BOARD OF GOVERNORS OF THE FEDERAL RESERVE SYSTEM
WASHINGTON, DC 20551

Supervisory Stress Test Documentation

Final 2026 Global Market Shock Component

February 2026

This document summarizes the models and the processes of the global market shock (GMS) component of the severely adverse scenario that the Board of Governors of the Federal Reserve System (Board) used to produce certain values in the scenarios for the 2026 Supervisory Stress Test. There were no revisions to this document from the version proposed in October 2025 other than typographical fixes.[1] Documentation on the other final and proposed models associated with the Board's 2026 Supervisory Stress Test is available at the following link: https://www.federalreserve.gov/supervisionreg/dfa-stress-tests-2026.htm.

[1] *See* 90 FR 51762 (November 18, 2025). This proposal was posted to the Board's public website on October 24, 2025. Board (2025), "Federal Reserve Board requests comment on proposals to enhance the transparency and public accountability of its annual stress test," press release, October 24, 2025, https://www.federalreserve.gov/newsevents/pressreleases/bcreg20251024a.htm.

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A. Statement of Purpose

The global market shock component for the severely adverse scenario is a set of hypothetical shocks to a large set of financial risk factors, such as stock market indices, currencies, commodities, interest rates, and credit securities, reflecting general financial market distress and heightened uncertainty. A firm subject to the annual Dodd-Frank Act supervisory stress tests that has significant trading activity must incorporate the global market shock into the severely adverse scenario.[2] In addition, certain large and highly interconnected firms must apply the same global market shock to project losses under the counterparty default scenario component. The losses associated with the global market shock are recognized in the first quarter of the scenario horizon, and no changes to losses or gains are assumed in subsequent quarters of the scenario. The global market shock is applied to positions held by the firms on a given *as-of date*, which was, for example, October 11, 2024, for the 2025 stress test cycle.[3] The stress test scenarios should not be regarded as forecasts; rather, they are hypothetical paths of economic and financial variables used to assess the strength and resilience of the companies' capital in various economic and financial environments.

The design and specification of the global market shock component differ from the design and specification of the severely adverse macroeconomic scenario for several reasons. First, informed by U.S. Generally Accepted Accounting Principles (U.S. GAAP), profits and losses from trading and counterparty credit positions are measured in mark-to-market accounting

[2] Firms that are subject to the global market shock are those with aggregate trading assets and liabilities of $50 billion or more, or with aggregate trading assets and liabilities equal to 10 percent or more of total consolidated assets; and are not a Category IV firm under the Board's tailoring framework. See 12 C.F.R. § 238.143(b)(2)(i); 12 C.F.R. § 252.54(b)(2)(i).

[3] A firm may use data as of the date that corresponds to its weekly internal risk reporting cycle as long as it falls during the business week of the as-of date for the global market shock (e.g., October 7–11, 2024 for the 2025 stress test).

terms in the global market shock, while revenues and losses from traditional banking activities, as generated under macroeconomic scenarios, are generally measured using the accrual accounting method. Another key difference between the global market shock and the severely adverse macroeconomic scenario is the timing of loss recognition. The global market shock has an impact on losses in the first quarter of the severely adverse scenario's projection horizon, whereas the severely adverse macroeconomic scenario moves over a nine-quarter projection horizon. This timing is based on a scenario assumption that market dislocations can happen rapidly and unpredictably at any time during the scenario horizon. Applying the global market shock in the first quarter of the stress test projection horizon ensures that potential losses from trading and counterparty exposures are incorporated into banks' capital ratios in each quarter of the severely adverse scenario. In addition, the severely adverse macroeconomic scenario currently has an as-of date of December 31 of each year, whereas the global market shock as-of date changes every year and does not necessarily coincide with the year-end.[4] The global market shock component is referred to as "scenario" or "global market shock scenario" hereafter.

As discussed in greater detail in Section B below, the global market shock scenario comprises a large set of financial risk factors. An exhaustive list is provided in the global market shock template.[5] The risk factors of the global market shock scenario include, but are not limited to:

[4] The Board has proposed changes to the global market shock as-of date and other aspects of the supervisory stress test in the Board's Enhanced Transparency and Public Accountability proposal, published in the fall of 2025.

[5] The template is available from the website of the Board. For example, for the 2025 stress test: https://www.federalreserve.gov/supervisionreg/dfa-stress-tests-2025.htm.

- Public equity returns from key advanced economies as well as from developing and emerging market economies, along with selected points along term structures of equity option-implied volatilities;
- Exchange rates of foreign currencies, along with selected points along term structures of foreign exchange option-implied volatilities;
- Government yields at selected maturities (e.g., 10-year U.S. Treasuries), swap rates, and other types of interest rates for key advanced economies as well as from developing and emerging market economies;
- Implied volatilities on interest rate options for selected maturities and expiration dates, which are key inputs to the pricing of interest rate derivatives;
- Futures prices at various expiration dates for commodity products such as energy, oil, metals, and agricultural products; and
- Credit spreads or prices for selected credit-sensitive products, including corporate bonds, credit default swaps (CDS), securitized products, sovereign debt, and municipal bonds.

B. Overview of Scenario Design Process

The Board generates shock values for all exposures in the global market shock template. Shock values represent the magnitudes of changes to the financial risk factors in the global market shock template, and they reflect the severity of market stress that these risk factors experience in the scenario. Table B1 provides an overview of the shock definitions by asset class as well as the horizons over which the shocks are calibrated, as discussed further in the following section. Throughout this document, the terms “financial risk factor shocks” and “shock values” are used interchangeably.

Table B1 - Overview of Shock Values Generated by the Global Market Shock Scenario Design Framework

Asset class	Spot/futures curve shocks	Option-implied volatility shocks	Horizon
Agencies	Option adjusted spread changes to U.S. residential agency products, U.S. commercial agency products, and non-U.S. agency products across various ratings.	N/A	1 month
Commodities	Arithmetic returns to spot prices and futures contract prices across maturities for commodities.	Changes to implied volatilities of commodities.	1 month
Foreign exchange rates	Arithmetic returns to spot exchange rates of various currencies against the U.S. dollar. Cross-currency spot exchange rates.	Changes in implied volatility of foreign exchange options across various maturities.	1 month
Interest rates	Absolute changes to term structures of government bond yields and swap rates for various countries. Absolute changes in inflation, cross-currency versus the U.S. dollar basis, and EUR tenor basis risk.	Changes to interest rate implied volatilities across various swaption maturities.	1 month
Public equity	Arithmetic returns to public equity across regions (markets).	Changes in implied volatilities of public equity options across various maturities.	1 month
Public equity dividends	Relative yield shocks on dividend derivatives (e.g., dividend swaps and dividend futures) across various regions (markets) and tenors.	N/A	1 month
Sovereign credit	Changes to five-year credit default swap spreads for various countries.	N/A	1 month
Corporate credit	Spread changes to corporate bonds, covered bonds, indices, index tranches, and index options across credit ratings.	N/A	3 months
Municipal credit	Spread changes to municipal bond indices and other municipal credit products across credit ratings.	N/A	3 months
Other fair value assets	Arithmetic returns to other securities held under fair value accounting rules. Examples include illiquid fair value securities, which cannot be grouped into	N/A	3 months

Asset class	Spot/futures curve shocks	Option-implied volatility shocks	Horizon
	another asset class, such as public welfare investments covering housing credit, tax credit, and energy investments.		
Securitized products	Market value haircuts (price declines), expressed in percentage terms, to value-weighted portfolios of mortgage-backed securities and other asset-backed securities (ABS).	N/A	3 months

i. Risk Factor Liquidity Horizons

Financial risk factor shocks are calibrated based on assumed time horizons that reflect several scenario design considerations. One consideration is the liquidity characteristics of the different asset classes (as listed in Table B1) that constitute risk factors. More specifically, the calibration horizons reflect the variation in speed at which banks could reasonably close out, or effectively hedge, the associated risk exposures in the event of market stress. The horizons are generally longer than the typical times needed to liquidate exposures under normal conditions because they are designed to capture the unpredictable liquidity conditions that prevail in times of stress.[6] Another consideration is maintaining consistency between the assumed time horizons used to calibrate risk factor shocks and the timeline for attributing the losses stemming from these risk factors. Specifically, losses associated with the global market shock component are attributed to one quarter of the stress test horizon, which implies an upper bound of three months for calibrating the shocks.

[6] The liquidity of previously well-functioning financial markets can undergo abrupt changes in times of financial stress. For example, prior to the Global Financial Crisis, AAA-rated private-label residential mortgage backed securities (RMBS) would likely have been considered highly liquid, but their liquidity deteriorated drastically during the crisis period.

Given these considerations, the shock liquidity horizons are chosen to be broadly consistent with the proposed standards in the Fundamental Review of the Trading Book (FRTB).[7] The horizons in the FRTB are specified based on recommendations from consultations with the financial industry and its regulators. Therefore, they are considered a reasonable benchmark for defining the shock horizons used in the global market shock.

The liquidity horizons used in the market shock scenarios are not perfectly matched with the FRTB liquidity horizons due to granularity differences between the FRTB standards and the global market shock template. The FRTB specifies horizons at a more granular level, often using different horizons within each asset class. For example, the FRTB specifies sovereign risk factor horizons by credit rating. In contrast, the global market shock template specifies sovereign shocks by country to capture country-specific risks not reflected by credit ratings. Moreover, the Board uses the same liquidity horizon for all risk factors within each asset class, whereas the FRTB allows for different horizons within asset classes. Given these differences, the global market shock scenario aims at aligning with the horizons specified by the FRTB by using a weighted average of the FRTB horizons within each asset class. The weights are determined using aggregate firm exposures.[8] For example, FRTB horizons for equity risk factors vary between ten and 60 business days, and the global market shock horizon for this asset class is assumed to be four weeks. Since the Board imposes an upper bound on global market shock horizons of one quarter, there are cases where the range of FRTB horizons is longer than the global market shock horizon. For example, FRTB horizons for corporate credit risk factors vary

[7] The FRTB standards are published as part of the "Minimum capital requirements for market risk" published by the Bank of International Settlements (2019).

[8] Exposures are taken from FR Y-14Q, schedule F.

between 60 and 120 business days, but the Board uses a horizon of three months for corporate credit.

ii. Scenario Design Process

The process for designing the global market shock consists of three stages: 1) the development of scenario narratives, 2) scenario generation, and 3) scenario selection. Three different parties with relevant decision authority vet this process before scenarios are finalized. The parties involved are market risk functional leads, an oversight committee of stress testing program leaders, and division directors for Supervision & Regulation and Financial Stability at the Board.

In the first stage, scenario narratives, which are thematic summaries of potential episodes of market distress that are summarized by a few risk factors, are developed for further consideration. These risk factors, denoted as *primary* risk factors, broadly characterize market conditions under a chosen narrative. After scenario narratives and their primary risk factors are developed, they are first reviewed by market risk functional leads. After this review is completed, narratives and primary risk factors are presented to the oversight committee, which decides on the candidate scenario choices of each stress testing cycle.

The second stage uses data-driven models to generate the full set of published global market shocks conditional on the primary risk factor shocks for each narrative. The models are based on the historical co-movements between risk factors under stressed market conditions. The output is a set of candidate scenarios, each consisting of a narrative and a complete set of risk factor shocks that make up the global market shock. This step emphasizes the process of *expanding* the primary risk factor shocks out to the full set of shocks. As in the first stage of the scenario design process, the oversight committee reviews and signs off on candidate scenario choices before they are presented to division directors for further evaluation.

In the third stage, a few scenarios are selected from among all the candidate scenarios for final global market shock scenario consideration by the Board. After reviewing all presentations and perspectives of stress test program leadership, the Director of Supervision & Regulation and the Director of Financial Stability decide on the final global market shock with the concurrence of the Chair of the Committee on Bank Supervision. The following three subsections (B.ii.1-B.ii.3) describe these stages in further detail.

1. Scenario Narratives

The first stage of global market shock scenario design consists of drafting several narratives of potential episodes of market distress. The Board creates multiple scenario narratives at this stage to encompass different types of market stresses. A scenario narrative consists of a qualitative description along with shock values for a few key risk factors that characterize different financial market conditions. These risk factors, denoted *primary risk factors*, represent the following five financial markets: equities, credit, interest rates, commodity, and foreign exchange rates. The selected primary risk factors are the S&P 500 equity market index, Moody's Baa-Aaa credit spread, the level and slope of U.S. Treasury interest rates, energy and metal commodity indices, and the U.S. dollar-to-Euro exchange rate. This set of primary risk factors can be augmented with additional risk factors if a particular scenario narrative requires additional risk aspects to provide a full characterization. For instance, a scenario narrative focusing on financial and economic conditions in Europe may require more Europe-specific risk factors, such as European stock market indices.

The Board considers multiple sources of information for specifying scenario narratives, including supervisory experience and forward-looking expert judgment, as well as statistical analysis of historical and recent financial data. Incorporating expert judgment in scenario design is a widely accepted practice by industry practitioners and in academic and regulatory literature. For example, the Committee on the Global Financial System (2005) classifies stress scenarios into historical and hypothetical scenarios, where the latter involves considerable judgment.[9] Aikman et al. (2024), Breuer et al. (2018), and Alfaro and Drehmann (2009) also emphasize the role of expert insights in scenario design.[10]

Expert judgment is based on a screening of emerging risks and current market conditions identified from various sources including, but not limited to, financial stability reports from government agencies, supervisory information, and internal and external assessments of potential sources of distress, such as geopolitical, economic, and financial market events.

The Board uses statistical analysis of historical data to specify primary risk factor shocks. One analysis uses percentiles of historical data, where the percentiles reflect the severity of stress for each primary risk factor. For example, the narrative may categorize shocks as "mild," "moderate," "large," "severe," or "unprecedented." In determining a given cycle's global market shock component, the Board may consider unprecedented shocks because times of stress can feature events that have not been observed previously. These qualitative characteristics are mapped to quantitative shocks as follows:

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[9] See Committee on the Global Financial System (2005): Stress Testing at Major Financial Institutions: Survey Results and Practice.

[10] See Aikman, D., R. Angotti, and K. Budnik (2024): Stress Testing with Multiple Scenarios: A Tale on Tails and Reverse Test Scenarios, European Central Bank Working Paper, No 2941.; Breuer, T., M. Jandačka , J. Mencía, and M. Summer (2012): A Systematic Approach to Multi-Period Stress Testing of Portfolio Credit Risk, Journal of Banking and Finance, 36(2), 332–340.; Alfaro, R., Drehmann, M., 2009. Macro Stress Tests and Crises: What Can We Learn? In: BIS Quarterly Review. Bank of International Settlements, pp. 29–41.

- "Mild" shocks fall within the 15th (including) and 85th (including) percentiles of the historical realizations of the primary risk factors.
- "Moderate" shocks fall within the 5th (including) and 15th (less than) percentiles or within the 85th (greater than) and 95th (including) percentiles.
- "Large" shocks fall within the 1st (including) and 5th (less than) percentiles or within the 95th (greater than) and 99th (including) percentiles.
- "Severe" shocks fall within the historical minimum (including) and the 1st (less than) percentile or within the 99th (greater than) percentile and the historical maximum (including).
- "Unprecedented" shocks are greater in magnitude than the historical maximum or lower than the historical minimum.

The above mapping is the Board's general guidance for standardizing the qualitative severity of the scenario narrative rather than using a statistical model. These percentiles are applied to historical primary risk factor shock values computed over ***h***-week windows of data, where ***h*** is the time horizon over which the shock is calibrated. The final adjustment may combine the shock values from these percentiles with adjustments from other considerations, such as the current levels of risk factors. For example, large negative shocks to interest rates could be avoided during times when interest rates are near the zero lower bound.

The shock values of primary risk factors may also be chosen to target specific secondary risk factors that are deemed particularly important for a given scenario narrative. For example, the shock value for the energy commodity index may be chosen to target a specific shock value for a shock to the price of West Texas Intermediate crude oil, which is a secondary risk factor.

Another analysis used to help inform the production of narratives considers historical shocks in combination with past firm risk exposures. For this exercise, the Board generates a large number of scenarios using historical simulation and estimates the trading gains and losses associated with them using firm exposures from previous quarterly FR Y-14Q submissions.[11] Using this approach, stressful historical shocks are defined by those scenarios that result in tail losses. This analysis generates a set of historical stressful episodes from which historically observed shock values to primary risk factors can be collected. This method is described in further detail in Section D.

Percentiles of historical data and the historical simulation method offer two alternative sets of primary risk factors that the Board can choose from to describe the narrative. The Board reviews both sets collectively and selects the primary risk factors that are most appropriate for the scenario narrative based on emerging risks and current market conditions. Finally, the Board checks primary shock calibrations for consistency using historical correlations to ensure that the final combination of shocks representing the key aspects of a given scenario remains plausible when considered jointly.

2. Scenario Generation

In the second stage of the scenario design, scenario shocks are quantified for all other risk factors in the published global market shock scenario, including the secondary risk factors. For this purpose, as a starting point, the Board uses a modeling approach that produces candidate shocks for all risk factors conditional on the scenario shocks to the primary risk factors.

[11] FR Y-14Q reporting forms and instructions are available at the website of the Board: https://www.federalreserve.gov/apps/reportingforms/Report/Index/FR_Y-14Q.

In designing the modeling approach, the Board adhered to two main objectives: (i) shocks should be internally consistent within each scenario (e.g., a severe shock to a security price should be accompanied by severe shocks to the implied volatility of options written on that security); and (ii) models should be flexible so as to incorporate emerging risks and targeted narratives in the global market shock (e.g., the set of primary risk factors should be flexible to incorporate narratives centered around risks arising outside the U.S.). To satisfy these objectives, the modeling approach uses a stepwise approach to modeling shocks similar to Abdymomunov, Duan, Hansen, and Misirli (2024, Section 3).[12] The models are designed to capture historical relationships between risk factors (primary, secondary, and all other), particularly between historical severe (tail) shock observations. The modeling approach is described in detail in Section C.

The Board evaluates the model-produced shocks and, if necessary, applies adjustments based on the Board's supervisory experience and expertise. These adjustments incorporate scenario narrative characteristics as well as emerging and ongoing risks highlighted in Financial Stability Reports and supervision reports of different agencies. Therefore, the narrative is not constrained by the limited number of the primary risk factors.

The adjustments to model output are motivated mainly by the following reasons:

- Data issues or internal model limitation: When model outputs are deemed to not accurately reflect periods of historical stress or would otherwise be unreasonable due to data issues or model limitations, model overrides are performed to bring them in line with market movement expectations under stress scenarios.

[12] See Abdymomunov, A., Z. Duan, A. L. Hansen, and E. U. Misirli (2024): Designing Market Shock Scenarios, Federal Reserve Bank of Richmond Working Paper Series, WP 24-17.

- Current market environment: Modeled shocks are based on long term historical data and historical correlations. Sometimes modeled shock outputs may be too extreme or too mild for current market conditions. For example, rate shocks calibrated to past periods of stress under a higher interest rate environment may lead to improper outcomes if directly applied to an extreme low-rate environment. In such cases, necessary adjustments to shock output are made in light of current market conditions.
- Bespoke narratives: Models are calibrated to historic periods of general market stress. Narratives may focus on differentiated shocks to specific asset classes or regions which may not be captured exactly or appropriately in such past historical events. In such cases, shocks are adjusted as needed to reflect the tailored scenario narrative.
- Cross-asset class and intra-asset class consistencies: Model output shocks sometimes may not reflect cross-asset class correlations or may not be internally consistent within the asset class due to primary-risk-factor shock specifications. As a result, the overrides are needed to reflect appropriate consistencies.

Consistent with the guidelines on stress testing published by the Bank of International Settlements (2018), judgment applied by the Board is subjected to rigorous review and validation aimed at ensuring that the judgments are properly justified.[13] Shocks to key risk factors, which include primary risk factors and potentially some secondary risk factors that are important for describing the narrative, are benchmarked to historical shock distributions. If any of the key shock values are unprecedented (i.e., exceed historical experience), they must be highlighted in the Board's internal processes, and their plausibility must be justified by the current environment and scenario narratives.

[13] Bank of International Settlements (2018): Stress Testing Principles.

All scenarios, including their judgment-based adjustments, are subject to independent quality assurance review. These reviews aim to ensure that the financial market shocks in the global market shock scenario are (1) severe, but plausible; (2) correspond to the scenario narrative; and (3) are internally consistent; that is, shocks to different product types co-move according to expectations of how the stress scenario would unfold.

3. Scenario Selection

In the third and final stage, the Board evaluates all candidate scenarios and selects one scenario for the global market shock component for the severely adverse scenario. The scenario selection process among candidate scenarios follows a governance process where inputs and feedback from various internal stakeholders are considered. In this process, the Board also collects feedback on scenario narratives from other federal regulatory agencies, such as the Office of the Comptroller of the Currency (OCC) and the Federal Deposit Insurance Corporation (FDIC). Ultimately, authority for the ultimate decision on scenario selection has been delegated jointly to the Directors of the Division of Supervision and Regulation and the Division of Financial Stability, with the concurrence of the Chair of the Committee on Bank Supervision.[14]

C. Detailed Modeling Approach

Generating financial market shock scenarios is a high-dimensional problem because there are thousands of potential risk factor shocks whose historical data are interrelated. Moreover, a scenario should endeavor to be internally consistent; that is, the directions and magnitudes of specific shocks must correspond to those of other shocks within their asset class and across other asset classes, as based on experience during financial turmoil. The shocks should also conform with the overall scenario narrative as a whole.

[14] 12 CFR 265.7(c)(11).

These features present challenges for the scenario design models. First, it is impractical to map scenario narratives directly to a large number of risk factors in one step. Second, it is challenging to ensure the consistency and joint plausibility of different shocks (i.e., the reasonableness of the cooccurrence of thousands of market risk factor shocks across a diverse set of asset classes). Both the modeling and the quality assurance review process of model outcomes increase in complexity as the number of risk factors increases.

To address these challenges, the Board adopts the following approach. First, the set of all risk factors is divided into three categories: (1) the primary risk factors, which characterize the scenario narratives at a very high level; (2) a subset of risk factors that specify the scenario narrative at a more granular level and, to the extent possible, can be described by the variation in the primary risk factors, called *secondary risk factors*; and (3) a large number of any remaining risk factors needed to complete the entire market shock scenario. Note that the set of remaining risk factors includes those that are in the global market shock template but not included in the sets of primary and secondary risk factors. The introduction of the secondary risk factors allows the Board to specify more detailed scenario characteristics than with the primary risk factors alone, while still maintaining a manageable number of risk factors. This approach gives the Board flexibility to design scenarios that target special vulnerabilities in key secondary risk factors while maintaining the model-driven consistency among all risk factors. While the Board seeks to choose secondary risk factors that can be modeled using the primary risk factors, there are cases where risk factors that are unrelated to the primary risk factors are necessary to describe the details of the scenario narrative. In those cases, the Board uses simpler methods such as mappings, multipliers, and percentile methods, as described further in Section C.iii.1.d.

As in Abdymomunov, Duan, Hansen, and Misirli (2024), the modeling approach links these three sets of risk factors in two steps.[15] In the first step, the secondary risk factors are modeled conditional on primary risk factors. These models generate shock values to secondary risk factors given the primary risk factor shocks determined from the scenario narratives. In the second step, any remaining risk factors needed are modeled conditional on the primary and secondary risk factors.

i. Choice of Primary and Secondary Risk Factors

As noted above, the risk factors of a financial market scenario are grouped into three categories: (1) primary risk factors that characterize broad market conditions, (2) secondary risk factors that describe a scenario narrative, and (3) the remaining risk factors that give a comprehensive description of a market scenario. The modeling approach proceeds stepwise to link these three sets of risk factors to one another. This section discusses the choice of primary and secondary risk factors.

1. Description and Rationale

The Board chooses the primary risk factors in accordance with three properties. First, primary risk factors should characterize a large part of the variation in asset prices across five broad asset classes; namely, public equities, traded credit, interest rates, exchange rates, and commodities. This property helps the Board to form economically and statistically significant relationships between the primary risk factors and other risk factors in their respective asset classes.

[15] See Abdymomunov, A., Z. Duan, A. L. Hansen, and E. U. Misirli (2024): Designing Market Shock Scenarios, Federal Reserve Bank of Richmond Working Paper Series, WP 24-17.

Second, primary risk factors should have available long time series of observations that cover major historical economic and financial crises, thereby capturing tail events that can be used for scenario analysis, such as with historical simulation. This property helps the Board to select and justify the magnitudes of risk factors from historical experience and to offer scenario options that differ in shock severity.

Third, primary risk factors should be observed variables with clear economic interpretation, which market participants would know and easily associate with a scenario narrative, as opposed to latent factors described indirectly by statistical methods from observed variables (e.g., principal component analysis). This property helps the Board to focus on the consistency of a small set of primary risk factors, to ensure the coherence of the scenario narrative and to communicate it to market participants.

Review of the available literature suggests that U.S. public equity returns, some measure of prevailing credit spreads, and a government bond term spread are among the key factors in explaining business cycle variation. For example, Diebold and Yilmaz (2009) show that U.S. public stock market returns spill over to global stock markets; Beaudry and Portier (2006) show that public stock price movements along with total factor productivity shocks jointly explain business cycle fluctuations; Jermann and Quadrini (2012) emphasize credit conditions as important contributors to economic downturns; and Estrella and Hardouvelis (1991) show that the term spread has predictive power for future real activity.[16] The literature also suggests that interest rate risk along the yield curve cannot be fully captured by a single factor; see, e.g.,

[16] See Diebold, F. X., and K. Yilmaz (2009): Measuring Financial Asset Return and Volatility Spillovers, With Application to Global Equity Markets, Economic Journal, 119, 158–171.; Beaudry, P., and F. Portier (2006): Stock Prices, News, and Economic Fluctuations, American Economic Review, 96(4), 1293–1307.; Jermann, U., and V. Quadrini (2012): Macroeconomic Effects of Financial Shocks, American Economic Review, 1, 238–271.; Estrella, A., and G. Hardouvelis (1991): The Term Structure as a Predictor of Real Economic Activity, Journal of Finance, 46(2), 555–576.

Litterman and Scheinkman (1991).[17] Given these observations, the scenario design framework employs the S&P 500 index return, Moody's Baa-Aaa credit spread,[18] the U.S. 10-year minus three-month Treasury term spread, and the U.S. 10-year Treasury bond yield as primary factors.

In addition to these business cycle variables, the scenario design framework needs primary factors specifically related to foreign exchange rates and commodities pricing to capture stress within these asset classes. The U.S. dollar-to-Euro exchange rate is central to developing foreign exchange shocks in scenarios based on stress within the U.S. and Europe. Given the strong interactions between U.S. and European financial markets, the Board determines that this exchange is particularly important to define for many scenario narratives. Therefore, the U.S. dollar-to-Euro exchange rate is included in the list of primary factors to represent the foreign exchange market. Commodity markets cover both energy and metal products. To describe the risks associated with these products, three risk factors are included in the set of primary risk factors: energy, gold, and "other metals". Gold is included separately from other metals due to its flight-to-quality property[19] during times of turmoil. Since the energy and metal primary factors are only weakly related, a primary factor for each group is needed. Specifically, the framework includes the Global Price Index of Energy and the Global Price Index of Metal, constructed by the International Monetary Fund. These variables are chosen because they describe common variation within energy- and metal-related primary factors.[20]

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[17] See Litterman, R., and J. Scheinkman (1991): Common Factors Affecting Bond Returns, Journal of Fixed Income, 1, 54–61.

[18] The Baa-Aaa spread is the difference between Moody's Baa-rated and Aaa-rated corporate bond yields.

[19] Flight-to-quality refers to a sudden shift in investment behavior during financial turmoil in which investors sell their risky assets such as stocks and purchase safe assets such as gold instead. This behavior is largely driven by investors' fear in the market, which makes them seek less risk in exchange for lower profits.

[20] The energy index is a price index of fuel-based commodities including crude oil (petroleum), coal, natural gas and propane. The metal index is a price index of base metals including aluminum, copper, iron ore, lead, molybdenum, nickel, tin, uranium and zinc. Index values represent the benchmark prices which are representative of the global

Although the eight primary factors are chosen in accordance with the three properties listed above, it remains unlikely that all scenario narratives can be properly described using this small subset of risk factors. Therefore, the secondary risk factors are introduced to provide a more detailed scenario narrative. Similar to Abdymomunov, Duan, Hansen, and Misirli (2024), the Board selects secondary risk factors from five broad asset classes. The secondary risk factors are chosen based on two principles.[21] First, the set of secondary risk factors must be able to broadly characterize scenario narratives. Second, the secondary risk factors must be statistically related to the primary risk factors, such that statistical models can be used to generate shock values for secondary risk factors conditional on primary risk factor shocks. It may be necessary to include secondary risk factors that do not satisfy this criterion to represent a detailed scenario narrative with the set of secondary risk factors. Using these principles, the set of selected secondary risk factors includes around 100 factors. Examples of chosen secondary risk factors, including their linkages to primary risk factors, are provided in Section C.ii. Unlike the primary risk factors outlined above, the set of secondary risk factors is not necessarily the same each year. Instead, the set of secondary risk factors is flexible and can be expanded to accommodate various market scenarios, as appropriate. For example, a scenario narrative specifically based on financial turmoil in, say, Asia, may require additional Asian equity market indices, interest rates, and foreign exchange rates in the set of secondary risk factors, all of which would not be needed to describe, for instance, a U.S.-centered scenario. Therefore, the set of secondary risk factors in Section C.ii represents a baseline that could be subject to adjustments year-over-year.

market. These prices are determined by the largest exporter of a given commodity and given in nominal U.S. dollars. The time series of the indices are retrieved from the Federal Reserve Bank of St. Louis' FRED online database with the following citations: Global Price of Energy Index [PNRGINDEXM] and Global Price of Metal Index [PMETAINDEXM].

[21] See Abdymomunov, A., Z. Duan, A. L. Hansen, and E. U. Misirli (2024): Designing Market Shock Scenarios, Federal Reserve Bank of Richmond Working Paper Series, WP 24-17.

2. *Assumptions and Limitations*

While summarizing scenario narratives by the small subset of primary risk factors is practical, this approach could overlook certain risks. For example, the Board models equity market risk based on the S&P 500 index as a primary risk factor, but equity market risk could arise from, for example, European or Asian markets. Also, even though simple and tractable models with straightforward interpretations are prioritized, the small number of primary risk factors may limit broad narratives. The Board attempts to mitigate this possibility by considering primary and secondary risk factor shocks as part of the detailed narratives. Moreover, the Board carefully evaluates shock values applied to the larger set of secondary risk factors that are produced from the primary risk factors and applies adjustments to model outcomes if necessary to properly describe the risks in the formulated scenario narratives. Such adjustments may include augmenting the set of primary risk factors with additional risk factors. Finally, the Board has extensive quality assurance processes in place to ensure coherence among all final risk factor shocks.

ii. Modeling the Relationship Between Primary and Secondary Risk Factors

This section explains the first part of the modeling approach, which generates secondary risk factors from primary risk factors using regression models.

The Board models each secondary risk factor separately, given one or more of the primary risk factors. Section C.ii.2 provides an overview of these linkages by asset class. With a few exceptions, the secondary risk factors are linked to primary risk factors from the same asset class; e.g., public equities secondary risk factors are modeled using the S&P 500 index, which is the only public equity primary risk factor. In some cases, these simple within-asset-class links are not sufficient for describing stressed conditions in the secondary risk factors. For example, municipal credit secondary risk factors are modeled using the credit primary risk factor (i.e., the Baa-Aaa credit spread). But in some scenarios, such as during the COVID-19 pandemic, one business cycle factor is not sufficient to generate severe stress in the shock values to municipal credit factors. Therefore, municipal credit is described using both the credit spread and the S&P 500 return. Finally, some secondary risk factors are modeled based on other secondary risk factors, which may describe the behavior under stressed conditions better than the primary risk factors. For example, the implied volatility of DAX index options is modeled using the DAX index return, which is also a secondary risk factor.[22]

Depending on the nature of the secondary risk factor data, the Board employs one of the following regression frameworks: quantile regression for secondary risk factor *shocks* (i.e., the log-return[23] of a risk factor or the difference of risk factor levels); quantile autoregression for secondary risk factor *levels* (e.g., index prices, interest rates, or exchange rates); ordinary regression for the secondary risk factor *shocks*; and ordinary autoregression for spreads between secondary risk factor *levels*.

1. Model Descriptions and Rationale

a. Quantile Regression Model

As discussed in Section C.ii.2, most of the secondary risk factors are modeled with quantile regressions. In market risk, extreme shocks tend to happen simultaneously during financial crises. This behavior is captured by the quantile regression model because it expresses the conditional quantiles of secondary risk factors as a function of primary risk factors. Let $\boldsymbol{r_{i,t,h}}$ denote a risk factor shock in month $\boldsymbol{t}$, such as an equity index log return, over a horizon of $\boldsymbol{h}$

[22] The DAX index is an index of 40 selected German blue chip stocks traded on the Frankfurt Stock Exchange.

[23] The $\boldsymbol{h}$-period log return of a product $\boldsymbol{i}$ with price $\boldsymbol{P_{i,t}}$ is defined as $\boldsymbol{r_{i,t,h} = \log P_{i,t} - \log P_{i,t-h}}$.

months. Specifically, let $\boldsymbol{r^S_{i,t,h}}$ denote a secondary risk factor shock, and let $\boldsymbol{r^P_{i,t,h}}$ denote a vector of $\boldsymbol{K_i}$ primary risk factor shocks chosen on a case-by-case basis for each secondary risk factor regression. The fitted $\boldsymbol{\tau}$'th percentile of the conditional distribution of $\boldsymbol{r^S_{i,t,h}}$ given $\boldsymbol{r^P_{i,t,h}}$ is given by:[24]

Equation C1 – Quantile Regression Model

$$\boldsymbol{Q_\tau(r^S_{i,t,h}|r^P_{i,t,h}) = h\alpha_\tau + r^P_{i,t,h}\beta_\tau},$$

where $\boldsymbol{\alpha_\tau}$ is the estimated constant, and $\boldsymbol{\beta_\tau}$ is the estimated coefficients on the primary factor shocks, both given the percentile parameter $\boldsymbol{\tau}$.[25] Quantile regression models are defined given a fixed percentile $\boldsymbol{\tau}$, which must be defined before the model can be estimated. This parameter controls the location on the conditional distribution of the secondary risk factor data for which the model is predicting shocks. In other words, $\boldsymbol{\tau}$ determines the extremity of the generated shock values. The Board chooses $\boldsymbol{\tau}$ based on the severity of the primary risk factor shock, relative to historical data. Specifically, let $\boldsymbol{\tilde{r}^P_{i,1}}$ denote the primary shock for factor $\boldsymbol{i}$ at a one-month horizon that is chosen in the stress-scenario,[26] and let $\boldsymbol{r^P_{i,1}}$ represent the one-month return on this primary risk factor. To a first approximation, $\boldsymbol{\tau}$ is chosen as the probability based on historical experience that the return for factor $\boldsymbol{i}$ is smaller than the primary factor shock:

Equation C2 – Targeted Quantile in Quantile Regression Model

$$\boldsymbol{\tau = \mathrm{Pr}\left(r^P_{i,1} \le \tilde{r}^P_{i,1}\right)}$$

In the expression, **Pr** denotes the empirical probability based on the history of time-series data for the return for primary factor $\boldsymbol{i}$. For example, if in the historical data the return for

[24] A conditional distribution of a random variable ***X*** given another random variable ***Y*** describes the distribution of ***X*** given that a particular value of ***Y*** has been realized.

[25] The parameters are estimated using the algorithm in Koenker and D'Orey (1987).

[26] Since this is a fixed value, it does not have a subscript ***t***.

primary factor $\boldsymbol{i}$ is less than the primary shock 85 percent (15 percent) of the time, then $\boldsymbol{\tau}$ would to a first approximation be 85 percent (15 percent), corresponding to an upper (lower) tail quantile. Upper or lower tails may correspond to stress depending on the variable being modeled; that is, lower tail shocks are associated with stress for equities, while upper tails are associated with stress for volatilities or credit spreads. As the time series data sample is limited for some secondary risk factors, it is difficult to accurately estimate the quantile regression for values of $\boldsymbol{\tau}$ close to either zero or one, because there are not sufficient observations to identify the possible extreme values of the distribution. The Board therefore introduces lower and upper bounds on $\boldsymbol{\tau}$ given by the 10th and 90th percentiles (i.e., $\mathbf{10\% \leq \boldsymbol{\tau} \leq 90\%}$) and rounds $\boldsymbol{\tau}$ estimates outside of this range to the closest bound. Imposing these bounds cuts off some possible very severe scenario choices for secondary factors. However, because the model already conditions on severe shock values for primary risk factors, the Board is not concerned that this limitation causes mild scenario shocks for secondary risk factors. Data limitations also restrict how granular $\boldsymbol{\tau}$ can be selected since the estimated coefficients will be statistically indistinguishable for small changes in $\boldsymbol{\tau}$. It is therefore sensible to limit the percentile to $\boldsymbol{\tau} \in \mathbf{\{10\%, 15\%, 20\%, . . . , 80\%, 85\%, 90\%\}}$. In sum, the framework uses Equation C2 to quantify $\boldsymbol{\tau}$ and then rounds it to the nearest five percentage-point interval, truncating it from below at 10 percent and from above at 90 percent.

b. Quantile Autoregressive Model

Implied volatility risk factors have strong autocorrelation properties in their time series data; that is, on average, data values observed in a given period are near the values observed in the previous period. For example, implied volatility levels of equity indices are often autocorrelated with a coefficient close to one. Such factors are therefore better characterized by quantile autoregression models (Koenker and Xiao, 2006) in levels with the underlying spot risk factor shock treated as an exogenous variable (i.e., a variable for which the variation is driven by factors that are outside of the model).[27] To state this model mathematically, consider a secondary risk factor that is an implied volatility of an option written on a security $\boldsymbol{i}$ with some maturity. Let $\boldsymbol{\sigma_{i,t}^{S}}$ denote the implied volatility at end of month $\boldsymbol{t}$; let $\mathbf{abs}(\boldsymbol{r_{i,t,1}})$ denote the absolute value of the one-month log return of security $\boldsymbol{i}$;[28] and let $\boldsymbol{Q_{\tau}(\sigma_{i,t}^{S}|\sigma_{i,t-1}^{S}, r_{i,t,1})}$ denote the $\boldsymbol{\tau}$'th percentile of the distribution of $\boldsymbol{\sigma_{i,t}^{S}}$ given $\boldsymbol{r_{i,t,1}}$ and $\boldsymbol{\sigma_{i,t-1}^{S}}$. The fitted value of the $\boldsymbol{\tau}$'th quantile of $\boldsymbol{\sigma_{i,t}^{S}}$ in the quantile autoregression is given by:

Equation C3 – Quantile Autoregressive Model

$$\boldsymbol{Q_{\tau}(\sigma_{i,t}^{S}|\sigma_{i,t-1}^{S}, r_{i,t,1}) = \alpha_{\tau} + \mathbf{abs}(r_{i,t,1})\beta_{\tau} + \sigma_{i,t-1}^{S}\rho_{\tau}}.$$

Shock values to $\boldsymbol{\sigma_{i,t}^{S}}$ are computed using the estimated coefficients from Equation C3 as follows. Let $\boldsymbol{\sigma_{i,0}^{S}}$ be equal to the implied volatility at the date at which the estimation sample ends. Starting from $\boldsymbol{t=1}$, compute recursively the risk factor level after $\boldsymbol{h}$ months, where $\boldsymbol{h}$ is the shock calibration horizon of the considered factor, using Equation C3 with $\boldsymbol{r_{i,t,1}}$.The risk factor shock is given as $\boldsymbol{\sigma_{i,h}^{S} - \sigma_{i,0}^{S}}$. To ensure high volatility levels in the stressed scenarios, the model in Equation C3 is implemented with the 90th percentile (i.e., $\boldsymbol{\tau = 90\%}$).[29]

[27] See Koenker, R., and Z. Xiao (2006): Quantile Autoregression, Journal of the American Statistical Association, 101(475), 980–990.

[28] The absolute value is used to model the idea that volatility increases with large return shocks in either direction.

[29] For example, consider a volatility risk factor with $\boldsymbol{\sigma_{i,0}^{S} = 500}$ volatility points. A one-month shock given parameters $\boldsymbol{\alpha_{0.90} = 100}$, $\boldsymbol{\beta_{0.90} = 0.05}$, and $\boldsymbol{\rho_{0.90} = 0.90}$, and a corresponding return shock equal to $\boldsymbol{r_{i,t,1} = -100}$ is computed as $\boldsymbol{\sigma_{i,h}^{S} - \sigma_{i,0}^{S} = 100 + 0.05 \cdot 100 + 0.9 \cdot 500 - 500 = 55}$ volatility points.

c. Ordinary Regressions

The stress scenario shock for some secondary risk factors is set equal to its expected value conditional on the primary risk factor shock(s) in the scenario, denoted $\boldsymbol{E(r^S_{i,t,h}|r^P_{i,t,h})}$. These models are preferred in cases in which tail realizations of the secondary and primary risk factors do not tend to coincide simultaneously. The relationship between the secondary factor and the primary factor shocks is given by:

Equation C4 – Ordinary Regression Model

$$\boldsymbol{E(r^S_{i,t,h}|r^P_{i,t,h}) = h\alpha + r^P_{i,t,h}\beta,}$$

where the parameters in the equation are estimated by ordinary least squares linear regression. $\boldsymbol{\alpha}$ is the estimated constant, and $\boldsymbol{\beta}$ is the estimated coefficient on the primary factor shock.[30]

For public equity index returns, the Board follows Ang et al. (2006) and adds an asymmetric component to the model to capture the idea that stock returns are more sensitive to downside risk than upside risk and, as a result, bear a downside risk premium.[31] The downside risk regression takes the form:

Equation C5 – Downside Risk Regression Model

$$\boldsymbol{E(r^S_{i,t,h}|r^P_{i,t,h}) = h\alpha + r^P_{i,t,h}\beta + 1_{r^P_{i,t,h}<0}r^P_{i,t,h}\gamma,}$$

where $\boldsymbol{1_{r^P_{i,t,h}<0}}$ is a function that takes value one if $\boldsymbol{r^P_{i,t,h} < 0}$ and zero otherwise. Finally, the ordinary regression counterpart to the autoregressive model in Equation C3 is given as:

Equation C6 – Ordinary Autoregressive Model

[30] The "ordinary least squares" method estimates parameter values $(\boldsymbol{\alpha, \beta})$ such that the sum of squared residuals, defined by $\sum_{t=1}^{T}\left(\boldsymbol{r^S_{i,t,h} - h\alpha - r^P_{i,t,h}\beta}\right)^2$ given the model in Equation C4, is minimized given data on $\boldsymbol{r^S_{i,t,h}}$ and $\boldsymbol{r^P_{i,t,h}}$ for **t=1,2,…,T.**

[31] See Ang, A., Chen, J., and Xing, X (2006) Downside risk. Review of Financial Studies, 19(4):1191–1239.

$$E(p^S_{i,t}|p^S_{i,t-1}, r^P_{t,h}) = \alpha + r^P_{i,t,h}\beta + p^S_{i,t-1}\rho.$$

In this framework, secondary risk factor shocks can be generated following the same procedure as described for Equation C3.

The full overview of regression links between secondary and primary risk factors by asset class is given below.

2. *Model Applications*

Shocks to secondary risk factors are computed using the models described above in Section C.ii.1, given the primary risk factor shocks as inputs. For each asset class, Table C1 shows how secondary risk factors are linked to the primary risk factors and which regression model is primarily used to generate shock values for that secondary risk factor asset class.

Table C1 – Overview of primary modeling choices for secondary risk factors by asset class

Secondary risk factor asset class	**Primary: r^P_t**	**Spot model**	**Implied volatility model**
Agencies	Baa-Aaa credit spread	Quantile regression, Equation C1.	N/A
Commodities:			
Metals	Global Price Index of Metal	Quantile regression, Equation C1.	Quantile autoregression, Equation C3.
Energy	Global Price Index of Energy	Quantile regression, Equation C1.	Quantile autoregression, Equation C3.
Foreign exchange	U.S. dollar-to-Euro exchange rate	Quantile regression, Equation C1.	Quantile autoregression, Equation C3.
Interest rates:			
10-year government bond	U.S. 10-year Treasury bond yield	Quantile regression, Equation C1.	Quantile autoregression, Equation C3.
3-month government bond	U.S. 10-year-minus-3-month Treasury term spread	Ordinary autoregression, Equation C6.	Quantile autoregression, Equation C3.
10-year swap	10-year government bond yield	Quantile regression, Equation C1.	Quantile autoregression, Equation C3.
3-month swap	10-year-minus-3-month government bond term spread	Ordinary autoregression, Equation C6.	Quantile autoregression, Equation C3.

Public equity	S&P 500 index	Downside risk regression, Equation C5.	Quantile autoregression, Equation C3.
Sovereign credit	Baa-Aaa credit spread	Quantile regression, Equation C1.	N/A
Corporate credit	Baa-Aaa credit spread	Quantile regression, Equation C1.	N/A
Municipal credit	Baa-Aaa credit spread	Quantile regression, Equation C1.	N/A
Securitized products	Baa-Aaa credit spread	Quantile regression, Equation C1.	N/A

3. Assumptions and Limitations

The quantile regression model in Equation C1 uses percentile parameters $\boldsymbol{\tau}$ derived from the severity of the primary risk factor shocks. For example, if the S&P 500 index return is in the 10^{th} percentile of the historical return distribution, the public equity index returns for secondary risk factor markets will be estimated as the model-implied 10^{th} percentile conditional on the S&P 500 index return taking values in the 10^{th} percentile. This assumption is justified if the secondary and associated primary risk factors realize severe shock values in the same periods. The Board has confirmed that this condition is satisfied in the data across many of the sets of primary and secondary risk factors.

All models are univariate; that is, they describe the dynamics of secondary risk factor shocks independently from other secondary risk factor shocks. This approach yields simple models with easy implementation but may abstract from important correlations across risk factor shocks. The Board has implemented multivariate models as alternative modeling approaches; see Section C.iv. The results from this exercise indicate that results do not change considerably when using multivariate models in place of univariate ones.

The list of potential models for linking primary to secondary risk factor shocks does not include a model specification for describing term structures of shock values. The set of secondary risk factors typically includes short-term and long-term government bond yields as well as swap rates. The Board ensures proper relationships between these rates of different maturities by first modeling the long-term rate and then modeling the spread between long- and short-term rates.

The use of quantile regression models requires an assumption on the targeted percentile parameter, $\boldsymbol{\tau}$. The Board chooses this parameter based on the severity of the primary risk factor shocks relative to historical data. However, short time-series data puts limitations on the upper and lower bounds of $\boldsymbol{\tau}$, which may cut off some very severe scenario choices for secondary risk factors. Moreover, data limitations restrict how granularly $\boldsymbol{\tau}$ can be chosen.

iii. Modeling the Relationship Between Secondary and All Remaining Risk Factors

This section explains the second part of the modeling approach which describes the link between, on the one hand, the set of primary and secondary risk factors and, on the other hand, the remaining set of risk factors in the global market shock template. This step involves the modeling of shocks to a large number of risk factors with widely different characteristics. Multiple modeling choices are, therefore, necessary to generate the remaining risk factor shocks.

The Board employs copula models, regression models, the Nelson-Siegel model, a suite of volatility models based on the Nelson-Siegel model,[32] and a set of simpler rules—such as mappings, multipliers, and averaging—as described below in Section C.iii.1. Copula models characterize the co-movement of a large set of risk factors by modeling their dependence

[32] The idea of applying the Nelson-Siegel model in the context of implied volatility term structures is developed in Guo, B., Han, Q., and Zhao, B. (2014): The Nelson–Siegel Model of the Term Structure of Option Implied Volatility and Volatility Components, Journal of Futures Markets, 34(8), 788-806.

structure separately from their time-series dynamics. These models are used as widely as possible when modeling the marginal distributions of different risk factors in their respective asset classes. However, in some applications, the joint modeling of a large set of risk factors may not be practical or beneficial. For example, securitized products backed by different types of loans are modeled separately. In some cases where copula models are not employed, ordinary regressions are applied. In other cases, such as capturing the structural relationship between implied volatility surfaces and factors that have a term structure, modeling using a copula is complex, which is inconsistent with the stress testing principle of simplicity. In these cases, Nelson-Sigel models are used to approximately capture term structure relationships. These models have parametric functional forms that describe term structures with just a few calibrated parameters and latent factors (for example, level, slope, and curvature factors). Finally, for risk factors where data is limited or risk factors that are immaterial, the Board uses simpler methods, such as multipliers and mappings.

1. *Model Descriptions and Rationale*

This section describes the models used to estimate the relationship between primary and secondary risk factors and all remaining risk factors. Ordinary regressions are described in Section C.ii.1.c.

a. Copula Model

Copula modeling is a statistical tool to generate multivariate distributions and to investigate the dependence structure between random variables. Specifically, a copula is a function that links univariate marginal distributions of random variables to their joint, multivariate distribution.[33] For a primer on copula modeling, see Fan and Patton (2014).[34]

Copula modeling offers several advantages over the quantile regression framework used for modeling secondary risk factors, which are particularly important for the purpose of mapping sets of secondary risk factor shocks into even larger sets of remaining risk factor shocks.[35] First, it does not require that all marginal distributions be the same. This feature is beneficial, for example, because it allows for different Generalized Autoregressive Conditional Heteroskedasticity (GARCH) specifications[36] across different risk factors based on the empirical data—where one risk factor might be best modeled with a GARCH(1,1) specification, another might require an exponential GARCH (EGARCH) model as defined by Nelson (1991) or even a different type of model altogether.[37] In contrast, traditional multivariate models often assume that all variables follow the same type of distribution. Second, copula models allow the separation of the modeling of risk factors' co-dependency structure, describing how different risk factors co-move at a given point in time, from the modeling of risk factors' marginal distributions. The time-series dynamics of each risk factor and the co-movement of all risk factors at a given point in time can therefore be modeled in two separate steps, using separate types of models. This feature offers additional flexibility in terms of choosing any joint model

[33] A multivariate distribution is a probability function describing the joint behavior of two or more random variables simultaneously. A univariate marginal distribution refers to the probability distribution of a single variable in a multivariate probability distribution, while disregarding the other variables.

[34] See Fan, Y., and A. J. Patton. (2014): Copulas in Econometrics, Annual Review of Economics, 6: 179–200.

[35] In contrast, the benefits of the copula model are less important for mapping one or a few primary risk factor shocks into a small set of secondary risk factor shocks, because it is a problem with much fewer variables.

[36] GARCH specifications refer to models of conditional volatility where the conditional volatility in period t depends on the conditional volatility in period $(t-1)$ and the innovation of the time series model in period $(t-1)$. A GARCH(1,1) model is defined below in Equation C9.

[37] See Nelson, D. B. (1991): Conditional Heteroskedasticity in Asset Returns: A New Approach, Econometrica, 59(2), 347-370.

that most closely resembles the tail-dependencies in the data, regardless of the choice of the marginal distributions. These advantages are important for modeling the relationship that the primary and secondary risk factors have with the remaining risk factors.[38] Copula models are also used by the European Central Bank to simulate adverse financial shocks for their stress test design (Rancoita and Ferreiro, 2019).[39]

The Board uses t-copulas, as defined below, to model risk-factor dependence because t-copulas better capture the clustering of extreme tail scenarios than copula models based on the multivariate Gaussian distribution.

To obtain the marginal distributions for each individual risk factor, the Board uses GARCH models. The GARCH framework is a widely used technique to account for conditional heteroskedasticity—i.e., time-varying conditional volatility, which is often exhibited in financial time series data (Bollerslev, 1986).[40] Using GARCH models is common practice in financial applications of the copula approach. For example, Patton (2006) uses a GARCH-t (1,1) specification when modeling the marginal distribution of the exchange rates.[41] Similarly, Bartram et al. (2007) suggest a GJR-GARCH-t(1,1) specification, as defined by Glosten et al. (1993), for European stock market returns.[42] The Board also follows this common practice and

[38] In contrast to the first modeling step, which involves modeling the relationship between primary and secondary risk factors in a low-dimensional setting, modeling the relationship between the remaining factors with the primary and secondary factors is a high-dimensional problem involving many risk factors.

[39] See Rancoita, E., and J. O. Ferreiro (2019): Technical note on the Financial Shock Simulator (FSS). European Systemic Risk Board. https://www.esrb.europa.eu/mppa/stress/shared/pdf/esrb.stress_test190402_technical_note_EIOPA_insurance~dcd7f1ed08.en.pdf.

[40] See Bollerslev, T. (1986): Generalized Autoregressive Conditional Heteroskedasticity, Journal of Econometrics, 31(3), 307–327.

[41] See Patton, A.J. (2006): Modelling Asymmetric Exchange Rate Dependence, International Economic Review, 47(2), 527–556.

[42] See Bartram, S.M., Taylor S.J., and Y.-H. Wang (2007): The Euro and European Financial Market Dependence, Journal of Banking and Finance 31(5), 1461–1481.; Glosten, L. R., R. Jagannathan, and D. E. Runkle (1993): On the

uses GARCH-t conditional volatility specifications as widely as possible when modeling the marginal distributions of different risk factors in their respective asset classes.

The following describes the copula framework mathematically. Let the modeled risk factor series $\boldsymbol{r_{i,t}}$ have the following specification:

Equation C7 – Copula Marginal Distribution Model

$$\boldsymbol{r_{i,t} = \mu_i + \rho_i r_{i,t-1} + \varepsilon_{i,t}},$$

where $\boldsymbol{\varepsilon_{i,t} = \sigma_{i,t} z_{i,t}}$. The innovation $\boldsymbol{z_t}$ is distributed according to a student's t-distribution with $\boldsymbol{v_i}$ degrees of freedom, standardized such that the distribution has mean zero and variance one—i.e., $\boldsymbol{z_{i,t} \sim t(0, 1, \nu_i)}$. The conditional variance $\boldsymbol{\sigma^2_{i,t}}$ is assumed to be measurable at time *(t-1)* and is described by one of the following models:

Equation C8 – Constant Variance Model

$$\boldsymbol{\sigma^2_{i,t} = \sigma^2_i}.$$

Equation C9 – GARCH-t Model

$$\boldsymbol{\sigma^2_{i,t} = \omega_i + \phi_i \sigma^2_{i,t-1} + \psi_i \varepsilon^2_{i,t-1}}.$$

Equation C10 – EGARCH-t Model

$$\boldsymbol{\log \sigma^2_{i,t} = \omega_i + \phi_i \log \sigma^2_{i,t-1} + \theta_i z_{i,t-1} + \psi_i \left(\left| z_{i,t-1} \right| - E \left| z_{i,t-1} \right| \right)}.$$

The parameters of these models are estimated for each risk factor using maximum likelihood methods.

Relation between the Expected Value and the Volatility of the Nominal Excess Return on Stocks, The Journal of Finance: 48(5), 1779–1801.Guo, B., Han, Q., and Zhao, B. (2014): The Nelson–Siegel Model of the Term Structure of Option Implied Volatility and Volatility Components, Journal of Futures Markets, 34(8), 788-806.

The dependence among $\boldsymbol{N}$ risk factors within a given asset class is modeled by assuming that their innovations z_i are linked through a t-copula. More specifically, for each $\boldsymbol{z_{i,t}}$, the framework defines a uniformly distributed random variable:

Equation C11 – Copula Uniform Variable Model

$$\boldsymbol{u_{i,t} = F_{v_i}\left(\sqrt{\frac{v_i}{v_i-2}}\, z_{i,t}\right)},$$

where from Equation C7, it follows that $\boldsymbol{z_{i,t} = \frac{r_{i,t}-\mu_i-\rho_i r_{i,t-1}}{\sigma_{i,t}}}$. In this representation, the joint cumulative distribution function of $\boldsymbol{u_i}$ is given by the copula

Equation C12 – Copula Model

$$\boldsymbol{C(u_1, u_2, \cdots, u_N, \nu, \Sigma) = F_{N,\nu,\Sigma}\,(F_\nu^{-1}(u_1), F_\nu^{-1}(u_2), \cdots, F_\nu^{-1}(u_N))},$$

where $\boldsymbol{F_\nu}$ is a univariate cumulative t-distribution with degree of freedom $\boldsymbol{\nu}$, and $\boldsymbol{F_{N,\nu,\Sigma}}$ is an N-dimensional multivariate cumulative t- distribution with degree of freedom $\boldsymbol{\nu}$ and correlation matrix $\boldsymbol{\Sigma}$ (see Fan and Patton (2014) for a detailed representation of the t-copulas).[43]

One of the benefits of working with a multivariate t-distribution is that its conditional distributional properties are well established. For example, partitioning a multivariate t-distribution $\boldsymbol{X}$ into two parts $\boldsymbol{X_1}$ and $\boldsymbol{X_2}$, Ding (2016) shows that the conditional distribution of $\boldsymbol{X_2}$ given $\boldsymbol{X_1}$ is also a multivariate t-distribution.[44] This property is useful for scenario design, because it allows the framework to describe the distribution of remaining risk factors in an asset class (e.g., public equity market returns) conditional on its secondary risk factors using a multivariate t-distribution.

[43] See Fan, Y., and A. J. Patton. (2014): Copulas in Econometrics, Annual Review of Economics, 6: 179–200.

[44] See Ding, P. (2016): On the Conditional Distribution of the Multivariate t Distribution, The American Statistician, 70 (3): 293–295.

Specifically, of the N risk factor shocks modeled by t-copula, assume that $\boldsymbol{N_S}$ shocks are labeled as secondary risk factor shocks, and $\boldsymbol{N_R}$ shocks are labeled as remaining risk factors shocks—i.e., $\boldsymbol{N_S + N_R = N}$. Following the research of Ding (2016), the distribution of $\boldsymbol{X_t^R = F_\nu^{-1}(u_t^R) = (F_\nu^{-1}(u_{1,t}^R), F_\nu^{-1}(u_{2,t}^R), \cdots, F_\nu^{-1}(u_{N_R,t}^R))}$ conditional on $\boldsymbol{X_t^S = F_\nu^{-1}(u_t^S) = (F_\nu^{-1}(u_{1,t}^S), F_\nu^{-1}(u_{2,t}^S), \dots, F_\nu^{-1}(u_{N_S,t}^S))}$ can be expressed as an $\boldsymbol{N_R}$-dimensional multivariate t-distribution with $\boldsymbol{\nu + N^S}$ degrees of freedom:

Equation C13 – Distribution of Conditional Multivariate Student's t Distribution

$$\boldsymbol{X_t^R | X_t^S \sim t_{N_R}\left(\mu_{R|S}, \frac{\nu + d_S}{\nu + N^S}\Sigma_{R,R|S}, \nu + N^S\right)}.$$

where $\boldsymbol{\mu_{R|S}}$ is the conditional mean and $\boldsymbol{\Sigma_{R,R|S}}$ is the conditional correlation matrix.[45] The computation of $\boldsymbol{\mu_{R|S}}$, $\boldsymbol{d_S}$ and $\boldsymbol{\Sigma_{R,R|S}}$ are explained in detail in Ding (2016) and requires the vector of $\boldsymbol{X_t^S = F_\nu^{-1}(u_t^S)}$ and the correlation matrix $\boldsymbol{\Sigma}$ as inputs. To construct $\boldsymbol{u_t^S}$, the framework assumes that the $\boldsymbol{h}$-week-horizon secondary risk factor shocks are evenly distributed across the $\boldsymbol{h}$ weeks.[46] To construct $\boldsymbol{\Sigma}$, the Board calculates the elements $\boldsymbol{\Sigma_{i,j}}$ (i.e., the correlation between the i^{th} and j^{th} risk factor shocks) using Kendall's tau formula with rank correlation $\boldsymbol{\tau_{i,j}}$:

Equation C14 – Kendall's Tau Formula

$$\boldsymbol{\Sigma_{i,j} = \sin\left(\frac{\pi}{2}\tau_{i,j}\right)}.$$

Kendall's tau formula is chosen because it is non-parametric and therefore does not assume linear or other pre-determined relationships between risk factors.[47] It is also often less

[45] See Ding (2016)

[46] For example, a one-month shock value of an equity market drop of 12 percent is decomposed into a path in which the market drops 3 percent per week for 4 weeks.

[47] This method for estimating t-copula models was suggested by Zeevi, A. and Mashal, R. (2002): Beyond Correlation: Extreme Co-Movements Between Financial Assets. SSRN Working Paper. See also Demarta, S. and McNeil, A. J. (2005): The t Copula and Related Copulas. International Statistical Review, 73(1), 111-129.

sensitive to outliers than other non-parametric methods, such as Spearman's rho (Kendall, 1970).[48] To calculate the degree-of-freedom parameter $\boldsymbol{\nu}$, the Board takes a conservative approach and sets it equal to the empirical tenth percentile of the estimated degree-of-freedom parameters from fitting bivariate t-copulas to each pair of factors whose dependency is modeled using the copula approach.[49] Setting the degrees of freedom in the copula to the tenth percentile among the bivariate copulas (instead of a more central value) ensures the copula builds in extreme tail dependence—i.e., the idea that extreme events are more likely to occur than would be expected under a Gaussian distribution. Failure to properly account for extreme dependence may result in inadequate stress in the expanded scenario shocks.

The Board uses the multivariate conditional distribution of $\boldsymbol{X_t^R}|\boldsymbol{X_t^S}$ to simulate the weekly path of the conditional mean $\boldsymbol{u_t^R}|\boldsymbol{u_t^S}$. These uniform variables are then transformed into standardized innovations $z_{i,t}$ using the t-distribution (Equation C11), which are then fed into the marginal model (Equation C7) to determine the weekly path of risk factor shocks $\boldsymbol{r_{i,t}}$. Summing these shocks over the assumed liquidity horizon yields the cumulative shocks of the modeled risk factors. The framework repeats the procedure up to 10,000 times and calculates the mean cumulative shocks of these modeled risk factors as their scenario shock values.[50] For asset

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48 As noted by Demarta and McNeil (2005), this method does not guarantee that $\boldsymbol{\Sigma}$ is positive definite. In practice, the Board does not encounter issues with non-positive definite correlation matrices. See also Kendall, Maurice G. *Rank Correlation Methods* Griffin, 1970.

49 For example, if the dependencies among 20 factors are modeled together in a single copula, then there are $\boldsymbol{20 \cdot 19/2 = 190}$ bivariate pairs of factors. A bivariate t-copula is fit to each pair, generating 190 bivariate degrees of freedom parameters. The 19th smallest degrees of freedom parameter is the tenth percentile among the estimates. This is then used as the degrees of freedom for the copula that models the dependence among the 20 factors.

50 The number of simulations is chosen as the smallest number (rounded to 1000, 5000, and 10,000, for example) such that randomization risk does not impact results considerably; that is, results should be roughly the same for every simulation without fixing the randomness or the seed.

classes in which shocks are defined as log returns (e.g., public equity), the last step involves the conversion of log returns into arithmetic returns for reporting purposes.

b. Nelson-Siegel Model

The Board uses the Nelson-Siegel functional form to generate term structures of risk factors shocks, including yield curves, futures curves, and implied volatility curves. Several academic studies highlight the model's success in describing such curves; see, e.g., Diebold and Li (2006) for yield curves and Guo, Han, and Zhao (2014) for implied volatility curves.[51] Term structures of risk factor shocks involve two dimensions: calendar time $\boldsymbol{t}$ and time of maturity $\boldsymbol{T}$ (or, equivalently, a tenor given by $\boldsymbol{T\text{-}t}$), which describes the maturity of a bond (for yield curves), a futures contract (for futures curves), or an option (for implied volatility curves).

The Nelson-Siegel model for a risk factor level at time $\boldsymbol{t}$ with maturity at time $\boldsymbol{T}$, $\boldsymbol{p_{i,t}(T)}$, is given by:

Equation C15 – Nelson-Siegel Model

$$\boldsymbol{p_{i,t}(T) = L_{i,t} + \left(\frac{1-e^{-\lambda_i(T-t)}}{\lambda_i(T-t)}\right) S_{i,t} + \left(\frac{1-e^{-\lambda_i(T-t)}}{\lambda_i(T-t)} - e^{-\lambda_i(T-t)}\right) C_{i,t}}.$$

This formulation is a three-factor model, defined as level ($\boldsymbol{L_{i,t}}$), slope ($\boldsymbol{S_{i,t}}$), and curvature ($\boldsymbol{C_{i,t}}$). The factor loadings are determined by a single parameter $\boldsymbol{\lambda_i}$, which controls the rate of decay of the term structure (i.e., how quickly interest rates decline as the maturity of debt instruments increases). Smaller values of $\boldsymbol{\lambda_i}$ generate a slower decaying curve.

The decay parameter $\boldsymbol{\lambda_i}$ is estimated by fitting the Nelson-Siegel functional form to the historical term structure data, e.g., yield curves and at-the-money implied volatility curves,

[51] See Diebold, F. X., and C. Li (2006): Forecasting the Term Structure of Government Bond Yields, Journal of Econometrics, 130, 337–364.; Guo, B., Han, Q., and Zhao, B. (2014): The Nelson–Siegel Model of the Term Structure of Option Implied Volatility and Volatility Components, Journal of Futures Markets, 34(8), 788-806.

where the level, slope, and curvature factors are obtained by regressing the data onto the factor loadings given a $\boldsymbol{\lambda_i}$ value. This procedure yields a time series for the level, slope, and curvature factors, which are subsequently used to calculate the risk factor shocks of the tenors in the published global market shock template.

Specifically, risk factor shocks are computed as the difference between pre- and post-shock risk factor levels. For the pre-shock risk factor levels, the Board uses the most recent data from the scenario as-of-date. For the post-shock risk factor levels, the Board uses various approaches for different types of risk factors. Two variations are outlined below. In both approaches, it is assumed for simplicity that the post-shock value of the curvature factor is constant and equals its historical time-series average. The only remaining factors in the Nelson-Siegel model are the level and slope factors; due to this assumption, post-shock values therefore only need to be estimated for the level and slope factors.

The Board uses the following two approaches. The first approach (Variation A) models the level and slope factors *jointly* using the copula model from which post-shock level and slope values can be generated. For some applications, e.g., public equity option-implied volatilities, the term structure is better described by estimating the level and slope factors in two subsequent stages (Variation B).

- *Variation A*: In the first variation, the level and slope factors of the Nelson-Siegel model are modeled using the copula. The modeled time series ($\boldsymbol{r_{i,t}}$) are the weekly logarithmic change in the level factor and the slope factor: $\boldsymbol{r_{i,t}} = (\Delta \boldsymbol{L_{i,t}},\ \ \Delta \boldsymbol{S_{i,t}})'$. The model uses the marginal model in Equation C7 with $\boldsymbol{\rho_i} = \boldsymbol{0}$ and the GARCH-t specification in Equation C9 to describe the time-series dynamics.

- *Variation B*: In the second variation, the model constructs the post-shock value of the level factor using a regression:

  Equation C16 – Regression Model for Level Factor Log Changes

  $$\ln \frac{\hat{L}_{i,t+1}}{\hat{L}_{i,t}} = \alpha_i + \beta_i \ln \frac{p_{i,t+1}}{p_{i,t}} + \varepsilon_{i,t+1}.$$

  This regression model gives the scenario-specific evolution and the post-shock value of the level factor using those of $p_{i,t}$. Next, the framework solves for the post-shock value of the slope factor by evaluating the post-shock values of the level factor, curvature factor, and $p_{i,t}$ in the Nelson-Siegel functional form.

  Given post-shock levels and slopes, the constant value for curvature, and the estimated decay factor, post-shock risk factor levels can be computed from Equation C15.

c. Models for Volatility Shocks

The Nelson-Siegel model in Equation C15 is used to generate shocks to term structures of implied volatility. While the Board aims at maintaining consistent model implementations where possible, the modeling of volatility shocks involves variation to accommodate differences in products and data limitations across asset classes. Specifically, three different approaches are implemented.

(a) Nelson-Siegel-GARCH volatility model: The first approach is applied to public equity implied volatilities, which have a considerable volatility feedback effect—i.e., a linkage between returns and implied volatilities. According to the volatility feedback effect (see for example, Campbell and Hentschel [1992] and Carr and Wu [2017]), with fixed future cash flow projections, an increase in a market's systematic business risk—captured by an unexpected increase in market volatility—increases the cost of capital and reduces the present value of the market portfolio.[52] This effect explains the negative contemporaneous relation between the volatility shock and the market return. It also implies that the larger the volatility shock, the more negative the market return.

The model assumes that risk factor dynamics are described by Equation C7 with a GARCH-t conditional volatility model as supplied in Equation C9. The model can be described in four steps:

1. For each primary and secondary market selected as the underlying index, calculate the call option price using the GARCH model-implied, one-week conditional volatility in a Monte-Carlo simulation.[53] Impute the implied volatility that produces the same call price using the Black-Scholes formula.[54] Repeat this computation in the pre- and post-shock periods. Volatility shocks are defined as the difference between implied volatility levels at the as-of date (pre-shock level) and ***h*** weeks after the as-of date (post-shock level). Post-shock volatility computation reflects the differences in spot shocks resulting from the copula model and generates the volatility feedback effect; that is, market indices that attain highly negative return shocks as copula model output also attain large volatility shocks.

2. Using the Nelson-Siegel model with variation B, generate a term structure of implied volatility shocks for different tenors starting at one week. Due to limitations in the availability of high-quality implied volatility data, this step is likely only feasible for a subset of risk factors; for

[52] See Campbell, J.Y., and L. Hentschel (1992): No News is Good News: An Asymmetric Model of Changing Volatility in Stock Returns, Journal of Financial Economics, 31(3), 281-318.; Carr P., and L. Wu (2017): Leverage Effect, Volatility Feedback, and Self-Exciting Market Disruptions, Journal of Financial and Quantitative Analysis, 52(5), 2119-2156.

[53] Monte-Carlo simulation prices the option by averaging random paths of the option's discounted payoff.

[54] The Black-Scholes formula for the call option is $\text{Call}^{\text{BS}}(S, r, K, T, \sigma^{\text{BS}}) = SN(d_1) - Ke^{-rT}N(d_2)$ where d_1 and d_2 are defined by: $d_1 = \frac{1}{\sigma^{\text{BS}}\sqrt{T}}\left(\log\left(\frac{S}{K}\right) + \left(r + \frac{(\sigma^{\text{BS}})^2}{2}\right)T\right)$ and $d_2 = d_1 - \sigma^{\text{BS}}\sqrt{T}$. In this formula, S is the stock price; K is the strike price; T is time-to-expiration; r is the risk-free rate; σ^{BS} is the volatility; and $N(.)$ is the standard normal cumulative distribution function.

example, the set of secondary and primary risk factors (i.e., the estimation subset). Denote this term structure of Nelson-Siegel model-implied volatilities by $\boldsymbol{\Delta\sigma_j^{\mathrm{NS}}(\tau)}$ for $\boldsymbol{\tau \geq 1}$ week and $\boldsymbol{j = 1, 2 \cdots, N}$, where **N** is the number of risk factors in the estimation subset.

3. Reconcile the one-week volatility shocks generated by step one and step two by computing adjustment factors – by comparing the one-week implied volatility shocks resulting from the Nelson-Siegel with the one from the GARCH models. The Board computed a simple adjustment factor to account for this difference, and it is computed for each of the risk factors in the estimation subset as:

Equation C17 – Nelson-Siegel-GARCH Model Adjustment Factor

$$\textbf{Adjustment Factor}_{j\in k_m} = \frac{1}{N_{k_m}}\sum_{j=1}^{N_{k_m}}\left(\Delta\sigma_j^{\mathrm{NS}}(1) - \Delta\sigma_j^{\mathrm{GARCH}}\right)$$

across developed and emerging markets ***k= {developed, emerging}***. $\boldsymbol{N_{k_m}}$ is the number of risk factors of type $\boldsymbol{k_m}$. $\boldsymbol{\Delta\sigma_j^{\mathrm{GARCH}}}$ is the volatility from step one and $\boldsymbol{\Delta\sigma_j^{\mathrm{NS}}(1)}$ is the volatility from step two. The one-week implied volatility shocks $\boldsymbol{\Delta\sigma_{i\in k_m}^{\mathrm{implied}}(1)}$ of market ***i*** belonging to market type ***k= {developed, emerging}*** is calculated as the sum of the volatility shock from the GARCH model and the adjustment factor:

Equation C18 – Formula for One-Week Implied Volatility Shocks

$$\Delta\sigma_{i\in k_m}^{\mathrm{implied}}(1) = \Delta\sigma_{i\in k_m}^{\mathrm{GARCH}} + \textbf{Adjustment Factor}_{j\in k_m},$$

for ***k={developed, emerging}***.

4. Generate a term structure of implied volatility for the risk factors outside the estimation subset. To accomplish this goal, the framework calculates an average of all available

implied volatility curves for the estimation subset in order to compute a scaling factor $\boldsymbol{\delta(\tau)}$ for volatility shocks at different tenors $\boldsymbol{\tau}$ relative to the one-week tenor:

Equation C19 – Term Structure Scaling Factor

$$\boldsymbol{\delta(\tau)} = \frac{1}{N}\sum_{i=1}^{N}\frac{\Delta\sigma_i^{\text{implied}}(\tau)}{\Delta\sigma_i^{\text{implied}}(1)}$$

where N is the number of risk factors in the estimation set. Using implied volatility shocks $\boldsymbol{\Delta\sigma_i^{\text{implied}}(1)}$ from Equation C18 and the scaling factor $\boldsymbol{\delta(\tau)}$ from Equation C19, the implied volatility shocks for the remaining market indices at all tenors are calculated as:

Equation C20 – Formula for Implied Volatility Shocks at Any Tenor

$$\boldsymbol{\Delta\sigma_i^{\text{implied}}(\tau) = \Delta\sigma_i^{\text{implied}}(1)\delta(\tau)}$$

<u>(b) The Nelson-Siegel-Copula Volatility Model:</u> The second approach includes spot and volatility shocks from an asset class in a copula model to generate the remaining volatility risk factor shocks in that asset class. This approach is applied for, e.g., foreign exchange-implied volatility curves. Spot shocks in this copula model are the subset of shocks for which spot and volatility risk factors are generated by the modeling framework of the asset class. Volatility shocks in the copula are weekly changes in one-month, at-the-money implied volatilities categorized as either secondary or remaining risk factors. The time-series dynamics of spot shocks are described consistently with the spot model, while the time-series model of the volatility shocks is given by Equation C7 with $\boldsymbol{\rho_i = 0}$ and the constant variance model supplied in Equation C8. Once the entire set of one-month volatility shocks are generated, the Nelson-Siegel model is used to populate the implied volatilities for the remaining maturities across the term structure. Data may not be available for the full set of risk factors. In such cases, this modeling approach is used for a subset of risk factors, and the remaining implied volatility

shocks are generated using the scaling approach described in step 4 of the Nelson-Siegel-GARCH model described in Section C.iii.1.c(a) above.

(c) The Five-Factor Nelson-Siegel Volatility Model: Finally, a third approach is needed to describe the term structures of swaption-implied volatilities, which involve an additional dimension. Specifically, modeling this volatility becomes a two-dimensional problem in which the volatility depends on both option maturity $\boldsymbol{T^O}$ and swap maturity $\boldsymbol{T^S}$. The Board models the swaption implied volatilities using a five-factor Nelson-Siegel functional form to describe both dimensions for secondary risk factors. This five-factor model is given by

Equation C21 – Five-Factor Nelson-Siegel Model

$$\begin{aligned}\boldsymbol{\sigma_{i,t}(T^S,T^O)} &= \boldsymbol{L_{i,t}} + \left(\frac{\boldsymbol{1-e^{-\lambda_i^S(T^S-t)}}}{\boldsymbol{\lambda_i^S(T^S-t)}}\right)\boldsymbol{S_{i,t}^s} \\ &+ \left(\frac{\boldsymbol{1-e^{-\lambda_i^S(T^S-t)}}}{\boldsymbol{\lambda_i^S(T^S-t)}} - \boldsymbol{e^{-\lambda_i^S(T^S-t)}}\right)\boldsymbol{C_{i,t}^s} + \left(\frac{\boldsymbol{1-e^{-\lambda_i^O(T^O-t)}}}{\boldsymbol{\lambda_i^O(T^O-t)}}\right)\boldsymbol{S_{i,t}^O} \\ &+ \left(\frac{\boldsymbol{1-e^{-\lambda_i^O(T^O-t)}}}{\boldsymbol{\lambda_i^O(T^O-t)}} - \boldsymbol{e^{-\lambda_i^O(T^O-t)}}\right)\boldsymbol{C_{i,t}^O}\end{aligned}$$

Two parameters $\boldsymbol{\lambda_i^O}$ and $\boldsymbol{\lambda_i^S}$ are obtained by minimizing the calibration error for each country's historical data over its sample period.

To calculate the change in implied volatility over the liquidity horizon $\boldsymbol{h}$, $\Delta\boldsymbol{\sigma_{i,h}(T^S,T^O)}$, for all $\boldsymbol{(T^S,T^O)}$ pairs, the model needs level, slope, and curvature factor shocks based on Equation C21. For simplicity, the Board assumes that the option curvature shock is equal to the swap curvature shock—i.e., $\Delta\boldsymbol{C_{i,h}^O} = \Delta\boldsymbol{C_{i,h}^S}$, which in turn reduces the number of factor shocks to four: $\Delta\boldsymbol{L_{i,h}}$, $\Delta\boldsymbol{S_{i,h}^O}$, $\Delta\boldsymbol{S_{i,h}^S}$, and $\Delta\boldsymbol{C_{i,h}}$. To generate these shocks, the framework uses four implied volatility shocks for the secondary risk factors, which are chosen from the U.S. dollar, Japanese yen, pound sterling, and Euro rates curves. Once these shocks are quantified, the framework uses different pairs of $(\boldsymbol{T^S}, \boldsymbol{T^O})$ and the parameters $\boldsymbol{\lambda_i^O}$ and $\boldsymbol{\lambda_i^S}$ in a first-differenced version of Equation C21 that reveals $\boldsymbol{\Delta\sigma_{i,h}}(\boldsymbol{T^S}, \boldsymbol{T^O})$ as a function of $\Delta\boldsymbol{L_{i,h}}, \Delta\boldsymbol{S_{i,h}^O}, \Delta\boldsymbol{S_{i,h,}^S}\Delta\boldsymbol{C_{i,h}}$. Volatility shocks $\boldsymbol{\Delta\sigma_{i,h}}(\boldsymbol{T^S}, \boldsymbol{T^O})$ for all swap and option maturities $(\boldsymbol{T^S}, \boldsymbol{T^O})$ for the U.S. dollar, Japanese yen, pound sterling, and Euro rates are computed from this specification. A multiplier method is used to construct rates volatility curve shocks for the remaining countries.

d. Multipliers, Mappings, Averaging, and the Percentile Method

Some risk factors are difficult to analyze within the scope of the above models due to either the nature of the data or lack of data. For such cases, the Board uses simpler methods, such as multipliers, mappings, averaging, and the percentile method.

Multipliers can be interpreted as regression models as described in Section C.iii.1.b, where the parameters are calibrated using alternative data sources determined from expert judgement. This approach is used when a statistical relationship between two variables is expected, but a lack of high-quality data prohibits proper estimation of this relationship.

For some risk factors, reasonable multipliers that can be properly justified may not exist, and better results can be obtained by other approaches. In such cases, a risk factor may be mapped to another risk factor—e.g., by setting the shock equal to another risk factor shock or by averaging a set of other risk factor shocks. These methods are subject to justification and scrutiny by subject-matter experts and leadership, following the same guidelines and processes for judgment-based adjustments to scenarios as described in Section B.ii.2.

Finally, the Board applies the percentile method used for generating values of primary risk factor shocks to set the shock values of some risk factors directly (see Section B.i.). This method may be used when there are data limitations and no mappings, multipliers, or averaging schemes deemed reasonable.

2. *Model Applications*

Shocks to the remaining risk factors are modeled separately for each asset class because risk factor shocks within asset classes are more similar and more correlated than risk factors across asset classes. The consistency and correlation of shocks across asset classes is captured by the correlations among shocks to primary and secondary risk factors. Table C2 provides an overview of the modeling approaches applied to spot and volatility shocks within each asset class. For each asset class, the model input is time-series data of risk factor returns, prices, or implied volatilities along with the shock values generated for the secondary risk factors. The output is the risk factor shocks outlined in Table B1. For corporate and sovereign credit shocks and shocks to the volatility of commodity and rates products, the Board publishes both absolute and relative shock values. The absolute shocks show either the change in the credit spread level (corporate and sovereign credit) or the implied volatility level (commodity and rates volatility). For these cases, the models produce absolute shocks, and the relative shocks are subsequently calculated as the absolute shocks relative to the corresponding levels on the as-of date. The following subsections provide additional details on model implementations for most asset classes.

Table C2 – Overview of modeling approaches for remaining risk factors by asset class

Risk factor asset class	Spot/futures curve shocks	Option-implied volatility shocks
Agencies	Percentile method.	N/A
Commodities	Nelson-Siegel model, Equation C15 with variations A and B. Weekly data for spot prices, as well as futures curve level and slope factors, are modeled in a GARCH-copula framework. Univariate log returns for spot/level factor follow an EGARCH model described by Equation C7 with no autoregressive term ($\boldsymbol{\rho_i = 0}$,) and C10. The slope factor is log-transformed, and the resulting series is modeled with equations C7 with an autoregressive term included and C10 as above. Dependence is modeled with the copula in Equation C12. Multipliers.	Copula model of one-month implied volatility shocks given Equation C7 with $\boldsymbol{\rho_i = 0}$, and C89. Categorical mapping.
Foreign exchange	Copula model of weekly log returns. Univariate returns follow a GARCH model described by Equation C7 with $\boldsymbol{\rho_i = 0}$ and C9. Dependence is modeled with the copula in Equation C12. Regional mapping.	Nelson-Siegel-copula volatility model. Regional mapping.
Public equity	Copula model of weekly log returns. Univariate returns follow a GARCH model described by Equation C7 with $\boldsymbol{\rho_i = 0}$[55] and C9. Dependence is modeled with the copula in Equation C12. Regional mapping.	Nelson-Siegel-GARCH volatility model.
Public equity dividends	Percentile method.	N/A
Interest rates	Nelson-Siegel model, Equation C15 with variation A, where level and slope factors are modeled in a copula model.	Five-factor Nelson-Siegel volatility model. Multipliers.

[55] Setting the autoregressive parameter to zero (i.e., $\boldsymbol{\rho_i = 0}$) reflects that log prices behave as a random walk; i.e., without forecastable patterns. Although subject to debate, this is a well-known assumption in the literature; see Campbell, J.Y., A.W. Lo, and A.C. MacKinlay (1997): "The Econometrics of Financial Markets," Princeton University Press.

Risk factor asset class	Spot/futures curve shocks	Option-implied volatility shocks
	Level and slope factors follow univariate EGARCH models described by Equation C7 and C10. Dependence is modeled with the copula in Equation C12. Percentile method.	
Sovereign credit	Copula model of weekly spread changes. Univariate spread changes follow the model described by Equation C7 with $\boldsymbol{\rho_i = 0}$ and C8. Dependence is modeled with the copula in Equation C12.	N/A
Corporate credit	Copula model of weekly changes in spreads between adjacent ratings. Univariate spread changes follow a GARCH model described by Equation C7 with $\boldsymbol{\rho_i = 0}$ and C9. Dependence is modeled with the copula in Equation C12. The model is implemented separately for bonds and single-name CDS. Multipliers.	N/A
Municipal credit	Copula model of weekly spread changes. Univariate spread changes follow the model described by Equation C7 with $\boldsymbol{\rho_i = 0}$ and C8. Dependence is modeled with the copula in Equation C12. Multipliers.	N/A
Other fair value assets	Mapping.	N/A
Securitized products	Ordinary regression models. Multipliers and mappings.	N/A

a. Commodities

Commodity spot shocks and the Nelson-Siegel level and slope factors of the futures curve, defined by Equation C15, are modeled jointly using the copula model due to the advantages of this approach, as discussed in Section C.iii.1.a. This method incorporates the variations A and B of the Nelson-Siegel model (see Section C.iii.1.b). Spot and future rates are modeled jointly to capture the dependence between these risk factor shocks. Future curves are described by the Nelson-Siegel model for parsimony and to impose a smooth term structure of futures rate shocks.

For commodities for which data is limited, the Board uses a multiplier method, where multipliers are computed based on the modeled risk factors. Such multipliers are constructed, for example, by the ratio of shocks at each maturity to the one-month shock.

The implied volatility of secondary risk factors is modeled using the copula model. Due to limited data availability, the implied volatility of the remaining risk factors is modeled using category averages, similar to the regional-average approach applied in foreign exchange.

b. Foreign Exchange

The majority of foreign exchange spot shocks are generated using the copula model due to the advantages of this approach discussed in Section C.iii.1.a. Implied volatility shocks are mainly generated using the Nelson-Siegel-copula model, described in Section C.iii.1.c, to capture the term structure of implied volatilities. Due to limited availability of foreign exchange option data, the volatility model is estimated for secondary risk factor implied volatilities only, whereas the remaining implied volatility shocks are obtained using scaling.

Due to data limitations, some shocks to exchange rate returns and their implied volatilities are generated using mapping and averaging procedures. Specifically, the Board groups currencies by regions that match risk characteristics or geography. Then, for each region and data-limited series, the model generates shocks to dollar-exchange rate returns and implied volatilities by mapping from the respective shock to a single U.S. Dollar exchange rate return or implied volatility from the copula model that represents that region, or by mapping to an average of exchange rate return shocks or implied volatility shocks obtained from the copula model. For exchange rates that do not involve the U.S. Dollar, exchange rate return shocks are computed based on shocks to underlying U.S. Dollar exchange rate returns.[56] In addition, implied volatility shocks are computed by averaging representative regional U.S. Dollar exchange-rate implied volatility shocks across regions. Finally, shocks for pegged currencies are determined by the shocks for the currencies to which they are pegged.

c. Interest Rates

Informed by academic research, such as Diebold and Li (2006), the Board uses the Nelson-Siegel model to describe the term structures of government bond yields and swap rates.[57] To preserve a relationship between government bond yields and swap rates within countries, swap rate shocks are generated by applying the Nelson-Siegel model to swap spreads (i.e., the difference between a swap rate and the corresponding government bond yield).

Swaption-implied volatility shocks for the secondary risk factors are modeled using the five-factor Nelson-Siegel model in Equation C21, because these shocks have both a dimension capturing swaption maturity and a dimension capturing the maturity of the underlying swap. Only implied volatilities for the secondary risk factors are modeled this way due to lack of swaption data for other risk factors. A multiplier method is used to construct rates volatility curve shocks for the remaining countries. These shocks depend intuitively on the relationship between these countries' government yield shocks and the secondary risk factors' government

[56] We can infer non-dollar exchange rates from dollar-exchange rates because the exchange rate between any two currencies is by a no-arbitrage relationship equal to the ratio of each currency's exchange rate against a third currency.

[57] See Diebold, F. X., and C. Li (2006): Forecasting the Term Structure of Government Bond Yields, Journal of Econometrics, 130, 337–364.

yield shocks. Other rates shocks (i.e., inflation and cross-currency versus U.S. dollar basis) are determined by the percentile method, as described in Section C.iii.1.d.

d. Public Equity

The public equity model generates index shocks using the copula model due to the advantages of this approach discussed in Section C.iii.1.a and regional mappings, where the copula cannot be implemented due to lack of data. Mapped risk factors are assigned by mapping the shock to a regional index that matches its risk characteristics or geography.

Implied volatility shocks are generated using the Nelson-Siegel-GARCH model described in Section C.iii.1.c, because equity returns have a considerable volatility feedback effect; see, e.g., Campbell, J.Y., and L. Hentschel (1992).[58] The model is estimated only for primary and secondary risk factors, because they have ample options data with various times-to-maturity and reflect the characteristics of major developed and emerging equity markets. The Nelson-Siegel model decay parameter for each market, $\boldsymbol{\lambda_i}$, is estimated using crisis-period[59] data only, as a full-sample estimation results in volatility curves with a slow decay for some markets, with the implication that the long-tenor volatility shocks for these markets are too severe relative to other markets.

e. Sovereign Credit

The sovereign credit model produces sovereign five-year credit default swap spread shocks to selected countries, e.g., Australia, Canada, and France, using the copula model due to

[58] See Campbell, J.Y., and L. Hentschel (1992): No News is Good News: An Asymmetric Model of Changing Volatility in Stock Returns, Journal of Financial Economics, 31(3), 281-318.

[59] For this purpose, crisis periods are defined as times where the S&P 500 index return (the public equity primary risk factor) experiences values in the bottom tenth percentile of its historical distribution.

the advantages of this approach discussed in Section C.iii.1.a. Due to lack of data, some countries' CDS spread shocks are mapped to other countries based on regional similarities.

f. Corporate Credit

The corporate credit model uses the copula model to describe shocks to advanced-economy bonds and single-name CDS across different ratings due to the advantages of this approach discussed in Section C.iii.1.a. Unlike the public equity and the foreign exchange models, spread shocks for credit ratings have a particular cross-sectional pattern; that is, shocks are monotonically increased as credit quality deteriorates (from AAA-rated securities to CCC securities). To preserve this desirable pattern in modeled results of both securities, the approach focuses on modeling the shock to the BBB-rated spread and the shocks to changes in spreads between adjacent ratings (spacings), e.g., AAA-AA, AA-A, and A-BBB.

Due to lack of data, the remaining corporate credit shocks are obtained using various multipliers on the model-generated shocks. For example, the emerging market bond shocks are calculated by weighing the advanced economy bond shocks by a multiplier given by the Corporate Emerging Markets Bond Index Credit Investment Grade Strip spread (a secondary risk factor) divided by the sum of advanced economy bond shocks across ratings AAA, AA, A, and BBB.

g. Municipal Credit

In municipal credit, changes in spreads on municipal AAA- and A-rated bond indices are modeled using the copula model due to the advantages of this approach discussed in Section C.iii.1.a. Changes in spreads bond, CDS, and other municipal credit products across different credit ratings are generated using multipliers due to data limitations.

h. Other Fair Value Assets

Other Fair Value Assets (OFVA) cover illiquid fair value securities that cannot be grouped into another asset class. Some examples for OFVA are public welfare investments covering housing credit, tax credits, and energy investments. Due to government guarantees, these investments are subject to low loss rates. Shocks to housing credit and tax credit are set at -4.9 percent and shocks to energy investments are set at -13.9 percent. For other OFVA, the Board sets a simple mapping rule in which equity shocks are equal to the S&P 500 return, and debt shocks are equal to the B-rated high yield leverage loan index shock from corporate credit.

i. Securitized Products

Securitized products include non-agency commercial and residential mortgage-backed securities, asset backed securities, and other products including corporate collateralized debt obligations, corporate collateralized loan obligations, and warehouse loans. All shocks are specified as market value haircuts, expressed in percentage terms, and applied over a liquidity horizon of three months. Market value haircuts are generated for representative portfolios constructed by sampling securities across three dimensions: product types (e.g., residential mortgage backed securities), vintages (e.g., ≤ three years), and ratings (e.g., AAA). The purpose of the representative portfolios is to provide a reflection of how the prices of current outstanding tranches of a given security type would respond to shocks to spreads. For a portfolio indexed by $\boldsymbol{r}$, the market value haircut is defined as:

Equation C22 – Definition of Market Value Haircut

$$\textbf{Market value haircut}_{\boldsymbol{r}} = \frac{\boldsymbol{P}_{\boldsymbol{r}}^{\mathbf{P}}(\textbf{post-shock}) - \boldsymbol{P}_{\boldsymbol{r}}^{\mathbf{P}}(\textbf{pre-shock})}{\boldsymbol{P}_{\boldsymbol{r}}^{\mathbf{P}}(\textbf{pre-shock})}$$

where $\boldsymbol{P_r^{\mathrm{P}}}(\textbf{pre-shock})$ denotes the portfolio value before the shock and $\boldsymbol{P_r^{\mathrm{P}}}(\textbf{post-shock})$ is the portfolio value after an appropriate spread-widening shock is applied.

The Board constructs these representative portfolios from individual securities by sampling a vendor-provided universe of asset-backed securities using a set of general rules (for example, the security status is active; pool factor[60] is greater than 10 percent; maturity is greater than or equal to one year; coupon data is valid, i.e., non-zero and non-N/A) and a set of product-specific rules (for example, including fixed-rate coupons for securities backed by auto loans). A representative portfolio typically contains at least ten securities.[61] In cases where an initial set of rules returns a portfolio with fewer than ten constituents, the portfolio definition is expanded to include one or more neighboring vintages.[62] Portfolio weights are given by the current amount outstanding on each underlying bond normalized by the total outstanding amount such that weights sum to one for each portfolio. Outstanding amounts are measured on the pre-shock date, as defined below. Specifically, for a portfolio $\boldsymbol{r}$ constituted by $\boldsymbol{N_r}$ securities with outstanding amounts[63] given by $\boldsymbol{B_i}(\textbf{pre-shock})$ for securities indexed by $\boldsymbol{i = 1, 2, \cdots, N_R}$, the portfolio weights are:

Equation C23 – Representative Portfolio Weights

$$\boldsymbol{w_i} = \frac{\boldsymbol{B_i}(\textbf{pre-shock})}{\sum_{\boldsymbol{i=1}}^{\boldsymbol{N_R}} \boldsymbol{B_i}(\textbf{pre-shock})}.$$

[60] The percentage of the original principal that remains outstanding.

[61] Portfolios are typically comprised of between ten and 30 securities.

[62] If vintage expansion does not yield a sufficiently large number of tranches, shock values are identified by mappings and multipliers, as described in Section C.iii.1.d. The portfolio definition is not expanded across rating grades because a portfolio covering several rating grades would fail to assign risk-sensitive shocks to lower-rated securities.

[63] Given for each bond by the number of outstanding bonds multiplied by its face value.

Portfolio values are computed as the weighted average of the prices of the constituent securities. Specifically, for a portfolio indexed by $\boldsymbol{r}$ constituted by $\boldsymbol{N_r}$ securities, the value is given as:

Equation C24 – Representative Portfolio Value

$$\boldsymbol{P_r^P}(\mathbf{t}) = \sum_{\boldsymbol{i=1}}^{\boldsymbol{N_r}} \boldsymbol{w_i P_i^{\mathrm{S}}(t)},$$

where $\boldsymbol{P_i^{\mathrm{S}}(t)}$ for $\boldsymbol{t}$ in {**pre-shock, post-shock**} defines the pre-shock and post-shock prices of individual securities. The remainder of this section describes how the Board obtains these prices to compute the market-value haircuts. This method is fundamentally different from other classes because of the specialized nature of securitized debt. We implement this methodology to ensure that pricing of asset-backed securities and the modeling of subsequent market value shocks properly accounts for the idiosyncratic waterfall structure of the securitized debt when computing discounted cashflows. The method involves product prices and spreads at the portfolio level (across product types, ratings, and vintages), for product indices (across product types and ratings), and for individual securities. To distinguish these levels, prices and spreads are denoted with superscripts as follows:

- $\boldsymbol{P_r^{\mathrm{P}}(t)}$ and $\boldsymbol{S_r^{\mathrm{P}}(t)}$ are prices and spreads for representative portfolios, indexed by $\boldsymbol{r}$.
- $\boldsymbol{P_{j,R}^{\mathrm{I}}(t)}$ and $\boldsymbol{S_{j,R}^{\mathrm{I}}(t)}$ are prices and spreads for a product index with rating $\boldsymbol{R}$, indexed by $\boldsymbol{j}$.
- $\boldsymbol{P_i^{\mathrm{S}}(t)}$ and $\boldsymbol{S_i^{\mathrm{S}}(t)}$ are prices and spreads for individual securities, indexed by $\boldsymbol{i}$.

The pre-shock price of a security $\boldsymbol{i}$, $\boldsymbol{P_i^{\mathrm{S}}}$(**pre-shock**), is defined as the security's price on a pre-determined, pre-shock date. Given this pre-shock date, prices are obtained from a third-party vendor. For liquid securities, the vendor price is typically close to commonly used pricing information available to market participants such as the Trade Reporting and Compliance Engine

(TRACE) prices. For illiquid securities, the vendor relies on model-based valuations that consider the last traded price, prices of similar traded securities, and broker input.

The post-shock price of a security $\boldsymbol{i}$, $\boldsymbol{P_i^{\mathrm{S}}}(\textbf{post-shock})$, is computed using a vendor cash flow engine, which computes, at time $\boldsymbol{t}$, the prices of an individual security $\boldsymbol{P_i^{\mathrm{S}}(t)}$ given a spread $\boldsymbol{S_i^{\mathrm{S}}(t)}$ over the spot rate curve that reconciles the value of the bond with its market price:

Equation C25 – Pricing (Cash Flow) Engine

$$\boldsymbol{P_i^{\mathrm{S}}(t) = f_{i,t}\left(S_i^{\mathrm{S}}(t)\right)}.$$

The function $\boldsymbol{f_{i,t}}(\cdot)$ represents the pricing (cash flow) engine. It has security-level and time subscripts ($\boldsymbol{i, t}$) because it uses security-level characteristics defined and provided by the vendor at a given point in time. The Board applies characteristics of the collateral at the pre-shock date for simplicity−that is, $\boldsymbol{f_{i,t}}(\cdot) = \boldsymbol{f_{i,\textbf{pre-shock}}}(\cdot)$−though these may change between the pre-shock and post-shock dates. The characteristics of the collateral capture credit risk (e.g., default rates, delinquencies, collateral quality as measured by loan ratings and corporate leverage, and subordination levels), interest rate risk (e.g., reference curves such as Treasury swap, and the Secured Overnight Financing Rate, prepayment speed, and duration), market risk (e.g., bid-ask spreads), and other risks (e.g., capital waterfall and issuer/servicer risk). The Board uses the vendor's default set of characteristics, which vary by product type. For example, for commercial mortgage-backed securities, the default prepayment values are set to zero as prepayments are typically restricted or penalized. In contrast, prepayments are a more important characteristic for residential mortgage-backed securities, and their default values are set by the underlying collateral.

The post-shock price of security $\boldsymbol{i}$ is computed by applying the cash flow engine on a stressed spread, given by the sum of a pre-shock spread $\boldsymbol{S}_{\boldsymbol{i}}^{\mathrm{S}}(\textbf{pre-shock})$ and a stressed term $\boldsymbol{\Delta S}_{\boldsymbol{i}}^{\mathrm{S}}$:

Equation C26 – Post-Shock Price of Individual Securities

$$\boldsymbol{P}_{\boldsymbol{i}}^{\mathrm{S}}(\textbf{post-shock}) = \boldsymbol{f}_{\boldsymbol{i},\textbf{pre shock}}\left(\boldsymbol{S}_{\boldsymbol{i}}^{\mathrm{S}}(\textbf{pre-shock}) + \boldsymbol{\Delta S}_{\boldsymbol{i}}^{\mathrm{S}}\right).$$

For operational efficiency, the Board applies the pricing engine over a non-uniform grid (i.e., 100, 200, 500, 1000, and so on basis points). This step allows the Board to pre-run these computations before the spread shocks are finalized. Specifically, prices are computed for ($\boldsymbol{S}_{\boldsymbol{i}}^{\mathrm{S}}(\textbf{pre-shock}) + \boldsymbol{\Delta}\mathbf{s}$) for $\boldsymbol{\Delta}\mathbf{s} = \{\mathbf{100, 200, 500, 1000\ldots}\}$ basis points.

The pre-shock spread $\boldsymbol{S}_{\boldsymbol{i}}^{\mathrm{S}}(\textbf{pre-shock})$ is computed from the pricing engine given the pre-shock price $\boldsymbol{P}_{\boldsymbol{i}}^{\mathrm{S}}(\textbf{pre-shock})$:

Equation C27 – Pre-Shock Spread of Individual Securities

$$\boldsymbol{S}_{\boldsymbol{i}}^{\mathrm{S}}(\textbf{pre-shock}) = \boldsymbol{f}_{\boldsymbol{i},\textbf{pre shock}}^{-\mathbf{1}}\left(\boldsymbol{P}_{\boldsymbol{i}}^{\mathrm{S}}(\textbf{pre-shock})\right),$$

where $\boldsymbol{f}_{\boldsymbol{i},\textbf{pre-shock}}^{-\mathbf{1}}(\cdot)$ refers to solving for the spread given a price in the pricing engine.

The Board models the stress in the spread of security $\boldsymbol{i}$, $\boldsymbol{\Delta S}_{\boldsymbol{i}}^{\mathrm{S}}$, at the level of representative portfolios. For a security $\boldsymbol{i}$ represented by portfolio $\boldsymbol{r}$, the spread shock is $\boldsymbol{\Delta S}_{\boldsymbol{i}}^{\mathrm{S}} = \boldsymbol{\Delta S}_{\boldsymbol{r}}^{\mathbf{P}}$. Stressed spreads of representative portfolios are modeled given spread shocks to AAA-rated securitized product *indices*: $\boldsymbol{\Delta S}_{\boldsymbol{j},\mathbf{AAA}}^{\mathbf{I}}$ where $\boldsymbol{j}$ denotes the index (e.g., the fixed-rate AAA-rated residential mortgage-backed security index). These AAA-rated index spread shocks are secondary risk factors,[64] chosen because they represent the bulk of issuance and trade in the most liquid markets.

[64] Shocks to these secondary risk factors are modeled using the Baa-Aaa spread as explained in Section C.ii.

The model has two steps. The first step uses ordinary regressions to project log-differences of spreads of indices for all other ratings $\boldsymbol{R}$ (e.g., AA, A, and BBB), $\boldsymbol{\Delta \ln \; S^{\mathrm{I}}_{j,R}(t)}$, onto the log-differences of spreads for AAA-rated indices for each product index $\boldsymbol{j}$, $\boldsymbol{\Delta \ln \; S^{\mathrm{I}}_{j,\mathrm{AAA}}(t)}$. Here, the argument $\boldsymbol{t}$ indicates the time dimension of the log-differenced spread data. Log-differences are defined over the three-month liquidity horizon.[65] This modeling approach is chosen as it is a simple method that captures how systemic risks impact securitized product tranche spreads.[66] The regression specifications are as follows:

Equation C28 – Regression Model for Index Spreads

$$\boldsymbol{\Delta \ln S^{\mathrm{I}}_{j,R}(t) = \alpha_j + \beta_j \Delta \ln \; S^{\mathrm{I}}_{j,\mathrm{AAA}}(t) + \varepsilon_j(t),}$$

where $\boldsymbol{\varepsilon_j(t)}$ is normally distributed with mean zero and variance $\boldsymbol{\sigma^2_{\varepsilon,j}}$. The coefficients are estimated using ordinary least squares, giving estimates $\boldsymbol{\widehat{\alpha_j}}$, $\boldsymbol{\widehat{\beta_j}}$, and $\boldsymbol{\widehat{\sigma}^2_{\varepsilon,j}}$ such that the change in the logarithm of the expected spread shock for index $\boldsymbol{j}$ with rating $\boldsymbol{R}$ given the spread shock to the corresponding AAA-rated index can be computed as:

Equation C29 – Post-Shock Index Spread

$$\boldsymbol{\Delta \ln S^{\mathrm{I}}_{j,R}(\text{post-shock}) = \widehat{\alpha}_j + \widehat{\beta}_j \ln\left(\frac{S^{\mathrm{I}}_{j,\mathrm{AAA}}(\text{pre-shock}) + \Delta S^{\mathrm{I}}_{j,\mathrm{AAA}}}{S^{\mathrm{I}}_{j,\mathrm{AAA}}(\text{pre-shock})}\right) + \frac{\widehat{\sigma}^2_{\varepsilon,j}}{2}},$$

[65] Specifically, letting $\boldsymbol{S^{\mathrm{I}}_{j,R}(t)}$ denote monthly spread data for index $\boldsymbol{j}$ with rating $\boldsymbol{R}$ at month $\boldsymbol{t}$, the log difference is given as: $\boldsymbol{\Delta \ln S^{\mathrm{I}}_{j,R}(t) \; = \ln S^{\mathrm{I}}_{j,R}(t) \; - \ln S^{\mathrm{I}}_{j,R}(t - 3\mathrm{M})}$ **.**

[66] Idiosyncratic risks are likely to average out because the model is considering indices rather than individual securities. Moreover, the literature (for example, Coval et al. [2009] and Hamerle et al. [2009]) has emphasized that the pooling and tranching process involved in securitization causes structured product tranches to load more heavily on systemic risks than non-securitized debt securities with comparable probabilities of default. See Coval, J. D., Jurek, J. W., & Stafford, E. (2009). Economic Catastrophe Bonds. American Economic Review, 99(3), 628-66.; Hamerle, A., Liebig, T., & Schropp, H. J. (2009). Systematic Risk of CDOs and CDO Arbitrage. Deutsche Bundesbank Discussion Paper Series 2. No. 2009, 13.

where $\boldsymbol{S}^{\mathbf{I}}_{\boldsymbol{j},\mathbf{AAA}}(\mathbf{pre\text{-}shock})$ is the spread of the AAA-rated index $\boldsymbol{j}$ on the pre-shock date. The variance term $\widehat{\boldsymbol{\sigma}}^{\mathbf{2}}_{\boldsymbol{\varepsilon},\boldsymbol{j}}/\mathbf{2}$ is added to account for the convexity term when taking the expectation over the logarithmic function in Equation C31.

The second step of the model calculates the spread shock, $\boldsymbol{\Delta S}^{\mathbf{P}}_{\boldsymbol{r}}$, under the assumption that the shock impacts the spread of representative portfolios by the same amount as its impact on the index. Specifically, for a representative portfolio ***r,*** for the product type described by index $\boldsymbol{j}$ with rating $\boldsymbol{R}$, the Board assumes that:

Equation C30 – Mapping Representative Portfolio Spread Shocks to Index Spread Shocks

$$\boldsymbol{\Delta} \ln \boldsymbol{S}^{\mathbf{P}}_{\boldsymbol{r}}(\boldsymbol{t}) = \boldsymbol{\Delta} \ln \boldsymbol{S}^{\mathbf{I}}_{\boldsymbol{j},\boldsymbol{R}}(\boldsymbol{t}).$$

This is a simplifying assumption that facilitates the computation of the post-shock spread of the individual securities, based on the indices for which data is available.[67] Given this assumption, the spread shock for representative portfolio $\boldsymbol{r}$ for the product type described by index $\boldsymbol{j}$ is given by:

Equation C31 – Representative Portfolio Spread Shock

$$\boldsymbol{\Delta S}^{\mathbf{P}}_{\boldsymbol{r}} = \boldsymbol{S}^{\mathbf{P}}_{\boldsymbol{r}}(\mathbf{pre\text{-}shock})\left[\exp\left(\boldsymbol{\Delta} \ln \boldsymbol{S}^{\mathbf{I}}_{\boldsymbol{j},\boldsymbol{R}}(\mathbf{post\text{-}shock})\right) - \mathbf{1}\right],$$

where $\boldsymbol{S}^{\mathbf{P}}_{\boldsymbol{r}}(\mathbf{pre\text{-}shock})$ is the pre-shock spread of the representative portfolio approximated using the pre-shock spreads for individual securities given by the pricing engine (see Equation C27) and portfolio weights:

Equation C32 – Pre-Shock Representative Portfolio Spread

$$\boldsymbol{S}^{\mathbf{P}}_{\boldsymbol{r}}(\mathbf{pre\text{-}shock}) = \frac{\mathbf{1}}{\boldsymbol{N}_{\boldsymbol{r}}} \sum_{\boldsymbol{i}=\mathbf{1}}^{\boldsymbol{N}_{\boldsymbol{r}}} \boldsymbol{w}_{\boldsymbol{i}} \boldsymbol{S}^{\mathbf{S}}_{\boldsymbol{i}}(\mathbf{pre\text{-}shock}).$$

[67] Due to lack of data, the assumptions cannot be justified numerically.

Using $\boldsymbol{\Delta S_r^{\mathrm{P}}}$ as proxies for $\boldsymbol{\Delta S_i^{\mathrm{S}}}$ in the pricing engine in Equation C26, the Board obtains the post-shock prices of individual securities, which are used in Equation C24 to compute post-shock values of the representative portfolios.[68] Finally, market value haircuts are computed using Equation C22.

This approach is not feasible for all products due to limited data availability. For products with lack of data (e.g., wholesale loans, ABS credit card and student loans, and portfolios with ratings lower than BBB), shocks are computed using multipliers and mappings as described in Section C.iii.1.d.

3. Assumptions and Limitations

Model-estimated correlations are assumed to reflect behavior under market stress. Models using the copula framework rely on estimating a correlation matrix of innovations. These correlations are based on long time series of data that capture a range of both stressed and non-stressed conditions. While numerous studies suggest that correlations between asset price movements are linked to crisis behavior (e.g., Junior and Franca, 2012), more recent samples might better reflect current market conditions.[69] The Board attempts to mitigate this risk by regularly assessing the current environment to make sure that the estimated models behave according to expectations. In the case that model estimation results do not adhere to expectations, the Board may apply overrides to model results to reflect the desired conditions.

[68] If $\boldsymbol{\Delta S_i^{\mathrm{S}}}$ falls within grid points, linear interpolation is used to compute the post-shock price. To illustrate, assume that $\boldsymbol{\Delta s_1 < \Delta S_i^{\mathrm{S}} < \Delta s_2}$, where $\boldsymbol{\Delta s_1}$ and $\boldsymbol{\Delta s_2}$ are grid points (i.e., $\boldsymbol{\Delta s_1, \Delta s_2 \in \{100, 200, 500, 1000, \cdots\}}$). Let $\boldsymbol{P_i^{\mathrm{S}}(\text{post-shock}) = f_{i,\text{pre-shock}}\left(S_i^{\mathrm{S}}(\text{pre-shock}) + \Delta s_1\right) + \frac{(\Delta S_i^{\mathrm{S}} - \Delta s_1)}{(\Delta s_2 - \Delta s_1)}\left[f_{i,\text{pre-shock}}\left(S_i^{\mathrm{S}}(\text{pre-shock}) + \Delta s_2\right) - f_{i,\text{pre shock}}\left(S_i^{\mathrm{S}}(\text{pre-shock}) + \Delta s_1\right)\right]}$. If $\boldsymbol{\varphi_i}$ exceeds the maximum grid point ($\Delta \boldsymbol{s}_{\mathbf{max}}$), the Board computes $\boldsymbol{P_i^{\mathrm{S}}(\text{post-shock}) = f_{i,\Delta S_i^{\mathrm{S}}}\left(S_i^{\text{pre-shock}} + \Delta s_{\max}\right)}$.

[69] See Junior, L.S. and I.D.P. France (2012): Correlation of Financial Markets in Times of Stress, Physica A: Statistical Mechanics and its Applications, 391(1-2), 187-208.

For circumstances that may represent structural shifts and therefore are likely to raise similar concerns in future stress test cycles, the Board may invest resources into developing models to account for such structural changes.

When simulating shock values in the copula model, the secondary risk factor shock is assumed to be distributed evenly across the $\boldsymbol{h}$-week horizon. This is a simplifying assumption. The Board has tested other distributional assumptions (e.g., assuming that the entire shock is realized over the first week of the horizon) but did not pursue them because they had a small impact on simulated shock values.

For products with credit ratings, the models assume, for simplicity, that securities do not undergo rating transitions, which is a reasonable assumption given the instantaneous nature of the global market shock component. However, if ratings changes do occur, abstracting from rating transitions when generating shocks may understate (overstate) risk if ratings are downgraded (upgraded). The Board prioritizes simple and tractable models and therefore accepts this limitation without attempts to mitigate the risk associated with it.

The pricing of asset-backed securities does not account for collateral performance and interest-rate risk along the term structure. The current approach may underestimate the market value haircuts that would occur under a combination of spread stresses and collateral performance degradation. The stressed collateral performance assumptions could be set as judgmental parameters and would aid in further increasing the severity of the market value haircuts. However, the magnitude of the haircuts, under the current approach, appear sufficiently severe. Interest-rate risk is not stressed at different tenor points due to the structure of the vendor data. This practice is inconsistent with the rates shocks produced by the global market shock.

The Board acknowledges this limitation and is planning to conduct future model development work to address this potential issue.

For some risk factors, data limitations hinder the application of econometric models, such as the copula model or regressions. In such cases, the Board applies simpler methods, such as mapping risk factors to other, similar, risk factors for which data exists. Other methods involve averaging across multiple risk factors or using multipliers to link risk factors. These approaches ignore the fact that relationships between risk factors may change over time and scenarios. The Board therefore revises these methods on a continuous basis.

iv. Alternative Modeling Choices

The dependence among risk factor shocks (from primary to secondary, or from secondary to all remaining) can be modeled using a myriad of different models. As part of an evaluation of models, the Board tested the performance of the models described above to that of other modeling choices. These tests evaluate the abilities of the models to generate shocks that match the severeness of past crises given the available data and conditions at the crisis' outset. For example, the Board considers the 2007-2009 financial crisis, the European sovereign debt crisis, and the COVID-19 pandemic.

The modeling choices can be varied in different dimensions. One dimension is to replace all models—both those used to translate primary risk factor shocks to secondary risk factor shocks and those used to subsequently translate secondary risk factor shocks into shocks for all other risk factors—with simpler models, such as ordinary regressions. The ordinary regression model, described in Section C.ii.1.c, is considered the simplest and most widely used statistical method. This alternative model is thus in line with the Board's stress test principle of simplicity. However, the Board generally finds that this alternative model would fail to generate sufficiently severe shocks to reflect market distress and heightened uncertainty; that is, this model would fail to satisfy the purpose of the global market shock as discussed in Section A, because ordinary regression models predict the expected shock value given historical shock values in both normal and stressed times. In contrast, models such as the copula model described in Section C.iii.1.a and the quantile regression model described in Section C.ii.1.a are designed to capture tail outcomes of the dependent variable.

Another alternative approach would be the use of more complex models. For example, the Board models secondary risk factors using univariate models (mainly, quantile regressions and quantile autoregressions), which characterize each risk factor independently from others. As an alternative, this univariate modeling approach could be replaced by multivariate models that describe all risk factors within each asset class jointly. This feature is more flexible and potentially captures broader risks than the application of univariate models, but it comes at the cost of having considerably more parameters that would need to be estimated. More parameters both increase computational cost and impair the accuracy associated with estimation.

The Board has tested the performance of two multivariate modeling frameworks for describing secondary risk factor shocks in order to better understand and weigh the effects of these tradeoffs. First, the Board considers the copula model, which has multiple advantages as described in Section C.iii.1.a and is used for modeling the shocks to the remaining risk factors. Despite the flexibility of the chosen copula model, it does not capture time-variation in correlations across risk factors. To test a model that captures this aspect, the Board considers the dynamic conditional correlation (DCC) model from Engle (2002) as a second multivariate approach.[70] The DCC model is designed to capture time-varying correlations between multiple time series. It decomposes the conditional covariance matrix $\boldsymbol{H_t}$, i.e., the variances and covariances of all risk factor shocks at time $\boldsymbol{t}$ given past realizations of risk factor shocks, as follows:

Equation C33 – Decomposition of Conditional Covariance Matrix

$$\boldsymbol{H_t = D_t R_t D_t},$$

where $\boldsymbol{D_t}$ is a diagonal matrix with standard deviations from univariate GARCH models on the diagonal, and $\boldsymbol{R_t}$ is a time-varying correlation matrix.[71] Since the covariance matrix is time-varying, this model captures that covariances in stressed times can be different from covariances in other times. To generate scenarios reflecting market stress, the Board applied the covariances sampled from periods coinciding with past crises.

In doing so, the Board found that using these multivariate methods to generate secondary risk factor shocks gives similar results as the chosen univariate models.[72] Therefore, given the increased complexity and computational cost associated with these alternative models, the Board has determined that the chosen models are more appropriate methods.

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[70] Engle, R. (2002): Dynamic Conditional Correlation: A Simple Class of Multivariate Generalized Autoregressive Conditional Heteroskedasticity Models. Journal of Business & Economic Statistics, 20(3), 339–350.

[71] More specifically, $\boldsymbol{D_t} = \begin{pmatrix} \boldsymbol{\sigma_{1,t}} & \cdots & \boldsymbol{0} \\ \vdots & \ddots & \vdots \\ \boldsymbol{0} & \cdots & \boldsymbol{\sigma_{N,t}} \end{pmatrix}$ with $\boldsymbol{\sigma_{i,t}}$ for i=1,…,N described by the GARCH-t model in Equation C9, and $\boldsymbol{R_t} = \begin{pmatrix} \boldsymbol{1} & \cdots & \boldsymbol{\sigma_{N,1,t}} \\ \vdots & \ddots & \vdots \\ \boldsymbol{\sigma_{1,N,t}} & \cdots & \boldsymbol{1} \end{pmatrix}$, where $\boldsymbol{\sigma_{i,j,t}}$ is a correlation parameter between risk factor i=1,…,N and risk factor j≠i.

[72] For example, the mean absolute error from fitting public equity returns during the Great Financial Crisis is only 4 percent lower when using the multivariate DCC model compared with the univariate downside risk regression models.

v. Examples

This section provides examples of estimated model parameters for certain key models—i.e., quantile regression, downside-risk regression, quantile autoregression, the copula model, and the Nelson-Siegel model. The examples are included to illustrate the different approaches numerically, rather than to report the full set of results comprehensively. As the global market shock scenario involves thousands of risk factors, a comprehensive report of all estimated model parameters is not feasible.

The examples focus on models for spread shocks to corporate bonds, returns on selected market indices, and the implied volatilities of selected equity index options. These risk factors are chosen because their modeling approaches cover the key models listed above, and they involve relatively few model parameters to report. Parameter estimates are reported for the global market shock component of the 2025 severely adverse scenario. For this scenario, all models are estimated on data available up to the estimation cut-off date of June 28, 2024. The Board chooses this cut-off date relative to the as-of date of the next stress test (October 11, 2024 for the DFAST 2025 exercise) such that the two dates are both sufficiently close to one another to allow the parameter estimates to reflect the recent changes in the market environment and sufficiently distant from one another to allow the Board to make an initial assessment of its scenario choices prior to the as-of date.

1. Quantile Regression

As discussed in Section C.ii.1.a, quantile regression is the primary model used by the Board to generate shock values to secondary risk factors given primary risk factor shocks. This section shows a numerical example for corporate credit bonds in developed markets. As shown in Table C1, the primary risk factor is the Moody's Baa-Aaa credit spread. This factor is used to generate shock values for two global corporate bond indices obtained from a third-party vendor: namely, a BBB-rated index and a BB-rated index.

The data are monthly (end-of-month) spread changes, which are calculated from index option-adjusted spreads collected at the daily frequency from December 1996 to June 2024. Missing data are forward filled.[73] Monthly spread changes are calculated by subtracting the previous month's end-of-month index spread from the current month's end-of-month index spread.

Given these data points, the parameters of the quantile regression in Equation C1 given the 90th percentile are estimated using the algorithm from Koenker and D'Orey (1987).[74] The 90th percentile is chosen to reflect the severity of the shock value for the Baa-Aaa spread change used for the global market shock component of the 2025 severely adverse scenario, following the procedure outlined in Section B.ii.1. Results are shown in Table C3. The estimated coefficients reveal a positive and statistically significant relationship between the Baa-Aaa spread changes and the changes in the corporate credit bond market indices.

[73] That is any missing observation is populated by the most recently available data point.

[74] See Koenker, R., and D'Orey, V. (1987): Algorithm AS 229: Computing Regression Quantiles, 36(3), 383-393.

Table C3 – Estimated Coefficients for the Quantile Regression Describing the Corporate Bond Spreads in the Set of Secondary Risk Factor Shocks.

The model is given by $\boldsymbol{Q_{0.90}}(\boldsymbol{r^S_{i,t,1}}|\boldsymbol{r^P_{i,t,1}}) = \boldsymbol{\alpha_{0.90}} + \boldsymbol{r_{i,t,1}\beta_{0.90}}$*, where* $\boldsymbol{Q_{0.90}}(\boldsymbol{r^S_{i,t,1}}|\boldsymbol{r^P_{i,t,1}})$ *is the 90th conditional percentile of the secondary risk factor shock,* $\boldsymbol{r^S_{i,t,1}}$*, given the primary risk factor shock,* $\boldsymbol{r^P_{i,t,1}}$*. The primary risk factor shock is the Moody's Baa-Aaa corporate credit spread. The quantile regression is estimated at the 90th percentile for the 2025 global market shock component of the Severely Adverse scenario because the three-month Baa-Aaa corporate credit spread is chosen at a severity corresponding in the top 10th percentile of the distribution of historical data. The data series are monthly option-adjusted spreads from December, 1996 to June, 2024. Standard errors are reported in parentheses. P-values (p) are indicated as follows: * p<0.10, ** p<0.05, *** p<0.01.*

	(i) BBB-rated index	(ii) BB-rated index
$\alpha_{0.90}$	0.163***	0.391***
	(0.015)	(0.022)
$\beta_{0.90}$	1.591***	2.914***
	(0.125)	(0.148)

2. Downside Risk Regression

The Board uses downside risk regressions to generate shocks to public equity secondary risk factors given the S&P 500 index return (see Table C1). Secondary risk factors include the DAX index, the FTSE 100 index, and the Nikkei 225 index to represent developed markets, and the MSCI Emerging Market (EM) Latin America Index to represent emerging markets.

The data are monthly (end-of-month) log-returns, which are calculated from index prices collected from a third-party vendor at the daily frequency May 1997 to June 2024. Missing data are forward filled. Monthly log-returns are computed by first-differencing the logarithm of end-of-month index prices.

Given these data points, the parameters of the downside risk regression in Equation C5 are estimated using ordinary least squares. The estimates are shown in Table C4. The estimation results emphasize the co-movements of market returns, particularly during market declines. The positive and statistically significant estimates of $\boldsymbol{\gamma}$ imply that the secondary market index returns co-move with the S&P 500 index return more in market declines than in market rallies. The parameter estimates are used in conjunction with a shock value for the S&P 500 index return to generate scenario shocks for the DAX, FTSE 100, Nikkei 225, and MSCI Emerging Market (EM) Latin America indices. All shocks are reported as arithmetic returns. These shocks are subsequently reviewed by the Board for plausibility and consistency with the scenario narrative.

Table C4 – Estimated Coefficients for the Downside Risk Regressions Describing Equity Index Returns in the Set of Secondary Risk Factor Shocks.

The model is given by $\boldsymbol{E}\left(\boldsymbol{r}^{\boldsymbol{S}}_{\boldsymbol{i,t,1}}\middle|\boldsymbol{r}^{\boldsymbol{P}}_{\boldsymbol{t,1}}\right) = \boldsymbol{\alpha} + \boldsymbol{r}^{\boldsymbol{P}}_{\boldsymbol{t,h}}\boldsymbol{\beta} + \boldsymbol{1}_{\boldsymbol{r}^{\boldsymbol{P}}_{\boldsymbol{t,h}}<\boldsymbol{0}}\boldsymbol{\gamma}$, *where* $\boldsymbol{r}^{\boldsymbol{P}}_{\boldsymbol{t,1}}$ *is the one-month S&P 500 index return. The data series are monthly log returns from May 1997 to June 2024. Standard errors are reported in parentheses. P-values (p) are indicated as follows: * p<0.10, ** p<0.05, *** p<0.01.*

	(i) DAX index	(ii) FTSE 100 index	(iii) Nikkei 225 index	(iv) MSCI EM Latin America index
α	0.004	0.001	0.007*	0.006
	(0.003)	(0.002)	(0.004)	(0.006)
β	0.947***	0.595***	0.494***	0.937***
	(0.092)	(0.062)	(0.104)	(0.154)
γ	0.251*	0.165*	0.507***	0.617**
	(0.145)	(0.098)	(0.165)	(0.244)

3. <u>Quantile Autoregression</u>

For each public equity market, the Board publishes not only the spot shocks but also implied volatility shocks with tenors ranging from one month to three years. These volatility shocks are the changes in the implied volatility level over the one-month liquidity horizon. To ensure the consistency between spot and volatility shocks within each market, the Board uses quantile autoregressions that receive spot shocks as input as specified by Equation C3.

The dependent variables of the quantile autoregressions are the implied volatilities of secondary markets with one-month tenor. For developed secondary market indices, the Board uses the price level and the at-the-money implied volatility level of the same index. For the emerging secondary market index, the Board maps the MSCI Latin America index to Bovespa (Brazil) market volatility because Bovespa constitutes more than 50 percent of the regional index. The Board collects these data series from a third-party vendor.

The quantile autoregressions are estimated using monthly volatility data, which are calculated as end-of-month volatility observations with missing data treated by forward-filling along with monthly log-returns. The data samples are May 1997 to June 2024 for the S&P 500 index; January 2002 to June 2024 for the DAX and FTSE 100 indices; May 2004 to June 2024 for the Nikkei 225 index; and March 2011 to June 2024 for the emerging market index.

Results are shown in Table C5. The results emphasize the persistence of implied volatility time series with the coefficients loading on past volatility ($\boldsymbol{\beta}_{\mathbf{0.90}}$) estimated close to one. The results also show the dependence of market volatility on the underlying index returns, as the coefficients $\boldsymbol{\rho}_{\mathbf{0.90}}$ are statistically significant for all indices. These coefficients are estimated with a negative sign, consistent with more extreme (i.e., more negative) returns being associated with higher implied volatility.

Table C5 – Estimated Coefficients for the Quantile Autoregression Describing Equity Index Option One-Month (1M) Implied Volatilities in the Set of Secondary Risk Factor Shocks

The model is given by $\boldsymbol{Q_{0.90}(\sigma^S_{i,t}|\sigma^S_{i,t-1}, r_{i,t,1}) = \alpha_{0.90} + r_{i,t,1}\beta_{0.90} + \sigma^S_{i,t-1}\rho_{0.90}}$, *where* $\boldsymbol{Q_{0.90}(\sigma^S_{i,t}|\sigma^S_{i,t-1}, r_{i,t,1})}$ *is the 90th conditional percentile of* $\boldsymbol{\sigma^S_{i,t}}$ *given* $\boldsymbol{\sigma^S_{i,t-1}}$ *and the one-month return of the index underlying the option,* $\boldsymbol{r_{i,t,1}}$*. The data series are one-month implied volatilities at the monthly frequency from May 1997 (S&P 500), January 2002 (DAX and FTSE 100), May 2004 (Nikkei 225), or March 2011 (CBOE Brazil ETF) to June 2024. Standard errors are reported in parentheses. P-values (p) are indicated as follows: * p<0.10, ** p<0.05, *** p<0.01.*

	(i) DAX 1M Implied Volatility	(ii) FTSE 100 1M Implied Volatility	(iii) Nikkei 225 1M Implied Volatility	(iv) CBOE Brazil ETF 1M Implied Volatility	(v) S&P 500 1M Implied Volatility
$\alpha_{0.90}$	0.035***	0.041***	0.060***	0.020	0.042***
	(0.007)	(0.002)	(0.017)	(0.027)	(0.006)
$\beta_{0.90}$	1.001***	0.953***	0.921***	1.138***	0.962***
	(0.064)	(0.042)	(0.164)	(0.135)	(0.053)
$\rho_{0.90}$	-0.682***	-0.840***	-0.531***	-0.698***	-0.769***
	(0.038)	(0.065)	(0.133)	(0.085)	(0.070)

4. Copula

For an example of estimation results for the copula model, consider the developed market corporate credit bond spreads for which Section C.v.1 illustrated the generation of secondary risk factor shocks using the quantile regression. The remaining risk factors within this group of securities are corporate bonds rated AAA, AA, A, B, and lower than B. Shock values are generated by estimating the copula model explained in Section C.iii.1.a on weekly changes in credit spacings between various ratings. Specifically, the copula model is estimated on the following spacings: AA-AAA, A-AA, BBB-A, BBB, BB-BBB, B-BB, and CCC-B.

The data are weekly spacing changes computed from option-adjusted spreads on corporate bond indices with ratings from AAA to CCC. These data series are sourced from a third-party vendor between January 1, 2005 and June 28, 2024 at the daily frequency. First, daily spacings are computed by taking the difference between index spreads with adjacent ratings. Then, the daily spacings are converted into weekly changes in spacings by differencing the spacings between Wednesdays.[75]

The copula model is estimated in two steps. First, the marginal model for each spacing, given by Equations C7 and C9, is estimated using maximum likelihood. These parameter estimates are shown in Table C6. The results show significant GARCH effects as the parameter loading onto past volatility in the GARCH Equation C9, $\boldsymbol{\phi_i}$, is statistically significant for all spacings. The estimation results also show the fat-tailed behavior of the data as the degree-of-freedom parameter, $\boldsymbol{\nu_i}$, is estimated in the range of about 2.5–4.[76] In the second step, the parameters of the dependence structure, the joint degree-of-freedom parameter and covariance matrix ($\boldsymbol{\nu}$ and $\boldsymbol{\Sigma}$ in Equation C12), are estimated. These results are shown in Table C7.

[75] If Wednesday data is missing, the difference is taken as of Tuesday. If Tuesday data is missing, the difference is taken as of Thursday. If Thursday is also missing, the corresponding observation is dropped from the sample.

[76] For $\nu_i \to \infty$, the Student-t distribution converges to a normal distribution, which, by definition, does not feature fat tails.

Table C6 – Estimated Coefficients for the GARCH-t Model Describing Corporate Bond Spacing Spreads.

The model is given by $\boldsymbol{r_{i,t} = \mu_i + \varepsilon_{i,t}}$, *where* $\boldsymbol{\varepsilon_{i,t} = \sigma_{i,t} z_{i,t}}$ *with the GARCH-t model* $\boldsymbol{\sigma^2_{i,t} = \omega_i + \phi_i \sigma^2_{i,t-1} + \psi_i \varepsilon^2_{i,t-1}}$ *and* $\boldsymbol{z_{i,t} \sim t(0, 1, \nu_i)}$. *The GARCH-t model is estimated with variance targeting, introduced by Engle and Mezrich (1996), where the term* $\boldsymbol{\omega_i}$ *is calculated as the unconditional sample variance multiplied by* $\boldsymbol{(1 - \psi_i - \phi_i)}$.[77] *The data series are weekly spreads from January 1, 2005, to June 28, 2024. Standard errors are reported in parentheses. P-values (p) are indicated as follows: * p<0.10, ** p<0.05, *** p<0.01.*

	(i) AA-AAA	(ii) A-AA	(iii) BBB-A	(iv) BBB	(v) BB-BBB	(vi) B-BB	(vii) CCC-B
μ_i	0.009	-0.053*	-0.224***	-0.464***	-0.679**	-0.629**	-0.914**
	(0.035)	(0.031)	(0.062)	(0.118)	(0.280)	(0.244)	(0.423)
ω_i	0.511	0.117	0.489	1.747	7.953	9.514	17.271
ψ_i	0.340***	0.201***	0.198***	0.294***	0.180***	0.213***	0.087***
	(0.001)	(0.001)	(0.024)	(0.005)	(0.030)	(0.016)	(0.000)
ϕ_i	0.647***	0.787***	0.779***	0.684***	0.788***	0.757***	0.903***
	(0.001)	(0.002)	(0.028)	(0.006)	(0.371)	(0.017)	(0.000)
ν_i	3.112***	3.583***	3.084**	4.178***	4.279***	3.487***	2.585***
	(0.191)	(0.304)	(1.271)	(0.333)	(0.418)	(0.326)	(0.062)

Table C7 – Estimated Correlation Matrix and Degree of Freedom Parameter for the Copula Model of Corporate Bond Spacing Spreads

The correlation matrix is estimated by using Kendall's tau formula in Equation C14. To determine the degrees-of-freedom parameter, bivariate copula models between all pairs of bond spacing spreads are first estimated. The degrees-of-freedom estimate is the tenth percentile of estimated degree-of-freedom parameters from these bivariate copulas. The data series are weekly spreads from January 1, 2005 to June 28, 2024. The Board's calibration of these parameters does not include measures for conducting inference, and standard errors are therefore not reported in the table.

	AA-AAA	A-AA	BBB-A	BBB	BB-BBB	B-BB	CCC-B
AA-AAA	1	-0.034	0.107	0.218	0.122	0.147	0.141
A-AA	-0.034	1	0.288	0.632	0.417	0.336	0.264
BBB-A	0.107	0.288	1	0.796	0.504	0.449	0.384
BBB	0.218	0.632	0.796	1	0.643	0.527	0.442
BB-BBB	0.122	0.417	0.504	0.643	1	0.524	0.422
B-BB	0.147	0.336	0.449	0.527	0.524	1	0.303
CCC-B	0.141	0.264	0.384	0.442	0.422	0.303	1

Degrees of freedom:
5.498

[77] See Engle, R., and Mezrich, J, (1996): GARCH for Groups. RISK, 9(8), 36-40

5. Nelson-Siegel Model

The Nelson-Siegel model is used to generate shocks along the term structure as explained in Section C.iii.1.b. For example, the Board uses the Nelson-Siegel model to generate shock values to implied volatilities of primary and secondary public equity indices for different tenors, given the one-month implied volatility shocks generated using quantile autoregression as explained in Section C.v.3. For this purpose, the Nelson-Siegel model is estimated using daily at-the-money implied volatilities[78] sourced at daily frequency from a third-party vendor. Missing data points are forward filled. The estimated decay parameter ($\boldsymbol{\lambda_i}$) and the means and standard deviations of the estimated level, slope, and curvature factors are shown in Table C8. The table also shows the coefficient from regressing the log-changes of the estimated level factors onto log-changes in one-month implied volatilities. All parameters are statistically significant.

[78] That is the implied volatility of options for which the current market price of the underlying asset is close to or equal to the option's strike price.

Table C8 – Estimation Results for the Nelson-Siegel Model Describing the Term Structures of Equity Index Option Implied Volatilities

The table reports results for indices belonging to the set secondary risk factors, which comprise the estimation subset. The model is given by $\boldsymbol{p_{i,t}(T) = L_{i,t} + \left(\frac{1-e^{-\lambda_i(T-t)}}{\lambda_i(T-t)}\right) S_{i,t} + \left(\frac{1-e^{-\lambda_i(T-t)}}{\lambda_i(T-t)} - e^{-\lambda_i(T-t)}\right) C_{i,t}}$, *where* $\boldsymbol{\lambda_i}$ *is the decay parameter and* $\{\boldsymbol{L_{i,t}, S_{i,t}, C_{i,t}}\}$ *are level, slope, and curvature factors. The decay parameters are estimated separately for each market by numerical minimization of the sum of squared errors using implied volatility data sourced from a third-party vendor. The public equity implied volatility model uses Variation B to generate post-shock values, which involves regressing log-changes in the estimated level factor* $\boldsymbol{\hat{L}_{i,t}}$ *onto log-changes in* $\boldsymbol{p_{i,t}(t+1M)}$: $\boldsymbol{log\frac{\hat{L}_{i,t+1}}{\hat{L}_{i,t}} = \beta_i log\frac{p_{i,t+1}(t+1M)}{p_{i,t}(t+1M)} + \varepsilon_{i,t+1}}$. *The data series are at the daily frequency from May 1, 1997 (S&P 500), January 1, 2002 (DAX, FTSE 100, and Nikkei 225), or March 1, 2011 (CBOE Brazil ETF) to June 28, 2024, with maturities between one month and 30 years. Robust standard errors, computed using numerical derivatives, are reported in parentheses. P-values (p) are indicated as follows: * p<0.10, ** p<0.05, *** p<0.01. The table also report the mean and standard deviations of the estimated level, slope, and curvature factors as of June 28, 2024 (*$\{\boldsymbol{\overline{L_i}, \overline{S_i}, \overline{C_i}}\}$ *and* $\{\mathbf{sd}(\boldsymbol{L_i}), \mathbf{sd}(\boldsymbol{S_i}), \boldsymbol{sd(C_i)}\}$*). These factor time series are estimated by regressing the implied volatility data onto the factor loadings given the estimated decay parameter, for each day in the sample*

	(i) DAX	(ii) FTSE 100	(iii) Nikkei 225	(iv) BOVESPA	(v) S&P 500
λ_i	3.577***	3.474***	5.647***	5.225***	1.471***
	(0.000)	(0.047)	(0.011)	(0.098)	(0.000)
β_i	0.078***	0.072***	0.060***	0.131***	0.083***
	(0.002)	(0.003)	(0.002)	(0.011)	(0.002)
$\overline{L_i}$	0.228	0.205	0.209	0.262	0.254
$\overline{S_i}$	-0.031	-0.043	-0.003	-0.023	-0.089
$\overline{C_i}$	-0.061	-0.087	-0.029	-0.045	-0.089
$\text{sd}(L_i)$	0.036	0.038	0.024	0.023	0.026
$\text{sd}(S_i)$	0.068	0.061	0.062	0.076	0.067
$\text{sd}(C_i)$	0.063	0.069	0.054	0.082	0.081

D. Scenario Narrative Generation Tool

As discussed in Section B.1, the Board uses supervisory experience and expertise, including forward-looking expert judgment and statistical analysis of historical data in the "scenario narrative" stage of the scenario design process. This section describes one of the tools used to generate scenario narratives based on statistical analysis.[79] This tool provides a set of

[79] Other tools are described in Section B.ii.1.

scenarios based on past firm vulnerabilities and historical co-movements of key risk factors across various asset classes during stressed times in history. The Board may use this approach to generate a starting point for specifying scenario narratives. This approach suggests scenario narratives based on past stressful events. These past stressed times are identified based on firm vulnerabilities from FR Y-14Q submission data following the approach outlined in Abdymomunov, Duan, Hansen, and Misirli (2024, Section 3.4).[80] The output of this analysis is a set of shock values to representative risk factors, which include but may not be limited to primary risk factors. This output constitutes the basis for the formulation of scenario narratives. The shock values may, however, be adjusted further to account for emerging risks not captured by historical experience, as described in Section B.ii.1.

i. Description and Rationale

The approach has five parts: (i) identification of material risk factors to reduce the computational burden of analysis of the impact on profits and losses (P&L); (ii) hypothetical scenario generation using the historical simulation of material risk factors; (iii) selection of tail-loss scenarios that can have large impacts across many historical positions and all firms included in the global market shock component of stress testing; (iv) identification of similarities between tail-loss scenarios using statistical techniques; and (v) evaluation of these tail-loss scenarios by the Board and development of scenario narratives. This section describes these stages in detail.

Step (i): The first step of the analysis involves identifying risk factors that are deemed material for the P&L impact analysis. These material risk factors are identified for each asset class separately. For operational feasibility, the P&L impact of each risk factor in an asset class

[80] See Abdymomunov, A., Z. Duan, A. L. Hansen, and E. U. Misirli (2024): Designing Market Shock Scenarios, Federal Reserve Bank of Richmond Working Paper Series, WP 24-17.

is estimated using firm-specific P&L sensitivities and standardized shock values, such as parallel shifts in yield curve of ±200 basis points or ±20 percent changes in spot exchange rates. While these standardized shocks are arbitrarily chosen, they are sufficiently severe to approximate the relative contributions of each individual risk factor to the total P&L of all firms. This method offers a first-order approximation of risk factors' relative P&L impact contributions under stress. The risk factors are ranked according to the size of their P&L impact and gradually included in the set of material risk factors until cumulative P&L reaches a materiality threshold that reflects the balance between risk coverage and efficiency.[81]

Step (ii): Next, shock values for the material risk factors are simulated jointly using historical simulation over a long sample that includes periods of major financial crises. Given a calibration horizon of $\boldsymbol{h}$ months, the historical simulation uses $\boldsymbol{h}$-month non-overlapping data or rolling-windows of $\boldsymbol{h}$-*month* changes using daily data. Simulated shock values that are unsuitable for current levels of the risk factors, in the sense that they result in unprecedented values, are filtered out. For instance, historical realizations of the U.S. Treasury bond yield shocks that make post-shock rate levels negative are excluded from the simulated shocks. The simulated material risk factor shocks that pass this soundness test form the distribution of generated scenarios.

Step (iii): The third step selects scenarios from the set of historically simulated scenarios that have tail-loss impacts (i.e. large impacts across many historical positions and across all firms included in the global market shock component of stress testing). P&L distributions are constructed for each asset class, firm, and period for which firm-specific P&L sensitivities are

[81] In an application to interest rates, Abdymomunov, Duan, Hansen, and Misirli (2024, Section 3.4) show that a materiality threshold of 70 percent identifies 12 interest rate curves. Increasing interest rate risk capture to 80 percent involves expanding the number of curves from 12 to 20; that is, the computational cost is large relative to the gain in the risk capture.

available. This granularity of distributions ensures that (i) the model investigates scenario variation in less material asset classes; (ii) firms with small dollar exposures are included in scenario selection; and (iii) the model investigates the impact of future changes in firms' trading book portfolios, assuming that historical P&L sensitivities capture such potential changes in portfolios. Tail-loss scenarios are defined as those that result in losses below the first percentile of each firm's P&L distributions pooled across time and firm observations. The resulting set of scenarios covers vulnerabilities that are idiosyncratic for each firm or systematic for all firms.

Step (iv): Due to the abundance of sensitivity data across firms and firm submissions, the set of tail-loss scenarios may be large, and some of the scenarios may be similar. Hence, further reduction in the number of tail scenarios may be obtained by grouping similar scenarios and identifying representative scenarios. K-means cluster analysis as proposed by MacQueen (1967) is applied for this purpose.[82] This statistical method partitions observations into clusters to minimize the within-cluster variances. Given these clusters, a representative scenario for each cluster is selected to ensure the tail losses scenarios are captured across firms.

Step (v): Finally, the representative scenarios are evaluated and used to form scenario narratives. The set of representative scenarios shows past firm vulnerabilities; hence, they can indicate potential risks and directions for material risk factors. In the rates, commodities, and foreign exchange asset classes, for example, representative scenarios offer different directional risks, such as interest rates going up versus down, commodity prices going up versus down, and U.S. dollar appreciation versus depreciation against major currencies. These scenarios become a

[82] See MacQueen, J. (1967): Some Methods for Classification and Analysis of Multivariate Observations, Proceedings of the 5th Berkeley Symposium on Mathematical Statistics and Probability, 1, 281–297.

reference point for evaluating emerging risks and developing scenario narratives that encompasses past firm vulnerabilities.

ii. Assumptions and Limitations

The approach assumes that the historical firm exposure data contains information on future risks. However, future distributions of exposures may identify different scenario narratives than suggested by the approach given past data. Similarly, the approach also assumes that historical simulation of risk factors represents potential future risks. The Board addresses these limitations by using the scenario design narrative approach in combination with expert judgment, as discussed in Section B.ii.1.

In step (ii), P&L impact is approximated using standardized shock values such as a parallel shift in the yield curve. This approximation ignores non-parallel shifts in the curve, such as a yield curve steepening. The standardized shock values simplify the evaluation of P&L impact. As the approximated P&L impact is only used to identify material risk factors, and not for generating final results, the Board considers this simplification reasonable.

In step (iv) of the approach, the set of tail-loss scenarios are reduced using cluster analysis. While cluster analysis offers a statistical approach to group similar scenarios, it may overlook certain risks that only show up in a single period or for a single firm. To mitigate this risk, the Board compares the P&L impact of the final set of scenarios with the P&L impact of all tail-loss scenarios.

E. Scenario Design Process Limitations and Alternatives

This section discusses limitations and alternatives related to the overall scenario design process at a broad level. Limitations and alternatives related to the specific modeling choices are addressed for each modeling component throughout Sections C and 0. The main limitations to the overall scenario design process include: (1) the instantaneous nature of the global market shock; (2) limitations related to the choice of horizons over which scenario shocks are calibrated; and (3) limitations imposed by the global market shock template. This section elaborates on these limitations and discusses potential alternatives. While the Board acknowledges that the approach to the global market shock scenario design involves limitations, the Board has considered its resources and overarching policies to arrive at a sensible, yet feasible, design process.

i. Instantaneous vs. Dynamic Global Market Shocks

The global market shock scenario assumes that firms cannot change their position over asset-specific liquidity horizons. This assumption is implemented as if shocks, which are calibrated over assumed shock liquidity horizons, occur instantaneously. This design choice is motivated by the simplicity of implementation for firms in measuring shock impacts, and by the comparability of market losses across firms by avoiding the need to make assumptions regarding exposure dynamics during shock horizons. However, this choice ignores potential changes in trading positions due to the expiration of derivative contracts or trading risk management strategies. For example, an instantaneous global market shock scenario may result in unrealistic profits and losses if an instantaneous shock, which is calibrated to a one-month horizon, is applied to a position that will expire in 10 days.

The alternative to the instantaneous global market shock would be to dynamically model shocks through multiple time periods and assume that positions could change over that period. In this alternative, the Board would provide a common, quantitative method for deriving dynamic shocks. Firms would be required to apply appropriate horizon shocks to each position in their trading book. Using this alternative, the Board would mitigate the potential problem of implausible trading gains or losses from longer horizon shocks to short-dated positions.

The alternative would require firms to scale a base market shock into a position-based shock according to the remaining contractual term of each trade, so that firms could recognize the short-dated nature of their positions. This could, e.g., be implemented by scaling shocks using a square root of time rule. As a result, this alternative would also require changes to FR Y-14Q instructions and to firms' models so that they could estimate their profits and losses. Since the Board does not have the profits and losses estimated under the dynamic approach, the impact of this approach is unknown. While this alternative could resolve potential problems of implausible trading gains or losses, it has drawbacks. It adds implementation complexity and reduces comparability across firms due to the differences in firms' assumption on position dynamics. Since each firm would use its own adjustments to the market shocks for different contractual terms according to its firm-specific exposures, such adjustments could be independent of one another across firms. Under these circumstances, the Board would not be able to ensure uniformity in evaluating these adjustments.

ii. Calibration Horizon Granularity

The global market shock framework uses calibration horizons of one month for liquid asset classes and of three months for illiquid asset classes. These horizon choices are within the one-quarter horizon used for global market shock loss recognition in the stress capital buffer projections.

The horizons over which global market shock values are calibrated are determined separately for each asset class, but risk factors within each asset class are calibrated over the same horizon. Although the set horizons follow FRTB closely, there are differences arising from the fact that the FRTB determines horizons at a more granular level that allows for different horizons within each asset class. For example, the FRTB differentiates shock liquidity horizons to equity risk factors between two weeks and three months and credit risk factors between one and six months. In contrast, the global market scenario framework assigns a one-month liquidity horizon for all equity risk factors and a three-month horizon for all credit risk factors.

Allowing for different horizons within asset classes would necessitate a substantial change to the existing model structure and require additional layers of complexity that may outweigh the benefits and could conflict with the Board's stress test principle of simplicity. If undertaken, further alignment with FRTB would create additional burden for the Board due to the creation of new risk factor categories and the changes to the modeling framework. Firms would also be affected by these changes, as updates to Y14-Q instructions and firms' modeling methods would be needed.

iii. Missing Risks and Global Market Shock Scenario Simplification

The global market shock template covers most of the exposures to which firms have trading and counterparty credit vulnerabilities. However, the template is not granular enough to differentiate shocks to certain risk factors and to capture basis risks. For example, all U.S. equity spot risk factors are represented by a single shock to S&P 500 index. This practice may miss basis risk that arises from different shocks to concentrated positions in individual stocks or in industry sectors. To address this issue, the Board could add additional risk factors that would capture material basis and concentration risks that the current template does not.

At the same time, the global market shock template includes many risk factors, some of which may not improve the risk capture of the global market shock. The Board is therefore proposing to simplify the global market shock template, as described in detail below.

The immaterial risk factors were introduced as part of the original global market shock template, but over time risk profiles changed, and the Board has confirmed their limited risk capture in recent stress tests. The Board continued to publish these immaterial risk factors so that firms can easily use the original global market shock template in their operational systems. Yet, modeling these risk factors is burdensome and resource consuming for the Board because it involves maintaining additional data sets and models and performing additional qualitive assurance checks.

To align with the principle of simplicity and overcome the additional burden, the Board is proposing to substantially reduce the number of disclosed risk factors in the template, including relative shocks, and to increase the use of mapping, as described on p. 46, for non-disclosed risk factors. The Board would provide instructions for how this reduced set of shocks maps to most of the risk factor shocks contained in the original global market shock template. For example, in equity, the original template differentiated shocks for 23 advanced economy regions along with a shock for all "other cross-country indices" and "other advanced economies." In the simplified template, the number of differentiated shocks would be smaller, and the "other" categories would cover a broader set of risk factors. For volatility term structures, the simplified template may differentiate term structure shocks up to a tenor of three years, providing a single shock for tenors greater than or equal to three years for each term structure. All risk factors are therefore still captured under the simplified framework. The Board seeks public comments on its simplification approach.

Under this simplification approach, the Board would consider the materiality of the risk factors measured by P&L from firms' trading activity. Specifically, the Board would ensure that material risk factors are maintained in the global market shock template. The Board would also consider the importance of the risk factors for characterizing scenario narratives. The simplified template would therefore maintain risk factors that are categorized as primary and secondary. One example of such risk factors is the gold spot price shock, which is an indicator of market stress that the Board often cites in the scenario narrative, but for which firm exposures have historically been relatively small. Another example of a risk factor with lower materiality (in terms of aggregate P&L) that is important to characterize scenario narratives is the exchange rate between U.S. dollar and Japanese Yen.

Another consideration is maintaining consistency across asset classes. In this regard, where possible, the Board would generate shocks for the same set of countries, regions, and tenor points across different asset classes. For example, the template would include shock values for the same set of tenors for term structures in the equity, FX, and commodity asset classes. Such consistency would simplify risk factor comparison across different asset classes and help the Board to communicate its scenario narratives more effectively to the public. Finally, the Board would consider data quality and the availability of data such that the template maintains risk factors for which high-quality data is available, unless these risk factors are material, important for the scenario, or serve to ensure consistency across asset classes. For example, high-quality data may not be available for thinly traded securities. In addition, for some assets, such as certain securitized products, data may not be available at all, as these products have not been actively traded since the Global Financial Crisis.

Although the Board would provide instructions for all risk factors under the simplified approach, the simplification may involve a loss of risk capture by not differentiating all risk factor shocks. The Board has evaluated the impact of simplification on P&L of firms' trading activities, finding that the loss impact is immaterial.

An important consideration of simplification is the operational burden for firms to adjust their production processes to the changed template. To mitigate this burden and give firms sufficient time to operationalize the change, the Board would publish the undisclosed shocks for the 2026 stress test scenario in the same file format as was done for the 2025 stress test and map the undisclosed risk factors to the disclosed ones. In the 2026 stress test scenarios, there are approximately 2,300 disclosed risk factors, which are published in a separate template for ease of disposition among the 2026 stress test scenario materials. The Board proposed to publish only the disclosed risk factors used in future stress test scenarios, in the shorter-form template, for the 2027 stress test and beyond.