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The Development of Work-day Adjustment Factors to Improve the Quality of Seasonal Adjustments 17 pages

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The Development of Work-day Adjustment Factors to Improve the Quality of Seasonal Adjustments

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Summary

This note presents in as simplified a manner as possible the logic underlying a method of estimating work-day factors with variable weights for individual days of the week. An application of this method has been made with sufficiently encouraging results that wider understanding of the approach may be of value to those engaged in seasonally adjusting other types of time series. It is hoped that this benefit may outweigh the other shortcomings of this note and that a body of experimental data sufficient to test the worth of this method will become available.1/

It is fairly well known that seasonal adjustment factors can be improved by adjusting the original data to remove known sources of regular or irregular variation. One such source is variation in the number and kind of working days between months and between the same months in different years.

1/ Exploratory work on this method was done some years ago by Frederick R. Dahl and A. B. Hersey of the Board's Division of International Finance in developing the work-day factors now used by the Census Bureau for U. S. exports and imports. Independent work done at the Organization for European Economic Corporation was published in the proceedings of the 1960 O.E.E.C. conference, Seasonal Adjustment on Electronic Computers. The Census Bureau is currently carrying out further research on its applicability. This note has benefitted substantially from the comments and suggestions of A. B. Hersey, Robert Steinberg, and Allan Young, none of whom can be held accountable for those flaws the author insisted on preserving.

In seasonally adjusting some monthly series no explicit adjustment for the number of working days is made; implicitly each day is weighted equally and the derived seasonal factors compensate for the variation in the length of the month. In many cases, however, adjustment factors are designed to remove that part of the variation in the original series due solely to variation in the number of working days. For 5-day weeks, for example, this approach implicitly weights each weekday 1.400 (i.e., seven-fifths) and Saturdays and Sundays zero, and the work-day factor for a given month equals the weighted sum of its days divided by a standard month of 30-7/16 days. The logic underlying this approach seems to be based on casual inspection of work patterns in many industries, and the wide use of this system suggests that, despite its simplicity, it improves the seasonal adjustment factors.[2/]

Experimental work on a fairly volatile monthly series, U. S. merchandise exports, suggests that improved results can be obtained by closer examination of the pattern of daily weights used to derive the monthly work-day factors now in

2/ A list of work-day patterns by industry for many U. S. series is provided in Industrial Production, 1959 Revision, pp. S-4 ff. In the majority of cases a specific number of days per week is given; total activity for the month is divided by the number of working days to get a daily average value. No distinction between 28, 29, 30, or 31-day months need be made since only the total number of days worked is relevant; this is equivalent to using a standard average month of 30-7/16 days as the denominator of the type of work-day factor discussed in this paper.

use. A method which relies only on information generated during a first-round seasonal adjustment "run" and which may be susceptible to automation has been developed. This method seems to yield useful results and has the advantage that no detailed knowledge of the actual work pattern of the particular economic activity being measured is essential to the development of improved monthly factors for past periods, though of course knowledge of the activity and its statistics is needed to guarantee the continuing applicability of the derived weighting pattern.

This note presents a summary of the main points of the method together with the results of the work done on factors for U.S. exports and imports. Until wider application confirms the usefulness of this approach, the method should be regarded as experimental.

Some preliminary comments

A work-day factor is essentially the ratio of the number of "effective" days to the total number of days either in a given month or in a "standard" month. The numerator of this ratio is the weighted sum of the number of Sundays weighted by the relative importance of Sunday within the week plus the number of Mondays weighted by their relative importance plus the number of Tuesdays, etc., on through the number of Saturdays weighted by Saturday's relative importance. For convenience, the weights may be chosen so that their sum is 7 and the importance of the average day is therefore 1. There is no presumption that the daily weights are all equal or that they vary in some given way for all series; they are, however, non-negative.

It is clear from the weighting scheme that the smallest possible value for the numerator is 28, the 4 weeks of a non-leap-year February. For any thirty-day month, its value will be 28 plus the values of the 2 extra days; for a thirty-one-day month, 28 plus the values of the 3 extra days.

There is some scope for choice in the denominator. One approach preserves the differences that exist in the lengths of months. Factors for thirty-one-day months beginning on, say, Sunday have as their numerator 28 plus the weights for the extra Sunday, Monday, and Tuesday; as the denom inator they have 31, the number of days in the month. Similarly, all 30-day months would have factors whose denominator is 30. For Februarys, a standard month of 28-1/4 days is used. Alternatively, the denominator can be the number of days in an average or "standard" month for the whole year, 30-7/16, given by dividing 365-1/4 by 12. The first method lets the seasonal factor adjust for the average difference between 31-day and 30-day months, and the second method makes the work-day factor adjust for this difference.

Either method produces exactly twenty-two factors. One factor suffices for all thirty-day months of the same calendar composition, i.e., which begin on the same day, regardless of the name of the month; seven factors take care of all such cases. Another seven are required for all of the thirty-one-day months, one for each possible calendar variant. A single factor disposes of all non-leap-year Februarys. Finally seven more are needed for the leap-year Februarys beginning on each of the various days of the week.

To simplify subsequent exposition a few notational conventions are in order. Let the days of the week be numbered from 1 through 7 beginning with Sunday; let w_i $(i = 1, \ldots, 7)$ be the weights corresponding to each day of the week; and let m_i and m_i' $(i = 1, \ldots, 7)$ be the monthly factors for 31 and 30-day months beginning on the various days of the week.

Daily weights and monthly factors

From what has already been said it is clear that there is a strict correspondence between daily weights and monthly work-day adjustment factors. In particular, if either set is known, the other can be immediately calculated.

Using the notation set forth above, the relationship between the adjustment factor for a 31-day month beginning on any i'th day and the pattern of daily weights can be written:

$$m_i = \frac{28 + w_i + w_{i+1} + w_{i+2}}{31} \qquad \ldots\ldots\ (1)$$

Analogously, the work-day factor for a thirty-day month beginning on any i'th day of the week is written:

$$m_i' = \frac{28 + w_i + w_{i+1}}{30} \qquad \ldots\ldots\ (2)$$

There are seven equations (1), one for each day of the week on which a thirty-one-day month can begin. Writing the expressions for m_i and m_{i+3} and remembering that the sum of the seven w_i is equal to 7, the implied weight for the i+6th day can be determined.[3/]

As in (1) we have:

$$m_{i+3} = \frac{28 + w_{i+3} + w_{i+4} + w_{i+5}}{31} \qquad \ldots\ldots\ (3)$$

Adding (1) and (3) we have:

$$m_i + m_{i+3} = \frac{56 + w_i + w_{i+1} + \ldots + w_{i+5}}{31}$$

3/ In the following equations, the monthly factors are defined according to the system that preserves the differences in length between 28, 30, and 31-day months. This is done purely to simplify the exposition.

Recalling that the sum of the daily weights is 7:

$$m_i + m_{i+3} = \frac{56 + 7 - w_{i+6}}{31}$$

$$= 2 + (1 - w_{i+6})/31$$

So that:

$$w_{i+6} = 1 - 31(m_i + m_{i+3} - 2) \qquad \ldots\ldots\ (4)$$

More generally:

$$w_i = 1 - 31(m_{i+1} + m_{i+4} - 2) \qquad \ldots\ldots\ (5)$$

which gives the seven daily weights implicit in the seven factors for thirty-one day months of identical calendar composition.

Similarly, values for the daily weights can be obtained from the seven factors for thirty-day months.[4/] Ideally these should be identical to those obtained from the factors for thirty-one-day months. In general, however, they will differ slightly. This discrepancy can most easily be resolved by taking as the set of daily weights an average of the two sets with the thirty-one-day month set weighted 7 and the thirty-day set weighted 4. (February is neglected at this stage of the exercise.)

Experimental determination of the daily weights

The process of determining optimal weights for the various days of the week and hence optimal monthly adjustment factors for variation in the number of working days consists essentially of examining the results of the seasonal adjustment process for signs of bias in the daily weights, and then removing this bias by appropriate adjustments. Once no further improvement is possible, the derived weights are considered optimal.

4/ The general form of the relation between daily weights and monthly factors for 30-day months is:

$$w_i = 1 - 30(m'_{i+1} + m'_{i+3} + m'_{i+4} - 3)$$

Bias in the pattern of daily weights shows itself in the irregular component of the final seasonally adjusted series (given in table 23 of the Board's program). If the weight for some particular day is too low, the work-day factor for months with a high proportion of such days also tends to be too low and the irregular component of the final seasonally adjusted series tends to be too high. At least one other day's weight and corresponding monthly factors will of course tend to be too high since the sum of the weights is fixed.

In order to test for this bias, it is necessary to array the irregular components of all months of given calendar composition, i.e., obtain the distribution of irregulars corresponding to all thirty-one day months which begin on Sunday, Monday, and so on for the various possible combinations. Since the expected value of the randomly distributed irregular component is by definition 1.000 in a multiplicative model of seasonal variation, if there is bias in the pattern of daily weights, the distribution of irregulars will tend to be centered about some level other than unity. Low weights will tend to produce a distribution of irregulars whose center is above 1.000; high weights, the opposite.

A first approximation to elimination of this bias can be accomplished by simply multiplying the existing monthly factors by the observed average value of the irregulars corresponding to months of particular kinds of calendar composition. In practice several problems arise. The distribution of irregulars may have one or more extreme values which appear unrelated to the bulk of the distribution. These may be due to special circumstances in a particular month, for example inadequate adjustment for the effects of a holiday or strike or for some other singular occurrence. (Some time series, for example, were dramatically affected by the widespread interest in the televising of the first manned U. S. satelite; inappropriate adjustment for this event could produce an extreme in the distribution of irregulars.) In such cases it seems better to use some measure of central tendency other than the arithmetic mean as the estimate of the average irregular; either the medians of the distributions or perhaps modified means computed without regard to the highest and lowest elements can be used.

In the special case where no pre-adjustment of the original data was made, the implicit monthly factors were all equal to unity, and the tentative estimates of work-day adjustments are then simply the "averages" of the 14 (neglecting Februarys) distributions of irregulars. If the original data were pre-adjusted by work-day factors, the "averages" of the irregulars are used to correct these factors (multiplying the one by the other) and the products are then the new tentative adjustment factors. It is important, however, to ensure that the sum of the seven tentative monthly factors for 31-day months (or the seven for 30-day months) be 7; if necessary, the tentative factors should be adjusted to force their sum to 7.000, before going into the next step.

Using the set of tentative adjustment factors just derived, one set of daily weights can be determined from the seven factors for 31-day months by the methods earlier described. (See equation (5).) A second set of daily weights is obtained from the seven factors for thirty-day months. These two sets of daily weights are then combined in a weighted average, the daily weights derived from the seven thirty-one day months being given a relative weight of 7 and the daily weights from the four thirty-day months being given a weight of 4 in the final average set.

The new set of daily weights can then be used to compute monthly work-day factors which can be used to preadjust the original data. Ideally all of the bias in the distributions of irregulars in the new seasonally adjusted series due to calendar variation should have been removed by the procedure described above. In practice, however, this may not be the case, particularly since there is an unpredictable element in the replacement of extreme SI ratios at an early stage of the seasonal adjustment process. Hence it is necessary to re-examine the new distributions given in table 23 for remaining bias. If any is present, the whole procedure can be repeated until no further improvement -- i.e., removal of bias -- is possible. At this stage the derived daily weights are then the best obtainable.

The interpretation of daily weights

There are three aspects of the derived daily weights which require some comment. First, since the weights have been derived from the past behavior of a time series, they need not represent the pattern of work loads actually in effect in some particular industry. They are simply a set of numbers based on the deviations of the irregulars from their theoretical norm which can be used to shift the distribution of irregulars back toward this norm. They therefore include all of the special factors which recur regularly in the performance and reporting of particular kinds of economic activity, not just the work-loads. Independent evidence on the work patterns in particular industries would be a useful though not necessarily conclusive bit of evidence on the probable ability of the derived factors to similarly reduce deviations of the irregulars in the future.

Secondly, the derivation of the weights is likely to be based on a relatively small sample of irregulars and the results are therefore less likely to be significant (in a probabilistic sense) than they would be if longer series were used. In a typical ten-year series -- perhaps the longest time stretch over which working loads and reporting patterns might be expected to remain constant -- only about 10 observations of each of the seven thirty-one-day month combinations and less than 6 each of the thirty-day-month combinations would be available. (This explains the neglect of Februarys in this exercise; in any ten-year period, at most 3 leap-years will occur and all of the other Februarys will have the same work-day factor, viz. either 28/28.25.) This lack of data makes the use of ordinary statistical measures of reliability irrelevant, and places an added premium on whatever external evidence can be mustered to show the likelihood of continuing applicability for the derived daily weights.

Finally the possibility that the daily weights are not constant over the year or even over some longer period must be considered. No very satisfactory tests for variation in the daily weights have yet been devised, but several informal approaches are possible. The distributions of irregulars for the various calendar variations can be divided into two or more groups -- e.g., pre- and post-1956 -- to see if there has been any apparent change between the two periods.

The testing and evaluation of this approach to improving the seasonal process is obviously in its infancy. It is doubtful whether any significant tests can be devised until a greater body of experimental results has been amassed.

<u>Experimental results: U.S. foreign trade</u>

The method described in this note was applied to the monthly series of U.S. merchandise exports and imports. Work-day adjustment factors based on weights determined some years ago by a limited survey of the factors affecting the timing of export data by days of the week have been used to pre-adjust the raw data before seasonally adjusting the series. These weights are shown on line 1 of table 1.

The irregular component of the seasonally adjusted series was arrayed in accordance with the procedure outlined in the text. The medians associated with these distributions are shown in figure 1. The deviations of the centers of the distributions from 1.0 (the expected value of the average irregular if no bias were present) suggested the presence of systematic bias associated with the calendar composition of different months.

New daily weights were determined from monthly factors based on the products of the old factors and the deviations of the averages of the irregulars from 1.00. Their weights were combined into revised monthly factors which were used to preadjust the original data. After again seasonally adjusting the series and re-arraying the irregular component as before, the centers of the distributions of irregulars were found to have been shifted significantly toward 1.00. The new distributions are shown in line (d) of figure 1. Further iterations yielded no significant improvement.[5/]

An examination of the effect of the new weights on the ratios of the original (i.e., work-day adjusted) data to the trend cycle curve suggests that for some months the new weights seem to yield a slightly smoother series than either the old weights or no weights. (The details are shown in figure 3.) This is consistent with the reduction in the average irregular variation from 2.91 (using no weights) to 2.78 (using the old weights) to 2.27 (using the new weights).

The reduction in the average irregular variation using the new weights is distributed fairly evenly over the whole year. In particular, comparison of the average deviation of irregulars from 1.000 month by month shows the new weights yield a lower average deviation than either no weights or the old weights in eight months of the year, do second best in three, and have a slightly higher average deviation than the other two only in February.

5/ See table 1 for some of the intermediate calculations in the derivation of the new weights.

The same approach was used to examine the weights currently in use to pre-adjust data on U.S. imports. The present weights were obtained from the same kind of survey of the day-of-the-week pattern of the factors affecting the timing of imports as was used in the case of exports. The reduction in the average irregular component using these weights was much more dramatic in the case of imports than in the case of exports; the average irregular variation fell from 5.01 to 2.35.

The examination of the distributions of irregulars showed relatively little scope for improvement on these weights. As an exercise, however, the computations were made and confirmed the general appropriateness of the existing weighting pattern. A further pair of iterations were performed at the Census Bureau with very slight further reductions in the average irregulars.

A problem of interpretation of the daily weights arises in the case of import data which may arise in other applications of this method. Census Bureau procedures date imports of merchandise by the date on which the import documents are filed. Since the Customs offices are closed on Saturdays and Sundays no imports can occur (in the sense of entering the statistics) on those days. The successive iterations of the adjustments made to normalize the distributions of irregulars showed that small weights for Saturdays and Sundays (about .25) were needed to reduce the average irregular variation.

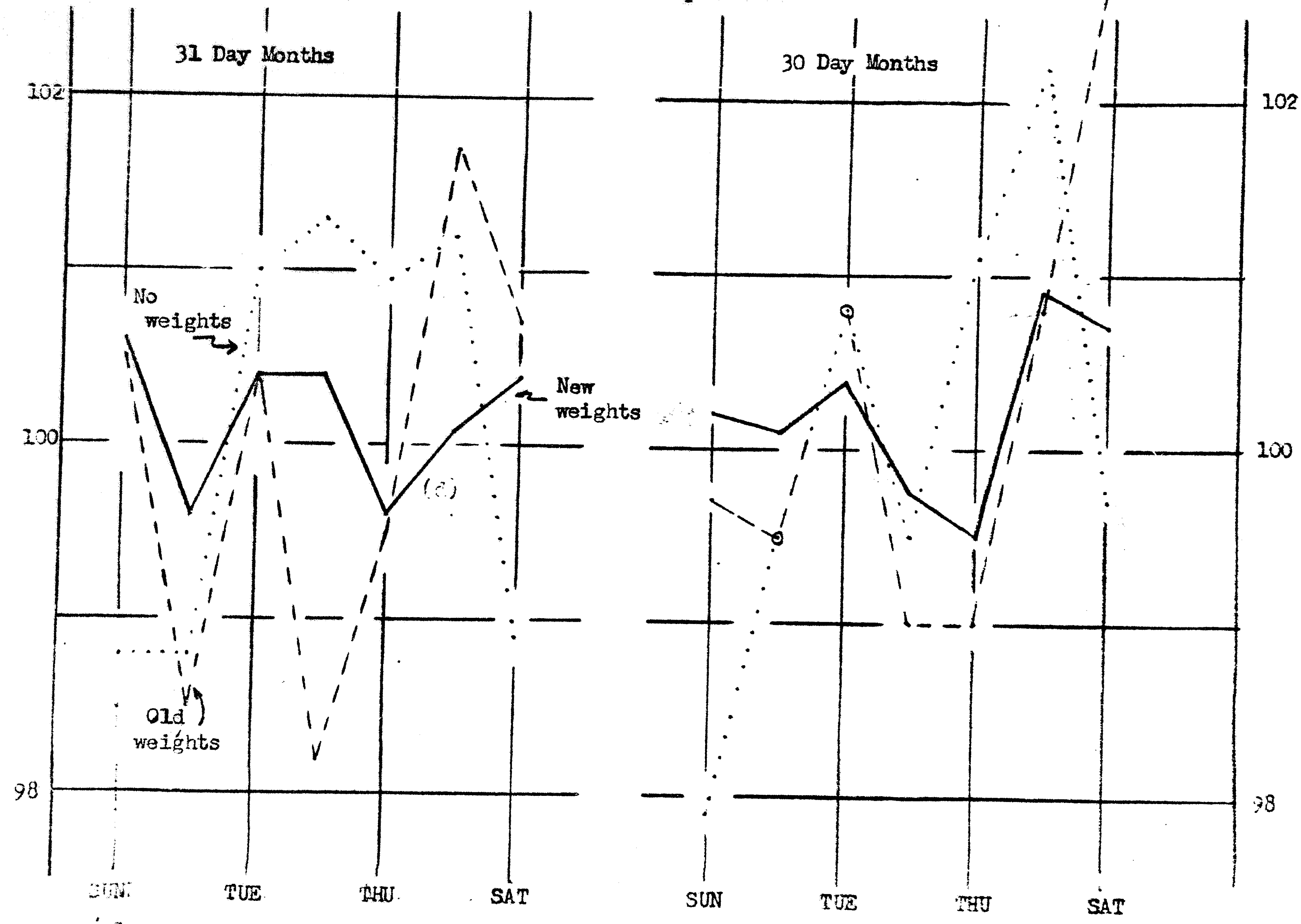
This conflict between internal and external evidence has not been resolved. (A similar problem has arisen at the Census Bureau in connection with some other series as well.) On the one hand, the emergence of positive weights for days on which no work is performed can be interpreted as an error resulting from the effect of the small sample-size of the distribution of irregulars; the offending weights could then be set at zero and the remaining weights adjusted appropriately. Alternatively the weights which emerge from the calculations may reflect not only the pattern of daily activity but also variations in reporting practices and perhaps other factors as well. Further work -- both applied and theoretical -- is needed to clarify this point.

Aside from these more sophisticated problems, the conlusion that a set of daily weights can be of some help in improving the quality of seasonal adjustments seems warranted from the work done on U.S. trade. The approach described in the above permits estimates of what appear to be optimal daily factors, although the interpretation of these weights -- and therefore their continuing applicability -- is still a matter for further investigation.

Table 1. Derivation of Revised Daily Weights, U.S. Exports

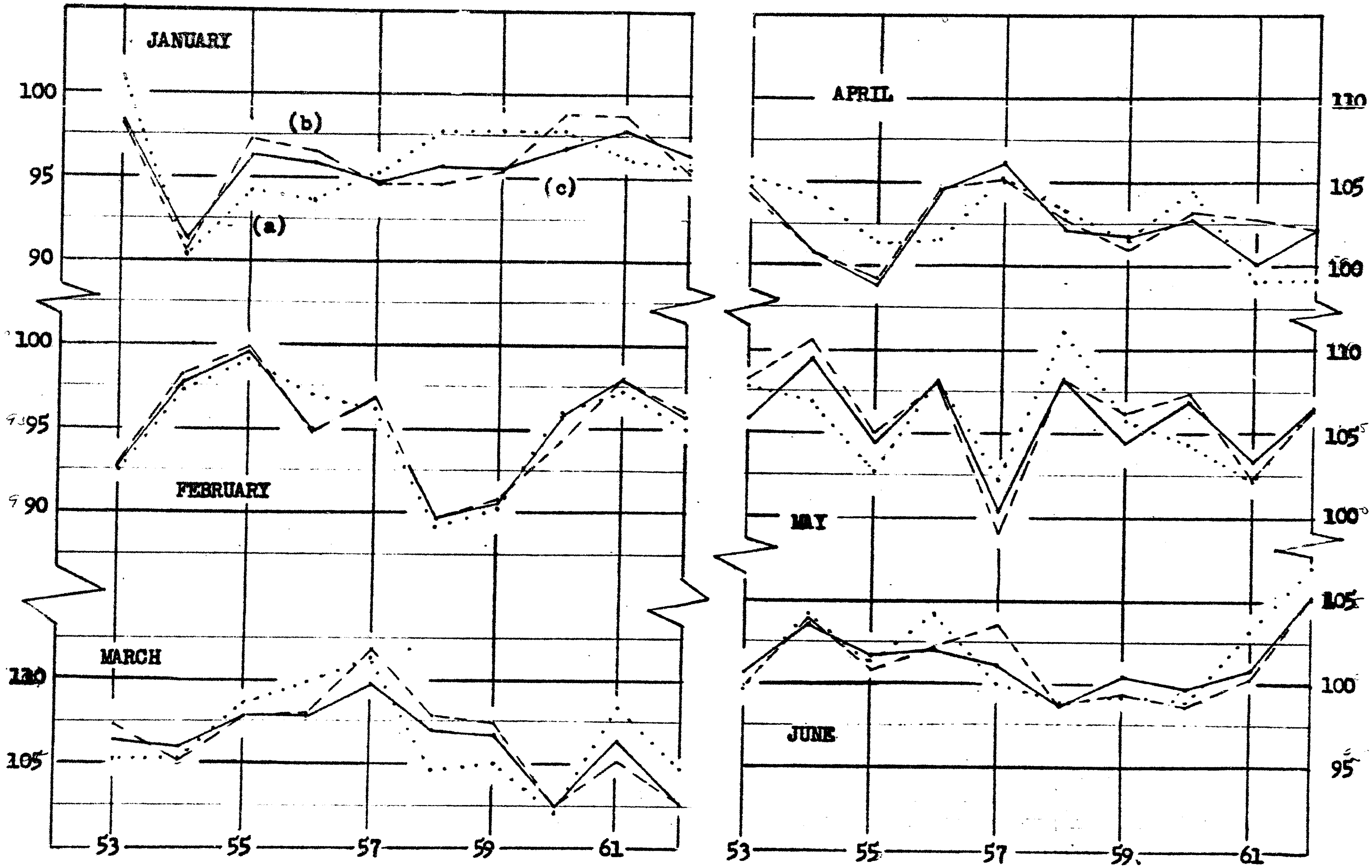
	Sun.	Mon.	Tues.	Wed.	Thur.	Fri.	Sat.
Old weights	.25	1.000	1.000	1.050	1.100	1.850	.750
Monthly factors (by beginning day of month)							
31-day months	.976	1.002	1.005	1.032	1.023	.995	.968
30-day months	.975	1.000	1.002	1.005	1.032	1.020	.967
-- These were then adjusted by the medians of the corresponding distributions of irregulars --							
New daily weights from adjusted monthly factors:							
31-day months	.726	.427	1.295	.906	1.026	1.535	1.086
30-day months	.728	.510	1.196	1.009	.728	1.820	1.009
Weighted average of new daily weights (rounded)	.70	.50	1.20	.95	.95	1.70	1.00
Revised monthly factors (by beginning day of month)							
31-day months	.981	.989	1.003	1.019	1.021	1.013	.974
30-day months	.973	.990	1.005	.997	1.022	1.023	.990

Medians of Distributions of Irregular Components Arrayed by Beginning Day of Months of Identical Calendar Composition[a]



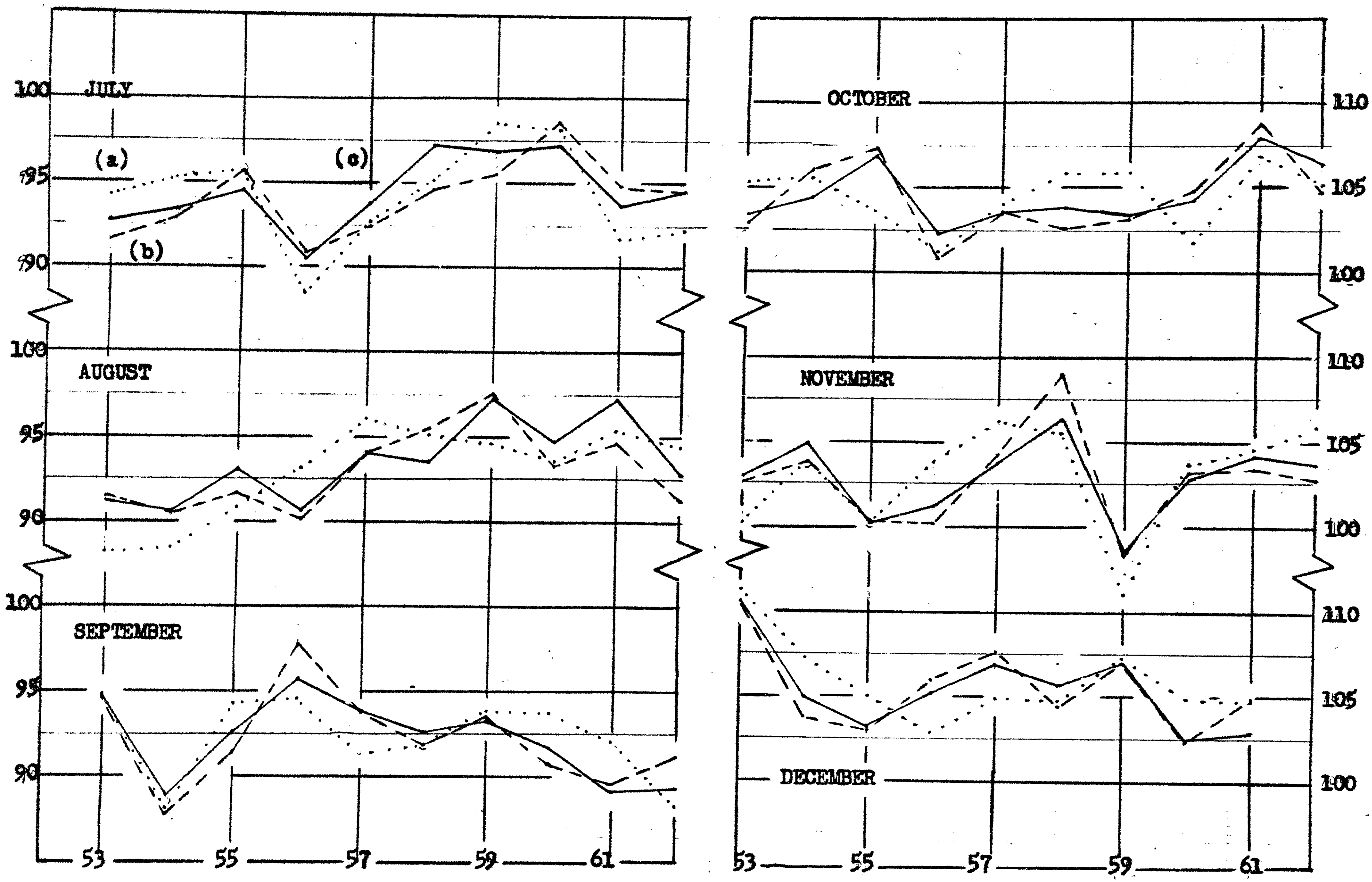
a/ Source: Table 23 of X-9 seasonal adjustment program.

U.S. EXPORTS : RATIOS OF ORIGINAL TO TREND-CYCLE (Table 9 of FR 125, X-9)



(a) = No preadjustment for working day variation.
(b) = Preadjusted using present weights.
(c) = Preadjusted using proposed weights.

U.S. Exports : Ratios of Original to Trend-Cycle (Table 9 of FR 125 - X-9)



(a) = No preadjustment for working day variation.
(b) = Preadjusted using present weights
(c) = Preadjusted using proposed weights.