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A Distinction Between Criteria for Assigning Policies in the World Economy

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A Distinction Between Criteria For Assigning Policies in the World Economy[1/]

Although final macroeconomic policy decisions are largely judgmental, they do conform to policy rules which, if our mathematics were sophisticated enough, could be given a formal statement. A less ambitious but, I think, worthwhile task is to codify (into simple mathematical form) some of the considerations involved in the reactions of policy-makers to target disequilibria. The purpose of this paper is to distinguish between two different criteria used in policy formation, to demonstrate how they can be translated into mathematical policy rules, and to draw out and compare some of the policy responses which they imply. Since one of the interesting differences in the rules implied by the two criteria concerns the reaction of policy-makers to foreign developments, the argument will be carried out in terms of a two-region model of the world economy. In addition, the growth of economic interdependence between national economies increases the importance of including a second country (the rest of the world) in our analysis of macroeconomic, multi-target policy rules.

1/ The paper has benefited from the author's lengthy discussions with John Morton and the suggestions of Robert Gemmill and Robert Mundell. Helpful comments were also received when previous versions of the paper were presented at the International Economics Seminar at Harvard University and a seminar of the Special Studies Sections at the Board of Governors. Any remaining errors are the author's responsibility.

Policy-makers use multi-target rules in the sense that they consider many economic variables before deciding whether to expand or contract. It is important to distinguish between the structure and scale of such rules -- between the decisions of whether to contract or expand and how much to contract or expand. This paper will be concerned with the former decision which can be viewed as the problem of determining the relative weights in multi-target policy rules.

One criterion or approach which policy-makers use to determine the structure of multi-target rules is to make the relative weights conform to the principle of comparative advantage. I will call this the Comparative Advantage Approach or the CAA.

The CAA is similar in spirit to Professor Mundell's [6] Principle of Effective Market Classification, or the PEMC. Whereas the CAA is a criterion for designing multi-target rules on the basis of comparative advantage theory, the PEMC is used to assign instruments to targets with single-target rules. In a world of perfect knowledge one can always construct multi-target rules which are superior to single-target rules,1/ but it is not clear that they

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1/ The single and the multi-target approaches to policy rules were referred to as the "decentralized" and the "centralized" approaches by Tinbergen [11]. Richard Cooper [1] has followed Tinbergen's use of the word "decentralized," but he refers to multi-target rules (the construction of which will be discussed below) as a "coordinated" approach.

are superior in a realistic setting of limited knowledge.1/ But even if multi-target rules required more knowledge than single-target rules (generated under the PEMC) to achieve the same degree of effectiveness, it is useful to examine the criteria for deriving the structure of multi-target rules since policy-makers frequently incorporate several targets in their decisions.

In addition to the CAA, there is a second criterion frequently used in policy formation which I will call the Offset the Disturbance Approach, or the ODA. The thrust of this approach is to locate the source of a disturbance and then proceed to offset it with the most effective instrument. The ODA embodies the idea that the cure (to

1/ It is useful to distinguish between the two-by-two case and the general n-by-n case. When we have two targets (such as internal and external balance) and two instruments (monetary and fiscal policy), the PEMC never requires more and frequently requires less information than the CAA or the ODA (which is discussed in the next paragraph in the text). This is demonstrated in Appendix C.

In the general case it is not at all clear which requires the most information although it is clear that more information is required when more than two variables are involved. The fact that the CAA and the ODA require more information can be demonstrated by simply analyzing the characteristic equation for the policy-endogenous system. In the case of single-target rules generated with the PEMC criterion, we do not know how much more information is required. Mundell (Chapter 21 in [6]) has pointed out that, when policies can be assigned (i.e., the rows of the multiplier matrix manipulated) such that the Hicksian conditions of perfect stability hold, then some of the weights may matter but stability (global as implied by a theorem derived by McFadden and local as implied by a theorem first demonstrated by Fisher and Fuller [3]) is always achievable. But to this author's knowledge, no one has shown whether any arbitrary, non-singular square matrix can be manipulated (the signs and order of rows changed) to obtain the Hicksian conditions. If and when this is accomplished, we will know much more about the information required to achieve stability in the general case under the PEMC.

the problem of being away from our economic goals) should be related to the cause. For instance, a shift in the investment function is thought to be more appropriately offset with fiscal policy, and a shift in the demand for money function is thought to be countered most effectively with monetary policy.1/ The problem is that policy-makers do not know where the disturbance occurs -- they can only observe manifestations of the disturbance in variables like interest rates, incomes, and balance of payments disequilibria. According to the ODA, we should design the structure of policy rules such that only those policies which are most effective in offsetting undesired disturbances are induced into action.

Given the fact that these two approaches to policy formation are frequently used in an intuitive or judgmental fashion, our first task is to give the CAA and ODA explicit formal interpretation. They will be interpreted in the dollowing sections by demonstrating how they can be used to determine the structure of policy rules. Since the following two sections differ in the instruments and targets that are considered, the CAA and ODA will yield different rules in each section, but the methods of deriving the rules will be the same.

1/ Academic economists are well as policy-making authorities have been concerned with the problem of locating the source of disturbances. For instance, Jerome Stein [10] uses the movements in interest rates and income to determine whether disturbances in the U.S. economy from 1919 to 1958 were real (in an expenditure function), monetary (in the supply of money function), or liquidity (in the demand for money function) disturbances. William Poole [8] discusses different rules depending upon whether disturbances are real or monetary. David Meiselman [4] discusses the difficulty of identifying the source of a disturbance if investment is sufficiently sensitive to income such that the IS curve slopes upward.

Monetary and Fiscal Policies Applied to Interest Rates and Incomes

Since IS-LM curves are familiar to most economists, we will employ the CAA and the ODA in a two-country version of the Keynesian model to develop rules for monetary and fiscal policies with real incomes and interest rates as our target variables.[1/]

In matrix notation the system can be written as $Sdy = dp + dz$ where S is the matrix of structural coefficients, y is the vector of target variables, and p is vector of policy variables. We will assume that the desired values of the target variables are the same as the initial equilibrium values of the target variables, y_o. A differential, like dY, represents the deviation of a variable from its target value such that $dY = Y - Y_o$ and $dy = y-y_o$. Since the equations $Sdy = dp + dz$ are assumed to hold for all values of time, $S\dot{y} = \dot{p}$ where $\dot{y}_o = 0 = \dot{p}_o$, dz is assumed to be a step function, and dots indicate time derivatives. The policy rules which relate $\dot{p}$ to dy need to be specified to complete the system.

A general statement of policy rules which distinguishes between their structure and scale is as follows:

1/ Since external diseconomies are so great in most industrialized economies, income can only be a proxy for employment. Interest rates might be considered as a proxy for growth (cf. Richard Cooper [1]). The model is developed in Appendix A.

$$\dot{G} = k_1(w_{11}dY + w_{12}dr + w_{13}dr' + w_{14}dY')$$

$$\dot{M} = k_2(w_{21}dY + w_{22}dr + w_{23}dr' + w_{24}dY')$$

$$\dot{M}' = k_3(w_{31}dY + w_{32}dr + w_{33}dr' + w_{34}dY')$$

$$\dot{G}' = k_4(w_{41}dY + w_{42}dr + w_{43}dr' + w_{44}dY') \qquad \text{or } \dot{p} = KWdy$$

where W is the matrix of weights which the CAA and ODA will be used to specify and K is a diagonal matrix (whose elements are positive) that determines the scale of the four macro-policies. The scale parameters are also used to keep the policy rules invariant to the dimensions in which we measure time: the scale factors assume the dimensions necessary to make the left and right-hand sides of the policy equations dimensionally equivalent.

Let us turn first to the CAA which says that the relative weights should conform to the comparative advantages of the policies. To give a mathematical interpretation to this idea we should first note that comparative advantage is found by comparing the ratios of multipliers. Multipliers can be denoted by $Y_{G'}$ (the multiplier of foreign fiscal policy on domestic income), r'_M (the multiplier of domestic monetary policy on the foreign interest rate), and so on. The comparative advantage between, say, M and M' on the targets Y and Y' is given by the comparison of $Y_m/Y'_{m'}$ to $Y'_m/Y'_{m'}$.

Consequently, we can formalize the CAA by setting

$$\frac{w_{21}/w_{24}}{w_{31}/w_{34}} = \frac{Y_m/Y'_{m'}}{Y'_m/Y'_{m'}} .$$

This equality assures us that, if domestic monetary policy has a comparative advantage (foreign monetary policy has a comparative disadvantage) in influencing domestic relative to foreign income, the relative weights will reflect the comparative advantages. When this is done for all pairs of targets and goals, the weights, w_{ij}, are determined only up to a scale factor. It is convenient to normalize by setting the weights equal to the (negative) values of the multipliers.1/ That is, set $w_{11} = - Y_G$, $w_{34} = - r'_{m'}$, etc. Since the multipliers of this model are found from the elements of the inverse of S, the weights generated by the CAA can be stated as $W = - (S^{-1})'$ where the prime denotes transposition. Consequently, the CAA rules2/ are $\dot{p} = - K(S^{-1})'dy$.

When the structure of policy rules are specified according to their comparative advantages, the policy-endogenous3/ system is stable. This can be shown by combining the policy rules,

1/ This is permissible since we are addressing the problem of the structure of policy rules and are taking the K-matrix as arbitrary.

2/ "The CAA rules" is a short-hand expression for "the rules developed with the CAA." Of course, the CAA (and the ODA) will imply different rules in different models such that "the CAA rules" is not as unambiguous as it appears.

3/ "Policy-endogenous" means that the money supplies, interest rates, and government deficits are endogenous variables although the policy rules are, so to speak, still exogenous.

$\dot{p} = - K(S^{-1})'dy$, with the structural equations, $\dot{y} = S^{-1}\dot{p}$, to obtain $\dot{y} = - S^{-1}K(S^{-1})'dy$. This system of differential equations is not only stable but the adjustment to equilibrium is direct: there are no complex roots to cause cyclical adjustment.[1/] Since these rules are found from a straight-forward application of comparative advantage theory and multiplier analysis, it is surprising that such a policy system has not, I believe, been previously derived and tested for stability.

Unlike the CAA, the ODA does not use multiplier analysis. The purpose of the ODA is to find weights which, when attached to the target variables in policy rules, will only bring those policies into action which are most effective in offsetting a disturbance. If there is an autonomous shift in an investment function, fiscal policy is the most effective stabilizing instrument since it enters the same equation as the disturbance. If we write the linearized version of equation (1) as

$$(1-E_y+m)dy - E_r dr - m'dY' = dG + dZ_1,$$

it is clear that setting $dG = dZ_1$ effectively offsets any undesired impact which dZ_1 would have on dy in the absence of fiscal policy.

1/ The product matrix $- S^{-1}K(S^{-1})'$ has only real negative roots because it is negative definite. It is negative definite, or $S^{-1}K(S^{-1})'$ is positive definite, because the latter contains a positive definite matrix, K, inserted between a matrix of full rank and its transpose. This stability theorem is not limited to this Keynesian model -- it holds for any square, non-singular matrix S and a diagonal matrix K with positive elements.

But the problem is that policies are frequently formulated without any direct knowledge of the source of the disturbances -- those responsible for policy decisions must be guided by movements in observable variables like incomes and interest rates.

The ODA weights are derived by approaching the problem from the back door; that is, by searching for those values of dy which allow fiscal policy to remain neutral. The ODA can be interpreted as requiring domestic fiscal policy to remain neutral when either (i) there has been no disturbance in the demand for domestic goods and services or (ii) if the disturbance has already been offset by the cumulation of previous changes in domestic budgetary deficits or surpluses. Algebraically, this means that fiscal policy should be neutral when

$$(1') \qquad (1-E_y+m)dY - E_r dr - m'dY' = 0.$$

If there has been an expansionary disturbance which fiscal policy has not yet completely offset, then the expression in (1') will be positive and fiscal policy should contract. Consequently, the ODA fiscal policy rule can be written as

$$(5) \qquad \dot{G} = k_1[(1-E_y+m)dY - E_r dr - m'dY'].$$

A similar analysis can be carried out for all policy rules and the final result in matrix notation is $\dot{p} = - KSdy$. Instead of using multipliers for the relative weights, the ODA makes use of, in a sense, the opposite kind of information, viz., the structural coefficients of the system.1/

The complete system is found by combining the rules $\dot{p} = - KSdy$ with the structural equations $\dot{y} = S^{-1}\dot{p}$ to obtain $\dot{y} = - S^{-1}KSdy$. Just as the policy-endogenous system with CAA rules proved to be stable with a direct approach to equilibrium, the ODA system is also stable without any complex roots.2/3/

A comparison of the two approaches of policy decisions yields several interesting results. As previously mentioned the ODA uses estimates of structural coefficients and the CAA uses

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1/ Structural and reduced-form coefficients are both partial derivatives, but the former is a derivative with respect to one target variable while other target values remain constant and the latter is a derivative with respect to one policy while other policies remain unchanged.

2/ The product matrix, $S^{-1}KS$, is similar to K and the latter is positive definite. Since similar matrices have identical roots, all the roots of $- S^{-1}KS$ are negative real numbers. Of course, this is a general theorem such that any linear, homogenous policy-endogenous system (with a non-singular structural matrix S) with ODA rules will adjust directly to equilibrium.

3/ Richard Cooper [1] and John Patrick [7] have recently employed ODA rules. Of course, they did not refer to the rules by this name. Cooper used the two-country model developed in Appendix A and Patrick used the single-country (internal-external balance) model developed in Appendix C. In the special case in which all the elements of the scale matrix are identical (unitary), Cooper (Patrick) demonstrated that the policy-endogenous system is stable. However, neither author gave the economic rationale for employing structural coefficients as weights in multi-target policy rules.

estimates of multipliers.[1/] However, there is an important asymmetry in the way the information is employed. Policy-makers explicitly take account of the effect that their policies have on economic targets. If their estimates of the size of multipliers is altered, they deliberately alter their reactions to given target disequilibria. On the other hand, although they do make use of their knowledge of structural coefficients to determine the location of unwanted disturbances,[2/] they would be surprised to learn that they are "weighting" targets with such coefficients. The point here is that the more consideration they give to the source of a disturbance the more the weights (as would be found, say, in the empirical estimation of reaction functions) begin to

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1/ The fact that the two approaches to policy formation employ different information may have an implication for the kind of coefficients a research staff for a policy-making body tries to estimate. It is true that once the matrix of structural coefficients is known, it can be inverted to obtain the reduced-form coefficients or multipliers. However, if errors contained in the estimates of the coefficients of one matrix are, in some sense, compounded when the matrix is inverted, it makes a difference which coefficients are estimated in the first place. A very tentative investigation of whether errors are compounded when a matrix is inverted has only yielded ambiguous results.

2/ This statement needs elaboration. Suppose we view the world through simple IS-LM curves for the closed economy and suppose that the interest rate and income are away from their target values. Our view of which curve is further away from its desired position would depend upon the slopes of the two curves which are, in turn, given by the structural coefficients. Thus, our estimates of these coefficients are used to determine whether a shift of the IS or LM curve is primarily responsible for the disequilibria.

approximate the structural coefficients.[1/] If locating and offsetting disturbances were their only consideration, the relative weights they, in effect, give to the various economic goals would be the structural coefficients.

Another result is that the weights implied by the CAA and the ODA are not only quantitatively different but can differ in sign as well. An example of this difference is found by comparing the ODA fiscal policy rule given in equation (5) with the CAA rule

(6) $\dot{G} = - k_1(Y_G dY + r_G dr + r'_G dr' + Y'_G dY')$.

Inspection of (5) and (6) reveals that the weight given to foreign income is positive in the ODA rule $(- k_1(- m') > 0)$ and negative in the CAA rule $(- k_1 Y'_G < 0)$.[2/] The ODA rule runs against our intuition since it implies that, if there are inflationary (deflationary) conditions abroad, then domestic fiscal policy should expand

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1/ It is useful to examine the case in which the weights are a combination of those found with the CAA and ODA. A set of rules which embodies both approaches is $\dot{p} = - K(wS + (1-w)(S^{-1})')dy$ where $0 \leq w \leq 1$. When combined with the equations for the economy, the policy-endogenous system is clearly stable with no complex roots.

2/ The weight in the CAA rule, $- k_1 Y'_G$, is negative because Y'_G is positive. The sign of Y'_G is found from theelement in the first row and fourth column of $(S^{-1})'$.

(contract) more than otherwise.[1/] The fact that we ordinarily regard a negative weight as "correct" suggests that we usually think in terms of rules generated with the CAA. Other differences between the CAA and the ODA will be brought out in the next section.

Responses of Monetary Policies to the Liquidity and Adjustment Problems

The preceding IS-LM curve analysis has been used because it highlighted the factor which motivates the ODA: only the policy which directly enters the equation in which a disturbance occurs should be used to offset the disturbance. In a model in which the number of equations is greater than the number of instruments (and targets), a disturbance can occur in an equation which does not contain the direct influence of an instrument. For instance, there may be a disturbing shift in preferences between stocks and bonds that cannot be directly offset by an open market operation which impinges directly on the

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1/ Implicit in the mathematical rules suggested by Richard Cooper [1] for coordinating macroeconomic policies between countries is the rule (equation (5)) which requires domestic fiscal authorities to pursue expansionary policies when inflationary conditions in the rest of the world increase. As strange as this implication of the ODA rule appears, there is an explanation for it (in the context of this Keynesian model!). That is, if domestic fiscal authorities only expand or contract when there has been an unwanted disturbance in the sector in which they directly operate (viz., the goods-bond market), then they will be led to react in this seemingly perverse way to foreign developments. One might want to regard this implication of (5) as a *reductio ad absurdum* argument to show that this Keynesian model is not useful for interpreting the world economy. We have employed the Keynesian model as a vehicle for the argument of this paper because economists are familiar with the model and this particular implication needed to be brought out.

money-bond market. The purpose of this section is to show that the distinction between the ODA and CAA rules is still maintained in a model in which disturbances can occur in equations which are not directly affected by policy instruments.

To separate our task from other problems, let us assume that the number of target variables is equal to the number of policy instruments and that the desired targets are consistent or simultaneously obtainable. The two-country model of the previous section qualifies for our purpose if we drop two of the targets and two of the policies. I have chosen to drop the interest rate targets and fiscal policies. This leaves world income, $V = Y + Y'$, and the balance of payments, b, as our two targets which correspond to the well known liquidity[1/] and adjustment problems in international monetary theory. The remaining instruments are the quantities of domestic and foreign money.

Using the same procedure as before, the policy rules can be derived as

$$(CAA) \quad \dot{p} = -KR'dy \qquad \text{and} \qquad (ODA) \quad \dot{p} = -KR^{-1}dy.$$

where p is the vector of policies (domestic and foreign money

1/ One reason for the importance of international reserves is that a shortage or an excess of "international liquidity" induces government authorities to contract or expand more than otherwise. Consequently, if the world level of central bank reserves is less or greater than desired, world income, V, will be deflated or inflated more than otherwise.

supplies), y is the vector of target variables (V and b), and R is the matrix of reduced-form coefficients.[1/] Using these equations we can plot iso-policy lines which will facilitate a further comparison of the two criteria. Iso-policy lines are found by setting $\dot{p}$ = constant. Of special interest are neutral-policy lines determined by $\dot{p} = 0$. The slopes of the lines are determined by their relative weights or the structure (as opposed to the scale) of the policy rules. The neutral policy lines for $-KR'dy = 0$ and $-KR^{-1}dy = 0$ are plotted in Figure I. The CAA lines are dotted and the ODA lines are solid.

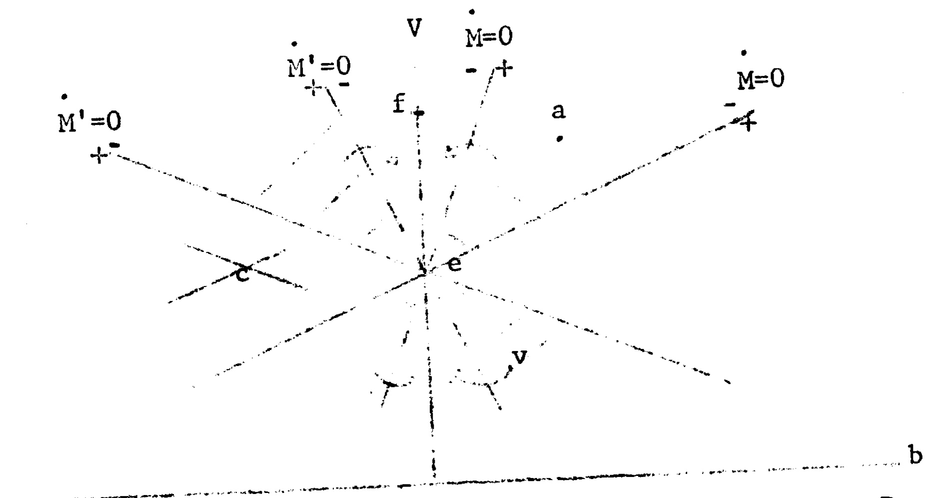

Figure I. CAA and ODA Rules for Adjustment and Liquidity Problems

Plus and minus signs are placed on each side of the neutral policy lines to indicate the direction of policy changes for values

1/ The complete model is derived in Appendix B.

of V and b not on the lines. The shaded areas represent those combinations of V and b which will induce qualitatively (as well as quantitatively) different policy responses under the CAA and ODA, For instance, if the system is at point a, the CAA implies that domestic monetary policy should expand and the ODA requires domestic money to contract.

The solid curves which intersect at point c are drawn for given levels of M and M' (as opposed to $\dot{M}$ and $\dot{M}'$). They represent the comparative-statics equations $R^{-1}dy = dp + dz$. For instance, the upward sloping line through point c is the first equation, $R^{11}\,dV + R^{12}\,db = dM + dZ_1 = \text{constant}$. Just as the iso-policy curves are drawn for constant rates of change of money, the comparative-statics lines are drawn for constant levels of money supplies. The intersection of the comparative-statics curves determines the actual values of V and b at all points in time, and c was selected as an arbitrary point at which $y \neq y_o$. The purpose of the monetary policies is to move the intersection, c, to the target levels of V and b at point e.

A comparison of the three sets of equations

$$\text{(CAA)} \quad \dot{p} = -\,KR'\,dy = \text{constant}$$

$$\text{(ODA)} \quad \dot{p} = -\,KR^{-1}dy = \text{constant}$$

$$\text{(Comparative-statics)} \quad R^{-1}dy = \text{constant}$$

makes it clear that the ODA lines are parallel and the CAA lines are perpendicular to the comparative-statics curves as drawn in Figure I.

The arrows in Figure I show the direction of adjustment under the CAA when the system begins from a shaded area. Each arrow crosses a CAA line which means that a policy must reverse itself in the process of adjustment. For instance, starting at point v, the CAA implies that foreign monetary policy will first contract and then expand before the desired values of b and V are obtained.

The paths of adjustment implied by the ODA rules are not drawn since the adjustment is always direct: there are no policy reversals with the ODA rules.[1/] Since the CAA rules may have a stop-go characteristic and the ODA rules do not, it appears that on this ground the ODA is a better criterion for policy formulation. However, we have compared the two criteria under the assumption of perfect knowledge. Suppose, with imperfect knowledge, that it were more difficult to derive the weights using the ODA than the CAA. Then the probability of a policy reversal using the ODA might exceed the corresponding probability under CAA rules. Since uncertainty has not been introduced in this paper, it would be inappropriate to judge the relative efficiency of the two criteria at this time.

1/ The maximum number of possible policy reversals is one less than the number of exponential terms in the solution of dp(t). The number of exponential terms in the ODA solution is one; therefore, it implies no reversals. The maximum number of exponential terms in the CAA solution is n (the order of the system); therefore, the maximum number of reversals is n-1.

Unlike the analysis in the previous section, one disturbance may call for more than one policy to remedy it. For instance, suppose an investment function shifts outward causing world inflationary pressures but no balance of payments disequilibrium such that the system moves to point _f_. Then _both_ foreign and domestic monetary policies must contract in order to relieve the inflationary pressure. This holds for both the ODA and the CAA rules.

It is useful to consider a major qualification to the distinction between the CAA and the ODA rules. Consider the economic goals of price stability and full employment in a closed economy. Suppose that Milton Friedman is correct -- that there is a natural rate of unemployment such that, in the long run, the effect of money on employment is zero.1/ Suppose, also that fiscal policy has a negligible impact upon the long-run rate of inflation but that it does affect the rate of employment. If the cross multipliers (from money to employment and budgetary policy to the inflation rate) are zero, the

1/ In [2] Friedman has defined the "natural rate of unemployment" to mean that rate which can be altered by real but not by monetary factors (i.e., the quantity of money) in the long run. As he points out, this follows Wicksell's concept of the natural rate of interest.

distinction between single and multi-target rules disappears. If we plot the employment rate on the abscissa and inflation rate on the ordinate, then the ODA and CAA lines coincide; the iso-fiscal-policy curves are vertical and the iso-monetary-policy curves are horizontal. Consequently, under Classical assumptions, the importance of this distinction appears to be diminished. However, the two-country model of this section has focused upon two important problems and two policies for which the cross multipliers are not zero regardless of whether one's views are Classical or Keynesian.

Conclusions and Qualifications

This paper has been given to the limited purpose of developing explicit interpretations of two criteria which frequently play a role (along with other considerations) in the decisions to contract or expand macroeconomic policies. It was demonstrated that both criteria (in the context of linear comparative-static models) imply policy rules which, when combined with the structural equations, yield a stable policy-endogenous system with direct approaches to equilibrium. It was also shown that the distinction between the CAA and the ODA is maintained in a more complicated model that allows for disturbances in equations in which policies do not directly operate.

Having given the CAA and the ODA simple mathematical interpretations, several interesting comparisons were possible.

In the first place they require different information. The CAA uses numerical estimates of multipliers as input for the policy decision process and the ODA requires the values of structural coefficients. Consequently, these approaches generally imply different weights for the target variables and different policy responses. In one case it was found that the ODA rule implied that, in a Keynesian model, inflationary conditions abroad would cause domestic fiscal policy to be more expansionary than otherwise. In the second place we noted that stop-go policies were consistent with the CAA but were ruled out by the ODA. And, finally, we noted that policy-makers are inadvertently led to weighting their target variables with structural coefficients when they allow considerations of the source of disturbances to influence policy responses. In contrast, with the CAA, they explicitly use the size of multipliers in determining the relative weights which they place on various economic goals.

APPENDIX A: A Simple Two-Country Keynesian Model

The model consists of four equations which are assumed to hold at all times. They are the equilibrium conditions for goods and money in the home country and the rest of the world and are written as follows:

(1) $Y = E(Y,r) + I'(Y') - I(Y) + G + Z_1$

(2) $L(Y,r) = M + Z_2$

(3) $L'(Y',r') = M' + Z_3$

(4) $Y' = E'(Y', r') + I(Y) - I'(Y') + G' + Z_4$

where Y = real national income; E = domestic private expenditures

r = interest rate; G = government expenditures

L = demand for money; $\dot{M}$ = supply of money

Z_i = disturbance in ith equation; I = domestic imports

Primed variables denote the foreign counterparts to domestic variables, and taxes are assumed constant such that dG represents the change in government deficit or surplus. The balance of payments has been omitted under the assumption that the effect of payments disequilibria on the distribution of the world money supply is being neutralized.[1/]

The linearized version of the equations can be written as

$$\begin{bmatrix} 1-E_y+m & -E_r & o & -m' \\ L_y & L_r & o & o \\ o & o & L'_r & L'_y \\ -m & 0 & -E'_r & 1-E'_y+m' \end{bmatrix} \begin{bmatrix} dY \\ dr \\ dr' \\ dY' \end{bmatrix} = \begin{bmatrix} dG \\ dM \\ dM' \\ dG' \end{bmatrix} + \begin{bmatrix} dZ_1 \\ dZ_2 \\ dZ_3 \\ dZ_4 \end{bmatrix}$$

where subscripts indicate partial derivatives, $m = \partial I/\partial Y$, and $m' = \partial I'/\partial Y'$. In matrix notation it can be written as $Sdy = dp + dz$.

1/ For a discussion of the full model and the sterilization assumption see [9].

APPENDIX B: A Model of the Liquidity and Adjustment Problems

The reduced-form model which relates domestic and foreign monetary policy to world income and the balance of payments can be derived from equations (1) - (4) in conjunction with the payments equation,1/

(7) $b = I'(Y') - I(Y) + T(r-r')$ = domestic country's surplus,

where T = net capital inflow. Assuming that the balance of payments is initially in equilibrium, $b_o = 0$, we can differentiate (7) to obtain

$$db = b-b_o = b = m'dY' - mdY + T_r(dr-dr').$$

To obtain the reduced-form equation relating b to dM and dM', we use the multipliers in S^{-1} to obtain

$$b_M = m'S^{42} - mS^{12} + T_r(S^{22} - S^{32}) < 0$$

and

$$b_{M'} = m'S^{43} - mS^{13} + T_r(S^{23} - S^{33}) < 0.$$

where S^{ij} is the ij^{th} element of S^{-1}. The reduced-form equation is

$$b = b_M dM + b_{M'}dM'.$$

The effect of dM and dM' on world income, V, is

$$dV = dY + dY' = (S^{12} + S^{42})dM + (S^{13} + S^{43})dM' > 0.$$

1/ For simplicity I have assumed that capital flows respond to the interest rate differential rather than its rate of change. We can justify this on the grounds that we are doing short-run analysis and portfolio stocks only adjust with a significant time lag. Another reason for not including $\dot{r}-\dot{r}'$ in T is that it would produce a second-order differential equation and complicate the analysis without, I believe, altering the conclusions.

Combining these equations we obtain

$$\begin{bmatrix} dV \\ b \end{bmatrix} = \begin{bmatrix} S^{12}+S^{42} & S^{13}+S^{43} \\ b_M & b_{M'} \end{bmatrix} \begin{bmatrix} dM \\ dM' \end{bmatrix} \quad \text{or} \qquad dy = Rdp$$

where dy and dp are defined as two-dimensional vectors. When the system is written to include a two-dimensional disturbance vector, $dy = R\ (dp+dz)$.

APPENDIX C: Information Required for Stability in the Two-by-Two Case for Single and Multi-Target Rules

Although the analysis here is cast in terms of the internal-external-balance problem, the results are general. That is, with any two consistent targets linearly related to any two independent instruments, the PEMC rules sometimes require less and never require more information to achieve stability than do CAA or ODA rules.

Definitions: Y_G = fiscal policy multiplier upon internal balance

b_m = monetary policy multiplier upon external balance

Y^*_m = our estimate of monetary policy multiplier upon internal balance

b^*_G = our estimate of the fiscal policy multiplier upon external balance

$$\Delta = Y_G\ b_m - Y_m b_G = |R| \quad \text{where } R = \begin{bmatrix} Y_G & Y_m \\ b_G & b_m \end{bmatrix}$$

$$dy = \begin{bmatrix} dY \\ db \end{bmatrix} \qquad dp = \begin{bmatrix} dG \\ dM \end{bmatrix} \qquad K = \begin{bmatrix} k_1 & 0 \\ 0 & k_2 \end{bmatrix} \quad (k_i > 0).$$

According to the PEMC, we can take the general statement of the policy rules, $\dot{p} = h\ dp$ where $h = \{h_{ij}\}$ $(i,j = 1,2)$, and set either $h_{12} = 0 = h_{21}$ <u>or</u> $h_{11} = 0 = h_{22}$ depending upon the comparative advantages of the policies (i.e., upon whether $|Y^*_G| / |b^*_G| > |Y^*_m| / |b^*_m|$ or $|Y^*_G| / |b^*_G| < |Y^*_m| / |b^*_m|$, respectively). In addition, the sign of the nonzero h_{ij}'s need to be set opposite the sign of (our estimate of) the multiplier that relates the policy to its assigned target. (E.g., if fiscal policy is assigned to internal balance and $Y^*_G > 0$, then h_{11} should be negative.)

When the PEMC rules, $\dot{p} = hdy$, are combined with the reduced-form equations of the system, $dy = Rdp$, we obtain $\dot{p} = hRdp$ with the characteristic equation

(PEMC) $\lambda^2 - \lambda[h_{11}Y_G + h_{22}b_m) + (h_{21}Y_m + h_{12}b_G)] + (h_{11}h_{22} - h_{12}h_{21})\ \Delta = 0.$

The characteristic associated with the CAA rules[1/] is

(CAA) $\lambda^2 + \lambda[k_1(Y_GY^*_G + b_Gb^*_G) + k_2(Y_mY^*_m + b_mb^*_m)] + k_1k_2\ \Delta^*\Delta = 0.$

A necessary and sufficient condition for the real parts of the roots to be negative is that the three coefficients (of λ^2, λ^1, and λ^o) have the same sign.

1/ This entire analysis could be carried out with the CAA rules replaced with ODA rules without altering the conclusions.

There are a great many combinations of assumptions we could make concerning the sign and magnitudes of the elements of R. Under each such combination, there is no important sense in which our estimates of the parameters must be closer to the "true" parameters in order to achieve PEMC stability than in order to achieve CAA stability.

On the other hand, there is an important and not infrequent case in which limited information or inaccurate estimates will render the PEMC system stable and the CAA system unstable. As an example suppose that[1/]

$Y_G > 0$	$Y^*_G > 0$	$\Delta < 0$
$Y_m > 0$	$Y^*_m > 0$	$\Delta^* > 0$
$b_G > 0$	$b^*_G > 0$	$\lvert Y^*_G\rvert / \lvert b^*_G\rvert > \lvert Y^*_m\rvert / \lvert b^*_m\rvert$
$b_m < 0$	$b^*_m < 0$	

In this case the PEMC system is stable (regardless of the true comparative advantage of the policies) and the CAA system is unstable ($k_1 k_2 \Delta^* \Delta < 0$).

In more general terms, the PEMC assignment may be stable even when our estimate of the policies' comparative advantages is wrong. This can occur when, unlike Mundell's [6] famous internal-external-balance diagram with both curves sloping in the same direction, the true iso-target curves have opposite slopes.[2/]

1/ John Morton [5] has a useful exposition of this case in which the internal balance curve slopes downward and the external balance curve slopes upward when plotted in M-G space. He derives these curves from the underlying IS-LM model.

2/ This helps explain why John Patrick (p. 36 in [7]) came to the conclusion that the <u>same</u> knowledge (i.e., that the estimate of the determinant have the correct sign) is necessary to obtain stability in the PEMC and ODA systems. That is, by defining monetary policy in terms of the interest rate, his internal and external balance curves both have negative slopes regardless of the absolute values of the parameters.

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