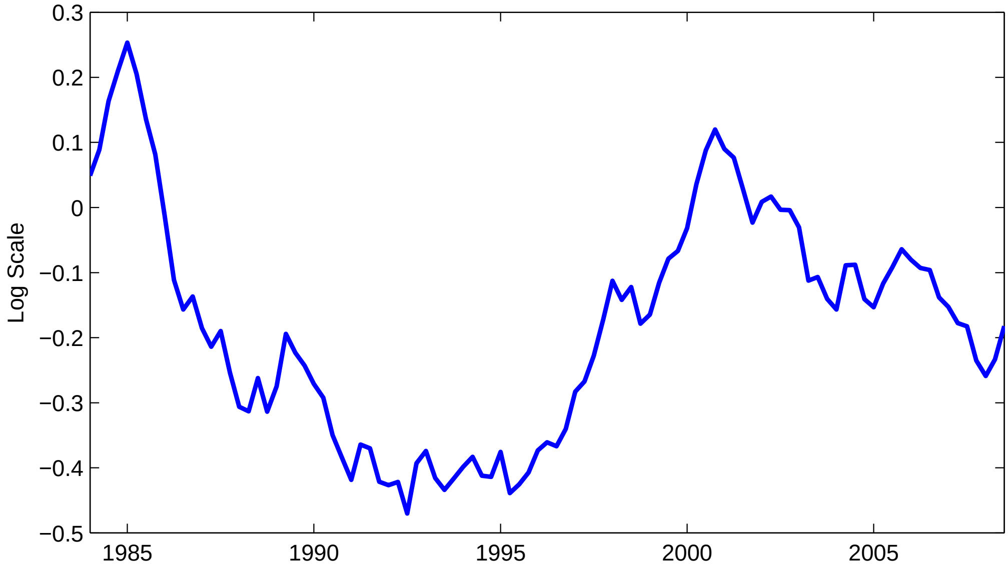


Figure 25: Data and Forecast Errors

Shock Process $Z^m_{2,t}$



Innovations to Shock Process $Z^m_{2,t}$

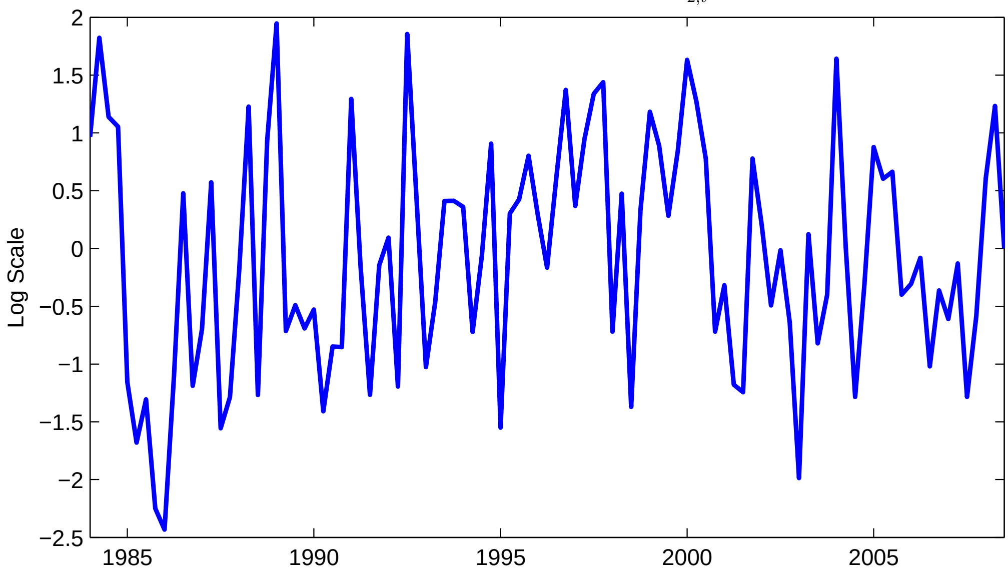
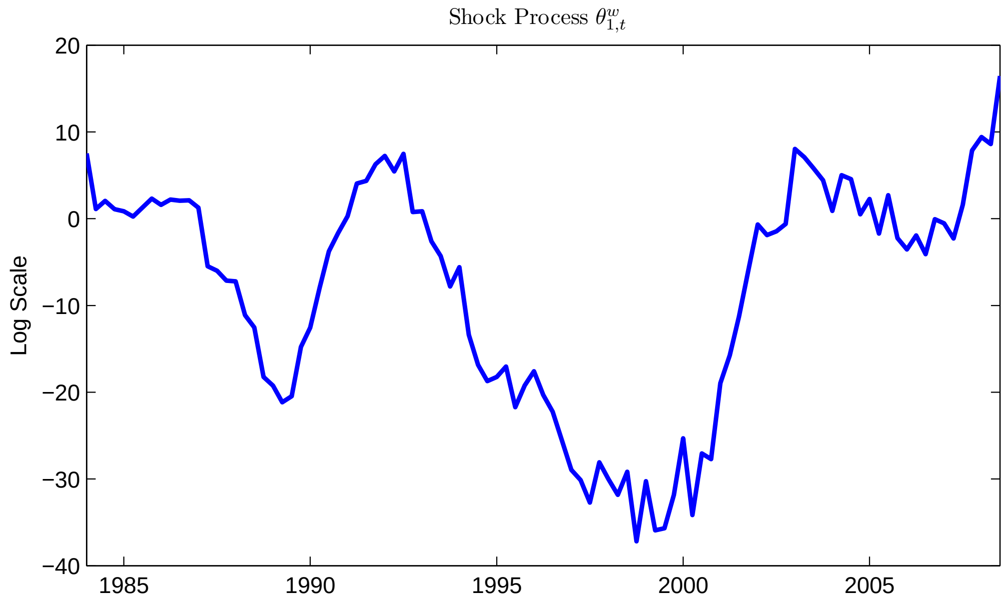


Figure 26: Data and Forecast Errors



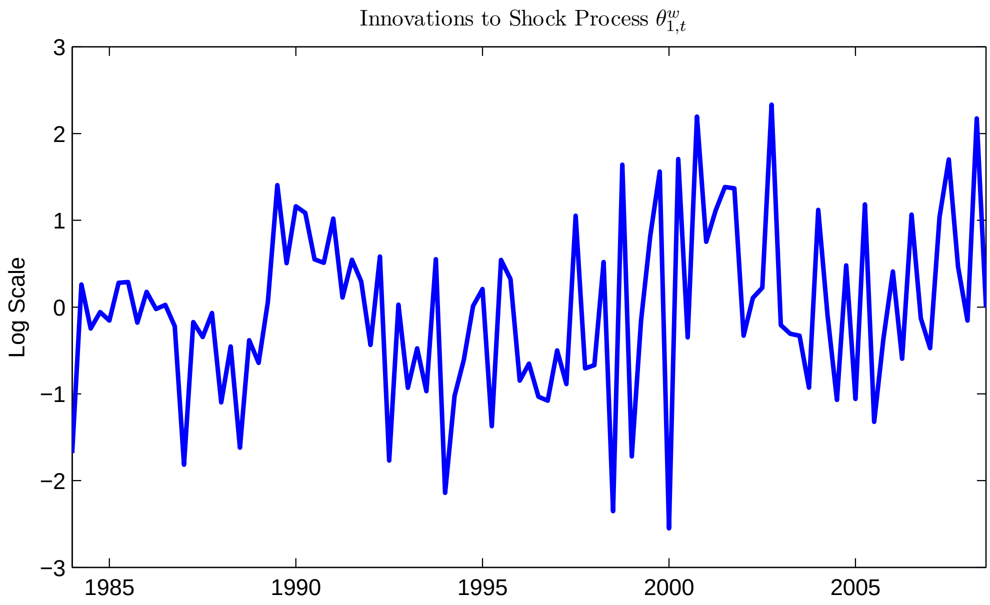
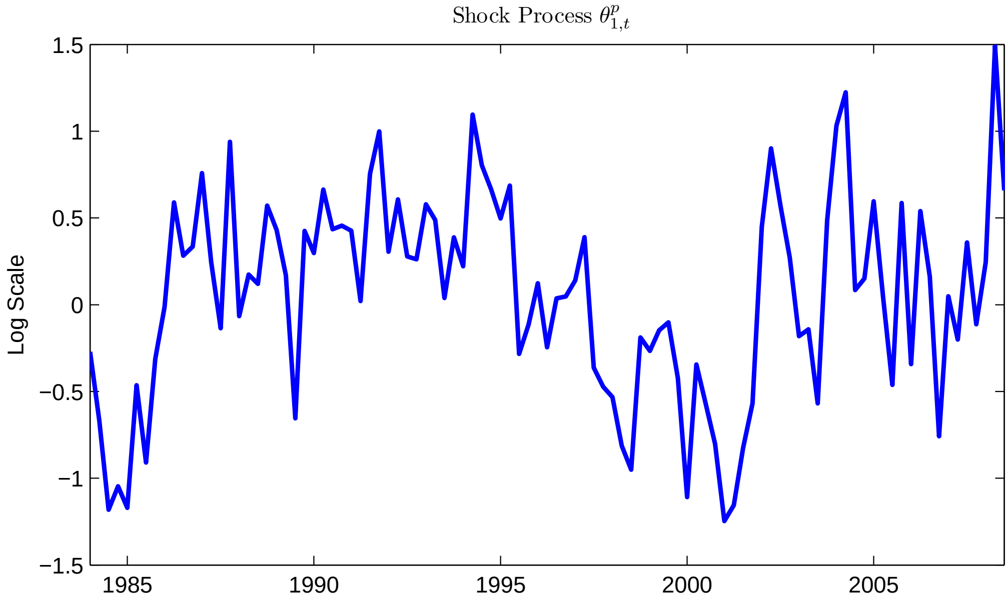


Figure 27: Data and Forecast Errors



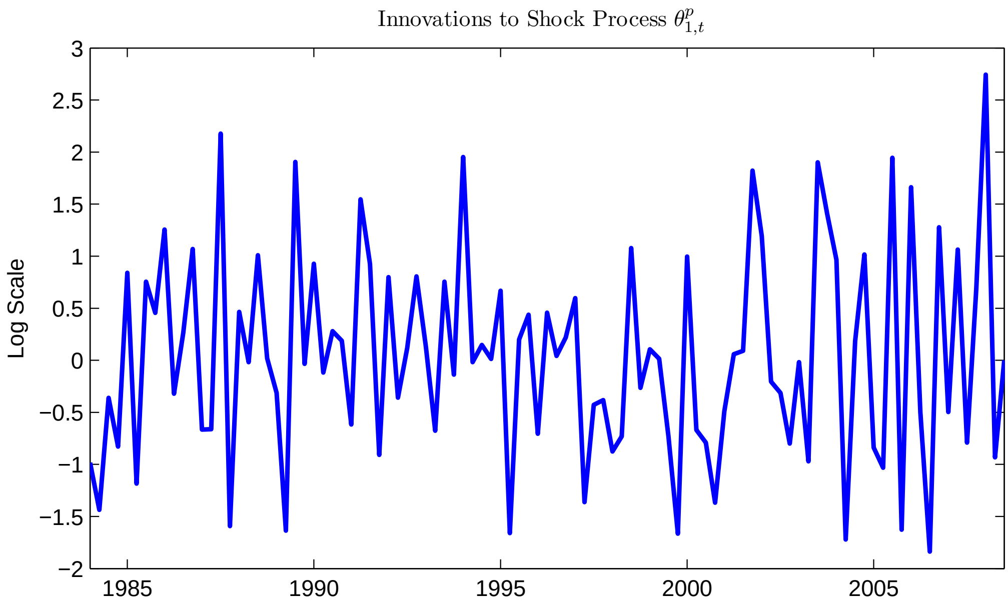
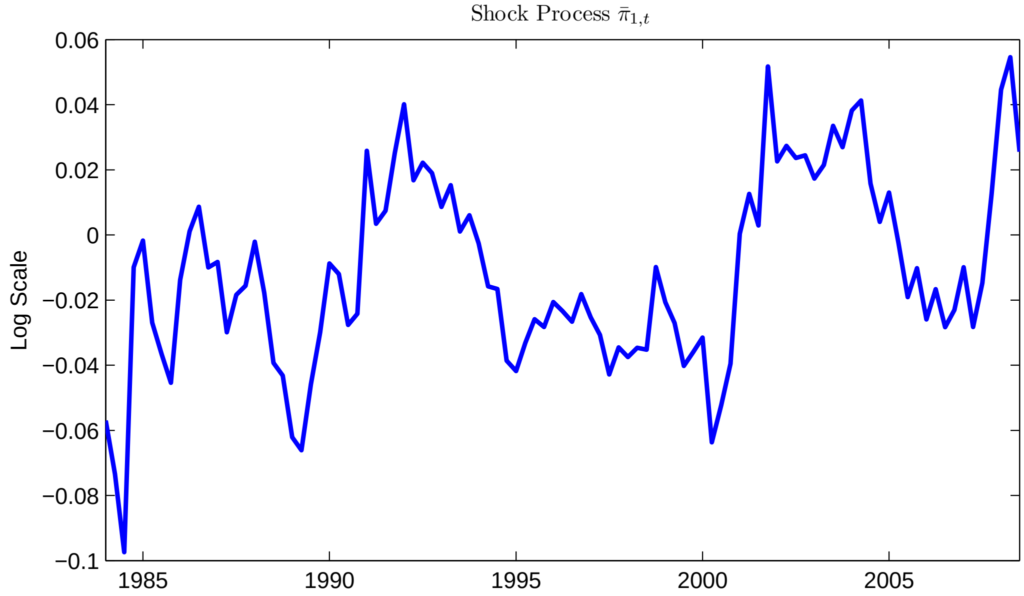


Figure 28: Data and Forecast Errors



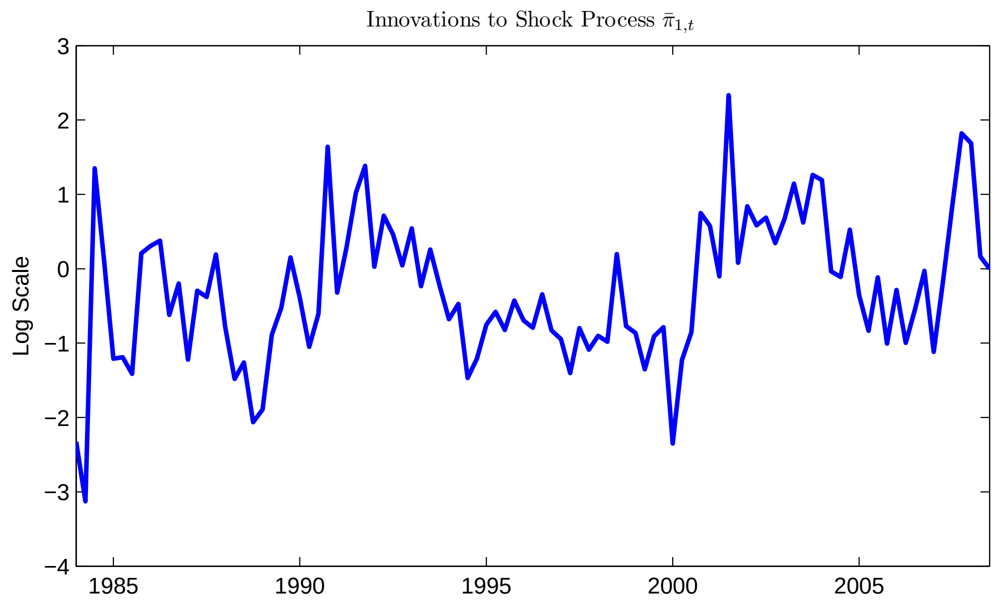
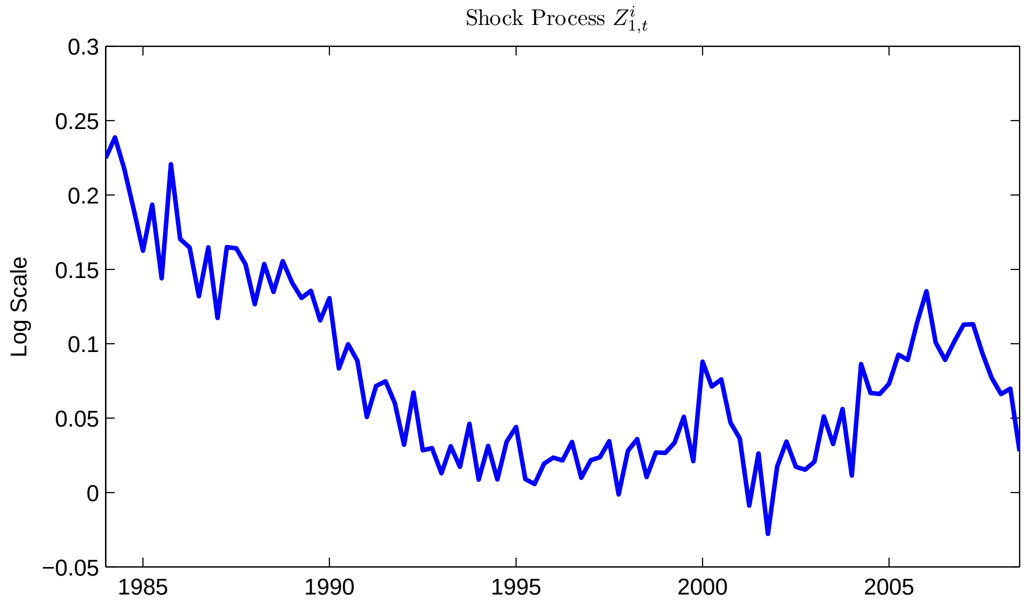


Figure 29: Data and Forecast Errors



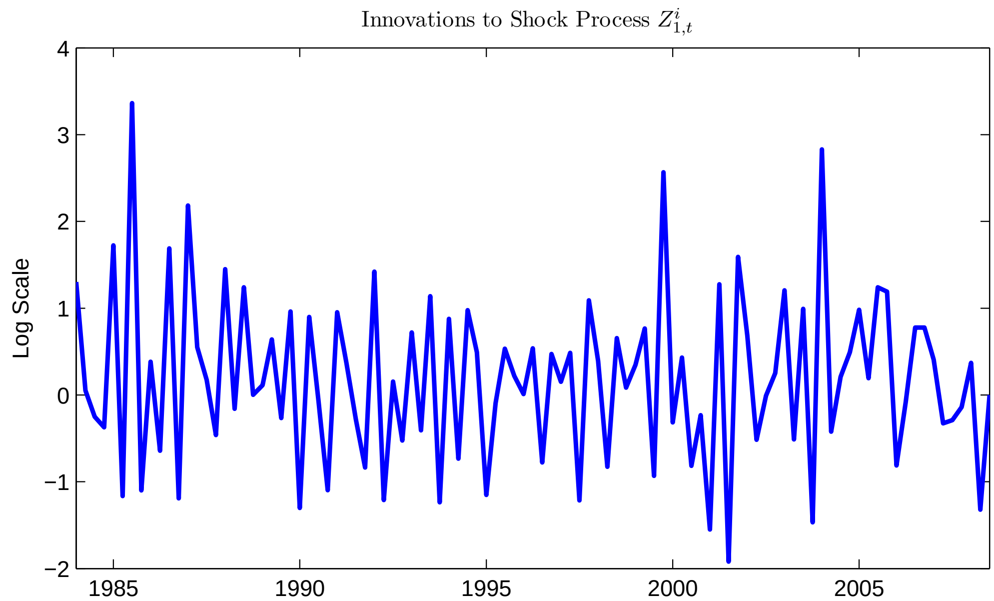


Figure 30: Data and Forecast Errors