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Consumption-Based Asset Pricing When Consumers Make Mistakes

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Abstract

I analyze the implications of allowing consumers to make mistakes on the risk-return relationships predicted by consumption-based asset pricing models. I allow for consumption mistakes using a model in which a portfolio manager selects investments on a consumer's behalf. The consumer has an arbitrary consumption policy that could reflect a wide range of mistakes. For power utility, expected returns do not generally depend on exposure to single-period consumption shocks, but robustly depend on exposure to both long-run consumption and expected return shocks. I empirically show that separately accounting for both types of shocks helps explain the equity premium and cross section of stock returns.

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1 Introduction

Consumption-based asset pricing models aim to answer one of the most important questions in asset pricing: How do asset returns relate to risk in the real economy? The simplest model, the consumption CAPM (CCAPM), predicts that an asset's expected return depends on its covariance with contemporaneous consumption growth. Intuitively, investors demand a higher return for buying assets which crash when they cut spending. However, the CCAPM has had limited empirical success. (Mehra and Prescott, 1985; Campbell, 2003) The literature responded by building new models linking risk and return which feature more complicated preferences, such as Epstein-Zin or habit preferences, and imposing additional assumptions on the data-generating process, such as persistent shocks to consumption or rare disasters during which consumption drops dramatically. (Epstein and Zin, 1989; Campbell and Cochrane, 1999; Bansal and Yaron, 2004; Barro, 2006)

In this paper, I introduce an additional way of improving consumption-based models. Almost all of these models, old and new, assume that consumers immediately and optimally adjust their consumption in response to all shocks. Consumers do not make common mistakes, such as ignoring information, reacting to news with a delay, or following suboptimal rules of thumb. Not accounting for these mistakes may lead to incorrect predictions, which motivates the main question of this paper: What risk-return relationship should we expect to see when consumers make mistakes? To my knowledge, this is the first paper to pose this question generally.

My main contribution is to derive a risk-return relationship that is robust to allowing for consumers who could make a range of mistakes. I focus on power utility preferences, however the results generalize. In this case, there is no guarantee that the standard CCAPM's predicted risk-return relationship will hold. Instead, expected returns robustly depend on exposure to two factors: Shocks to long-run consumption growth and expected returns. Assets have a higher expected return if they crash when long-run expected consumption falls or when expected returns rise. Due to the importance of long-run risk exposures, I call this model the "long-run CCAPM."

I do not focus on a specific consumption mistake. Instead, I flexibly allow for a range of mistakes by separating consumption and portfolio choice. In my framework, a consumer decides how much to consume (or, equivalently, how much to save) while a benevolent portfolio manager invests the savings on the consumer's behalf.[1] The manager optimally selects portfolio weights to maximize the consumer's utility, as in the standard model, but takes as given the consumer's possibly non-optimal consumption policy. I define a consumption mistake broadly as any consumption policy which deviates from the optimal one. The model allows for consumption policies which can incorporate many mistakes and also nests the standard case, in which consumers make no mistakes. In this sense, my model is a generalization of the standard model rather than an alternative.

For intuition, I provide examples of the importance of accounting for both long-run consumption and expected return shocks. In the first example, a consumer's delayed reaction to news breaks the CCAPM's prediction of a strong link between stock returns and contemporaneous consumption, similar in spirit to Gabaix and Laibson (2001) or Jagannathan and Wang (2007). Here, the stock market crashes immediately, but the consumer delays cutting spending. In equilibrium, the stock market commands a high risk premium despite its low co-movement with contemporaneous consumption. The portfolio manager recognizes that co-movement with contemporaneous consumption is a misleading indicator of the stock market's riskiness due to the consumer's delayed reaction and instead measures risk based on how stock returns co-move with long-run consumption. The second example features a risk premium driven entirely by exposure to a expected return shock to show that it isn't sufficient to only account for exposure to consumption shocks, whether contemporaneous or long-run. This result is impossible with a fully-optimizing power utility consumer and is only possible due to the introduction of consumption mistakes.

The key feature driving my results is a wedge between the marginal utility of consumption and the marginal value of wealth. In nearly all models, an asset earns a high expected

[1] While my discussion focuses on a separate consumer and portfolio manager, the model does not literally require delegated management. The important part is focusing on the partial equilibrium portfolio choice problem, taking the consumption policy as given. Section 3.1.4 discusses other possible interpretations in more detail.

return if it crashes when the marginal value of wealth rises. When consumers select their consumption optimally, they endogenously equate the marginal value of consumption with the marginal value of wealth. Thus, in the standard case in which consumers make no mistakes, shocks to single-period consumption perfectly reveal shocks to the marginal value of wealth.

When consumers don't perfectly optimize, the marginal utility of consumption and the marginal value of wealth can differ. I derive an alternative expression for the marginal value of wealth based on an accounting relationship which does not require any behavioral assumption about the consumer. Mechanically, the marginal value of wealth depends on future expected consumption growth and portfolio returns. Holding everything else constant, wealth is valuable when long-run consumption growth is low, because marginal utility is high, or when expected returns are high, because wealth grows at a faster rate.

I then compare the risk-return relationship from the long-run CCAPM to those from the long-run risks and ICAPM literatures, both of which assume a perfectly-optimizing agent with Epstein-Zin preferences. Broadly speaking, the long-run risks literature expresses risk premia in terms of exposure to news about long-run consumption (Hansen et al., 2008; Bansal et al., 2009, 2012), whereas the ICAPM literature expresses risk premia in terms of exposure to the return on wealth and news about future expected returns (Campbell, 1993, 1996; Campbell and Vuolteenaho, 2004; Campbell et al., 2018).[2] I show that there is a third way of expressing risk premia based upon exposure to both shocks to long-run consumption and expected returns. I show that this third expression is, to a first-order approximation, identical to the long-run CCAPM. Therefore, the price of exposure to long-run consumption and expected return shocks is the same for a perfectly-optimizing Epstein-Zin agent and a power utility agent who may make consumption mistakes.

This comparison reveals two key insights. The first is that consumer mistakes can provide a alternative and potentially more realistic explanation for why long-run risks are

[2]Later papers in the ICAPM literature, beginning with Campbell and Vuolteenaho (2004), apply the Campbell-Shiller decomposition to separate the return on wealth into cash flow shocks and expected return shocks. While these models may not explicitly include the contemporaneous return on wealth, it implicitly remains in the background.

priced. Long-run risks are priced by Epstein-Zin preferences because agents optimally choose consumption given a preference for early resolution of uncertainty (in standard calibrations). Some recent papers, most notably Epstein et al. (2014), argue that the magnitude of this preference is unrealistically high. In contrast, long-run risks are priced in the long-run CCAPM because an asset's exposure to single-period consumption is not a good proxy for its riskiness when consumers make mistakes.[3]

The second insight is that, when evaluating assets' exposure to long-run risks, it is important to jointly consider exposure to consumption and expected return shocks instead of only focusing on one or the other. Theoretically, jointly considering both shocks is valid both when consumers optimize perfectly and when they make mistakes, so there is little reason not to consider both shocks simultaneously.

While I focus on power utility, my approach of separating consumption and portfolio choice generalizes to other preferences. Standard models predict high expected returns for assets which crash when the single-period marginal utility of consumption falls. This risk-return relationship may not hold if consumers make mistakes. Instead, expected returns depend on exposure to long-run marginal utility and expected return shocks. In the case of power utility, long-run consumption shocks perfectly capture long-run marginal utility shocks since the level of consumption perfectly reflects marginal utility. However, in general, marginal utility could also depend on past consumption, durable goods, time preference shocks, or other factors. (Campbell and Cochrane, 1999; Yogo, 2006; Albuquerque et al., 2016)

I next turn to the empirical evidence. I demonstrate the importance of jointly considering shocks to long-run consumption and expected returns in the context of the equity premium and cross-section of stock returns. To my knowledge, this is the first paper to empirically evaluate long-run consumption and expected return shocks together.[4]

[3] There are other explanations for why long-run risks are priced, most notably Hansen and Sargent (2008), who show that ambiguity aversion provides a microfoundation for Epstein-Zin preferences with an elasticity of intertemporal substitution $\psi = 1$. Allowing for consumer mistakes is in no way in opposition to this explanation. Ambiguity-averse consumers could make mistakes too.

[4] The paper which comes closest to examining both is Campbell and Vuolteenaho (2004), which contains a specification which combines stock market cash flow shocks and expected return shocks. Campbell (1996) also contains a related specification which contains long-run labor income and expected return shocks.

I use aggregate consumption and measure expected return shocks to the aggregate stock market. I test the unconditional implications of the model, which link an asset's average return to its unconditional covariance with future realized consumption growth and market returns. I measure risk as the covariance of an asset's return with three-year-ahead realized consumption growth (including contemporaneous consumption growth) and stock market returns (excluding the contemporaneous market return, to capture shocks to expected returns only).

I first show that the long-run CCAPM introduces a novel source of risk that contributes to explaining the equity premium. While many papers have focused on the relationship between equity returns and long-run consumption, none have also examined the exposure of equity returns to long-run expected return shocks. The stock market crashes when expected returns rise, which should increase the equity premium under the long-run CCAPM. The point estimate for the increase is 1.3 percentage points.

Next, I examine the model in the context of the cross section of equity returns. I examine the model in the context of 34 equity market anomalies from Cho (2018). I show that exposure to expected return shocks is negatively priced among these anomalies, as the theory predicts. Additionally, I show that different momentum strategies are also exposed to expected return shocks and that these exposures are significantly priced. However, this exposure is not quantitatively large enough to explain the average returns on momentum strategies overall.

Consistent with results from earlier literature, I also document that the long-run CCAPM's inclusion of long-run consumption growth helps to match the data. This result is similar to those documented by Gabaix and Laibson (2001) and Parker (2003) in the context of the equity premium and Parker and Julliard (2005) and Jagannathan and Wang (2007) within the 25 Fama French portfolios sorted on size and book-to-market. I additionally check whether exposure to long-run consumption growth explains the average returns of more recent profitability- and investment-based strategies. Neither strategy can be explained by exposure to consumption or expected return shocks, deepening the puzzle of why they earn high average returns.

The paper is organized as follows: Section 2 provides examples of how consumption mistakes can break standard risk-return relationships and how exposure to long-run consumption and expected return shocks is a more robust measure of risk. Section 3 derives the general risk-return relationship for the case of power utility, which I call the long-run consumption CAPM, in which an asset's risk premium depends on its exposure to long-run consumption and expected return shocks. Section 4 derives long-run versions of models with more general preferences and examines the relationship between the long-run CCAPM and models with Epstein-Zin preferences. Section 5 empirically evaluates the long-run CCAPM. Section 6 concludes.

1.1 Related literature

While the robust risk-return relationship reflected in the long-run CCAPM is new, other papers have provided specific examples in which the standard CCAPM does not hold. For example, Jagannathan and Wang (2007) demonstrate that the consumption CAPM performs substantially better using 4th quarter to 4th quarter consumption growth, which is consistent with people optimizing at the end of each year rather than continuously throughout. Gabaix and Laibson (2001) examine a model in which people adjust their consumption occasionally, which leads to a delayed response of aggregate consumption in response to stock returns. Lynch (1996) considers how infrequent portfolio and consumption choice can explain the equity premium. Marshall and Parekh (1999) consider the implications of adjustment costs for consumption-based asset pricing models. The key difference is that my paper does not take a stand on the particular form of consumption mistake. Instead, I consider a general model which allows for a wide range of consumption policies and nests several of these models as special cases. I show that exposure to long-run consumption growth is consistently priced, as many of these papers show in special cases, but also show that long-run expected return shocks matter as well.

The long-run CCAPM closely relates to the consumption-based asset pricing literature using Epstein-Zin preferences. The two main strands correspond to the intertemporal CAPM (ICAPM) of Campbell (1993) and the long-run risk model of Bansal and Yaron

(2004).

The ICAPM literature primarily focuses on implications for risk premia from the perspective of market returns. Campbell (1993) develops an ICAPM using Epstein-Zin preferences, which yields a two-factor model in which shocks to the market and expected returns determine risk premia. Campbell (1996) further develops the ICAPM and includes shocks to future labor income. Campbell and Vuolteenaho (2004) develops a two-factor model which separates the market return into shocks to future dividends and expected returns. The long-run CCAPM is most closely related to this two-factor model. More recently, Campbell et al. (2018) builds an ICAPM which allows for stochastic volatility.

The long-run risks literature primarily focuses on implications of Epstein-Zin preferences for risk premia from the perspective of long-run consumption. Bansal and Yaron (2004) combines Epstein-Zin preferences with long-run predictability in consumption growth to explain the equity premium and other facts in asset pricing. Bansal et al. (2005) apply the long-run risks model to the cross section of stock returns. Hansen et al. (2008) demonstrates long-run predictability in consumption growth and also applies it to the cross section of stock returns.

My main contributions to these two literatures are twofold. First, I show that long-run consumption and expected return factors naturally emerge from a power utility consumer who might make a mistake in choosing consumption. In this sense, I provide an alternative microfoundation for Epstein-Zin risk premia, similar in spirit to Hansen and Sargent (2008). Second, to my knowledge, this is the first paper to combine long-run consumption and long-run expected return shocks within the same empirical specification. While there has been a large focus on the empirical importance of long-run consumption shocks, I show that expected return shocks also help to explain certain features of asset returns.

The empirical results of this paper are closely related to Parker and Julliard (2005) and Malloy et al. (2009), which both test consumption Euler equations involving long-run consumption growth. They show that using long-run consumption growth better explains the average returns of the 25 Fama French size and book-to-market sorted portfolios. My derivation of the long-run CCAPM provides a general theoretical argument for why long-run consumption growth is a more robust measure of risk. It also introduces expected return shocks as an additional risk factor. My empirical analysis confirms their earlier findings on the importance of long-run consumption growth. I build on their work by additionally showing the importance of expected return shocks and incorporating additional asset pricing anomalies.

More broadly, this paper relates to the large literature which empirically examines the importance of shocks to consumption growth for understanding asset returns. Kandel and Stambaugh (1990) is an early example providing evidence for changing means and variances of consumption. Bansal et al. (2012) summarize empirical results related to the long-run risks model showing evidence for variation in long-run consumption growth and volatility. Beeler and Campbell (2012) also summarize empirical results related to the long-run risks model, highlighting several areas of empirical difficulty. More recent empirical work by Schorfheide et al. (2018) provides evidence of persistence in consumption growth and volatility after correcting for measurement errors and applying Bayesian methodologies.

2 Examples of consumption mistakes

Here I provide two stylized examples of consumption mistakes with power utility preferences. In both of these examples, a benevolent portfolio manager decides how much to invest in a single risky asset on behalf of a consumer. The portfolio manager's goal is to maximize the consumer's utility, taking the consumption policy as given. My focus is on the equilibrium risk-return relationship, which derives from the portfolio manager's first-order conditions.

In a power utility environment, the standard result that risk premia depend on exposure to single-period consumption shocks does not hold in general. However, an asset's risk premium robustly depends on its exposure to shocks to long-run consumption and expected returns. The first example demonstrates the importance of looking at exposure to long-run consumption in a case involving an inattentive consumer who reacts to news with a delay. The second example demonstrates a risk premium arising entirely from exposure to an expected return shock when consumption is deterministic. This result is impossible when consumers optimize perfectly, but is possible when consumers make mistakes.

2.1 Example of a long-run consumption shock

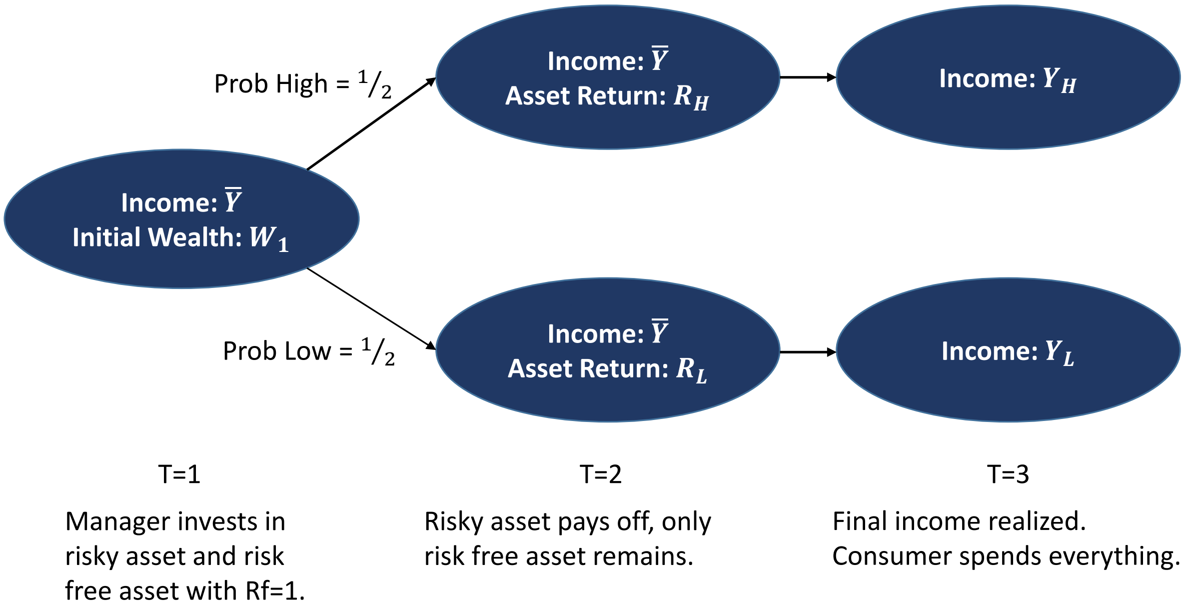
I begin by providing an example of a long-run consumption shock without any expected return shock. This example shows how allowing for consumers to make mistakes can break the standard risk-return prediction of the CCAPM and why it is necessary to use another risk-return relationship based on long-run consumption.

In this example, a negative shock causes the stock market to fall immediately, but labor income to fall with a lag. This relationship is consistent with empirical evidence from Campbell and Viceira (2002). In the standard case, in which the consumer makes no mistakes, the consumer immediately reduces consumption when a negative shock arrives. Because the consumer immediately reacts, there is a strong link between contemporaneous stock returns and consumption growth. The standard CCAPM holds, so that the expected stock return is related to the covariance between the contemporaneous stock return and consumption growth.

In this example, the consumer is inattentive. Specifically, once a negative shock arrives, the consumer does not immediately realize that future income will fall. Thus, the consumer only slightly reduces consumption. In this case, there is a weak link between contemporaneous stock returns and consumption growth. The standard CCAPM does not hold. However, there is a relationship between expected stock returns and the covariance between the contemporaneous stock return and long-run consumption growth.

While this example focuses on a particular form of consumption mistake, namely inattention, it does not depend upon it. Many other types of consumption mistakes will break the risk-return predictions of standard models. Later, Section 3 will show that the long-run risk-return relationship which I demonstrate in this example will hold for a

Figure 1: Environment for inattention to labor income



much broader range of consumption mistakes.

The example has three periods. A consumer has initial financial wealth W_1 and earns labor income in each period. Labor income in periods 1 and 2 is $Y_1 = Y_2 = \bar{Y}$. Labor income in period 3 is $Y_3 \in \{Y_L, Y_H\}$ with equal probability $\frac{1}{2}$, where $Y_H > Y_L$.

There is a single risky asset available for investment in the first period and a risk-free asset with a constant gross return $R_f = 1$ across all periods. For simplicity, the risky asset is no longer available in the second period. The risky asset has an exogenous gross return $R_2 \in \{R_L, R_H\}$ in the second period, where $R_H > R_L$. The return on the risky asset is perfectly correlated with future labor income. If $R_2 = R_H$, then $Y_3 = Y_H$, and similarly for low realizations. The model does not contain any shocks to expected returns.

Beginning-of-period financial wealth W_t (before labor income is received) accumulates according to

$$W_3 = W_2 - C_2 + Y_2 \tag{1}$$

$$W_2 = \underbrace{(1 + \theta_1 R_2^e)}_{R_{p,2}}(W_1 - C_1 + Y_1) \tag{2}$$

where θ_1 is the portfolio weight of the risky asset, $R_2^e = R_2 - 1$ is the excess return on the risky asset, and C_t is consumption. Figure 1 illustrates the environment.

The model features a division between consumption choice and portfolio choice. The consumer decides how much to consume (or, equivalently, how much to save) and delegates the portfolio allocation decision to a benevolent portfolio manager. The consumer tells the manager that she has power utility preferences with risk aversion parameter γ and discount factor $\beta = 1$. The portfolio manager selects θ_1, the allocation to the risky asset, based on the consumer's preferences. The portfolio manager has rational expectations and understands that the risky asset's return is related to future labor income.

I model the consumer making a mistake by assuming they follow the non-optimal consumption policy

$$C_3 = W_3 + Y_3 \tag{3}$$
$$C_2 = \frac{1}{2}(W_2 + 2\bar{Y}) \tag{4}$$
$$C_1 = \frac{1}{3}(W_1 + 3\bar{Y}) \tag{5}$$

which corresponds to smoothing out consumption based on current financial wealth and expectations of future labor income.[5] The consumer does not update her expectation of third period labor income based upon a negative shock occurring in the second period.

In period 1, the benevolent portfolio manager selects a weight in the risky asset θ_1 to maximize the consumer's expected utility

$$\max_{\theta_1} \sum_{t=1}^{3} E_1 \left[\frac{C_t^{1-\gamma} - 1}{1-\gamma} \right] \tag{6}$$

subject to the budget constraints and consumption policies described previously.

The manager's first-order condition is

$$E_1 \left[\left(\frac{1}{2} C_2^{-\gamma} + \frac{1}{2} C_3^{-\gamma} \right) R_2^e \right] = 0 \tag{7}$$
$$\implies E_1[R_2^e] = -Cov_1 \left(\frac{\frac{1}{2} C_2^{-\gamma} + \frac{1}{2} C_3^{-\gamma}}{E_1[\frac{1}{2} C_2^{-\gamma} + \frac{1}{2} C_3^{-\gamma}]}, R_2^e \right) \tag{8}$$

[5] For simplicity, the consumer discounts future labor income at the risk free rate. Applying a different discount rate to labor income does not materially affect the results of the model.

The equilibrium risk premium depends on covariance with contemporaneous *and* future consumption growth. The manager knows that the risky asset return predicts future labor income. The manager also knows that the consumer will not optimally adjust consumption in response to the risky asset's return.

To build intuition, I consider the specific parameterization

$$\gamma = 1 \qquad W_1 = 2 \qquad \bar{Y} = 1$$
$$Y_H = 1.5 \qquad Y_L = 0.5 \qquad R_H = 1.3 \qquad R_L = 0.8$$

which corresponds to a log utility consumer whose expected labor income is 50% larger than initial financial wealth.

Under this parameterization, the portfolio manager selects an optimal weight $\theta_1 = 0.43$ and consumption is

$$C_1 = 1.67 \qquad C_{2H} = 1.75 \qquad C_{3H} = 2.25$$
$$C_{2L} = 1.61 \qquad C_{3L} = 1.11$$

Figure 2 plots the consumption paths. The dark, solid lines show the inattentive consumer's spending over time.

Consider the inattentive consumer's response to a high risky asset return. In the second period, the consumer sees more money in her brokerage account and consumes some of it. But she does not realize that a high return means higher future labor income. She then further raises her consumption in the third period once labor income rises. Overall, consumption underreacts in the second period and overreacts in the third period.

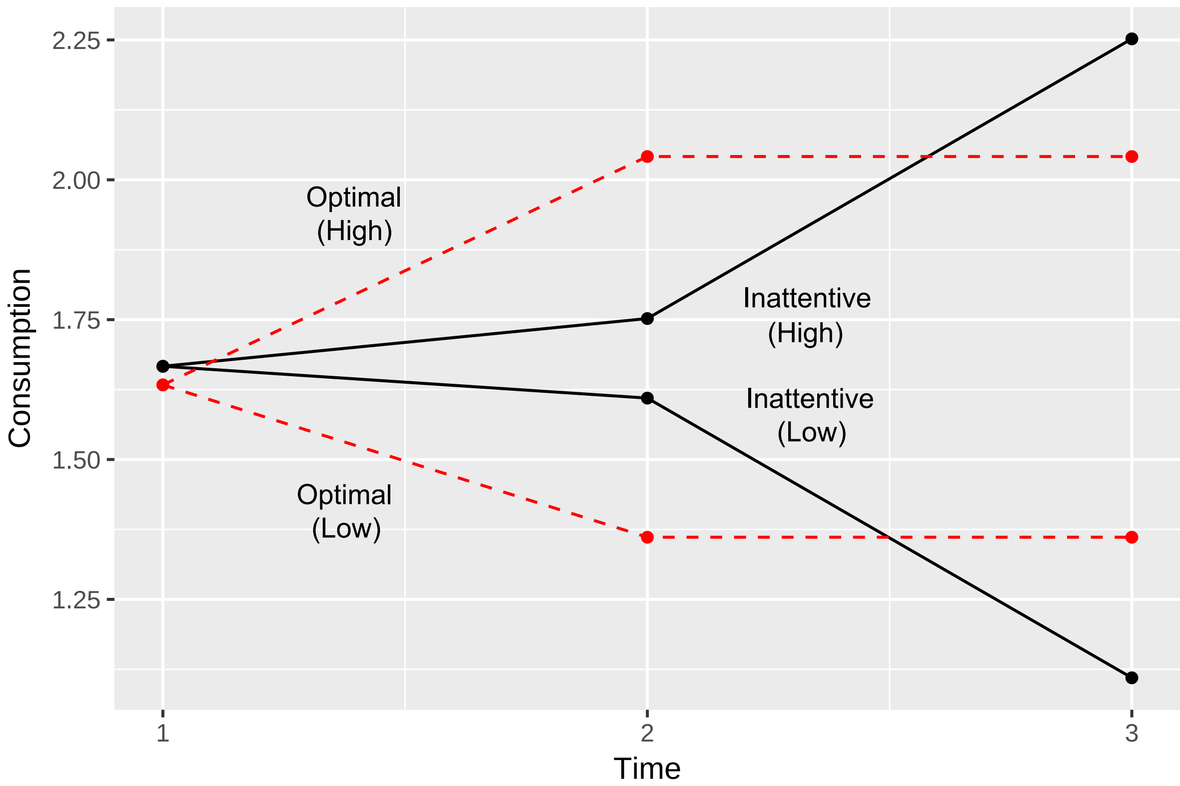
For comparison, I also consider the case in which consumption is chosen optimally. In that case, the portfolio manager selects $\theta_1 = 0.53$, which is slightly higher than in the inattentive case, and consumption is

$$C_1 = 1.63 \qquad C_{2H} = 2.04 \qquad C_{3H} = 2.04$$
$$C_{2L} = 1.36 \qquad C_{3L} = 1.36$$

The red, dotted lines in Figure 2 show the optimal consumption path over time.

When behaving optimally, the consumer fully reacts in the second period. Importantly, her consumption in the second period perfectly reveals her consumption in the

Figure 2: Consumption Paths in Parameterized Three Period Model



third. A general version of this point, where optimally-chosen current consumption reveals information about expected future consumption, is the rationale for why the standard risk-return relationship linking returns to contemporaneous consumption works. If consumers do not optimize perfectly, then their current consumption no longer reveals information about future consumption and it is instead necessary to use long-run risk measures. I will explore this point in more detail in Section 3.[6]

The standard predictions of the consumption CAPM are misleading when the consumer is inattentive. I estimate risk aversion from the perspective of an econometrician who believes that the standard CCAPM is the true model. The standard CCAPM predicts that the unconditional Euler equation

$$E\left[\left(\frac{C_2}{C_1}\right)^{-\gamma} R_2^e\right] = 0 \tag{9}$$

[6] Another interesting result is that the share invested in the risky asset, θ_1, is higher when consumption is chosen optimally. Intuitively, when the consumer optimally adjusts spending in response to shocks, an adverse shock will have less impact on the consumer's utility. Thus the portfolio manager can take on more risk.

holds. The econometrician estimates $\hat{\gamma}$ to minimize

$$E\left[\left(\frac{C_2}{C_1}\right)^{-\hat{\gamma}} R_2^e\right] = 0 \tag{10}$$

and obtains $\hat{\gamma} = 4.79$, even though $\gamma = 1$.

The econometrician sees that consumption in the second period does not respond strongly to the asset return. After seeing a high average return and a low contemporaneous consumption covariance, the econometrician infers that the consumer must be risk averse. However the econometrician misses the delayed reaction of consumption to future labor income.

If the econometrician has a prior belief that $\gamma = 1$, then the portfolio weight $\theta_1 = 0.43$ is puzzling in light of the asset's low correlation with contemporaneous consumption. The econometrician will think that the portfolio contains too little of the risky asset. But again, this is only because the econometrician neglects the additional risk reflected in the delayed reaction of consumption.

A natural reaction is to fit the two period Euler equation implied by the CCAPM

$$E\left[\left(\frac{C_3}{C_1}\right)^{-\hat{\gamma}} (R_f R_2 - R_f R_f)\right] = E\left[\left(\frac{C_3}{C_1}\right)^{-\hat{\gamma}} R_2^e\right] = 0 \tag{11}$$

where $R_f R_2 - R_f R_f$ is the two period excess return to investing in the risky asset followed by the risk-free asset, which collapses to R_2^e since $R_f = 1$. But this does not work either. Now the econometrician estimates $\hat{\gamma} = 0.57$, which substantially underestimates risk aversion. The logic is the reverse from before: Third period consumption overreacts to the risky asset. The econometrician observes the same average return with a high consumption covariance and infers that the consumer is risk tolerant.

The correct inference of $\hat{\gamma} = \gamma = 1$ only comes from fitting

$$E\left[\left(\frac{1}{2}\left(\frac{C_2}{C_1}\right)^{-\hat{\gamma}} + \frac{1}{2}\left(\frac{C_3}{C_1}\right)^{-\hat{\gamma}}\right) R_2^e\right] = 0 \tag{12}$$

which accounts for both the underreaction in the second period and the overreaction in the third period.

In this environment, the stock market's expected return depends on its exposure to long-run consumption growth, not contemporaneous consumption growth. In general, exposure to long-run expected return shocks will be priced as well, however this example does not include these shocks. The next example will demonstrate a risk premium arising entirely due to exposure to expected return shocks.

2.2 Example of a long-run expected return shock

In this example, the consumer has power utility and consumption is fully deterministic, yet a risk premium exists due to exposure to expected return shocks. This outcome is impossible in the standard model, in which consumers optimize perfectly, but it is possible when consumers make mistakes. The key takeaway of this example is the importance of considering expected return shocks in addition to consumption shocks.

The environment is similar to the previous example. There are three periods and identical budget constraints. In the first period, the portfolio manager decides how much to invest in a risky asset. In the second period, there is an equal chance of the state being H or L. The risky asset has an exogenous gross return $R_2 \in \{R_H, R_L\}$ in the second period, where $R_H > R_L$. As before, in the second period the risky asset disappears and only a risk-free bond remains. The first period's risk-free rate is 0%. However, now the risk-free rate changes in the second period to either $R_{f,H}$ or $R_{f,L}$, depending on the realized state. Since the risk-free bond is the only asset, the new risk-free rate is the entire portfolio's expected return. This change in the risk-free rate is the expected return shock. In the final period, labor income is either $Y_{3,H}$ or $Y_{3,L}$.

The consumer spends one third of initial wealth in the first and second period (i.e., $C_1 = C_2 = \frac{1}{3}W_1$) and then consumes all remaining wealth in the third period, which corresponds to the consumption policy

$$C_3 = W_3 + Y_3 \tag{13}$$

$$C_2 = \frac{1}{3}W_1 \tag{14}$$

$$C_1 = \frac{1}{3}W_1 \tag{15}$$

This consumption policy reflects a mistake since the consumer is not adjusting second period consumption in response to changes in the risk-free rate and information about future labor income. Since the consumer has pre-determined first and second period consumption, only third period consumption can potentially vary. However, in a moment I will calibrate the model so that third period consumption is endogenously deterministic.

As before, in period 1, the portfolio manager selects a weight in the risky asset θ_1 to maximize the consumer's expected utility

$$\max_{\theta_1} \sum_{t=1}^{3} E_1 \left[\frac{C_t^{1-\gamma} - 1}{1-\gamma} \right] \tag{16}$$

subject to the budget constraints and consumption policies. The manager's first-order condition is

$$E_1 \left[C_3^{-\gamma} \frac{\partial W_3}{\partial \theta_1} \right] = E_1[C_3^{-\gamma} R_{f,2} R_2^e (W_1 - C_1)] = E_1[R_{f,2} R_2^e] = 0 \tag{17}$$

where the elimination of $C_3^{-\gamma}$ follows due to consumption being deterministic in equilibrium.

I make third period consumption deterministic by precisely calibrating the expected return shock (the change in the risk-free rate) and the labor income shock. When the risk-free rate rises, next period's labor income falls to exactly offset the extra interest income. And vice versa when the risk-free rate falls. This is a knife-edge calibration that is not meant to be realistic, but rather to demonstrate an expected return shock independently of a consumption shock.

The exact parameterization I use is

$$\gamma = 1 \qquad Y_H = 0.12 \qquad R_H = 1.3 \qquad R_{f,H} = 0.8$$
$$W_1 = 3 \qquad Y_L = 0 \qquad R_L = 0.8 \qquad R_{f,L} = 1.2$$

As in the previous example, the risky asset earns a 5% risk premium. In the high state, the risky asset earns a 30% return, the risk-free rate falls by 20 percentage points, and third period's labor income will be a higher-than-average value of 0.12. In the low state, the risky asset crashes by 20%, the risk-free rate rises by 20 percentage points, and third period's labor income is zero. In this three-period model, a large change in risk-free rate is required to quantitatively match a 5% risk premium. In a longer-horizon model, a small, but persistent, change in rates could have a similar effect.

Under this parameterization, the portfolio manager selects an optimal weight $\theta_1 = 0.58$ and the consumption path is $C_1 = C_2 = 1$ and $C_3 = 1.06$ for both values of the expected return shock. Even though the portfolio manager endogenously selects the optimal portfolio weight, consumption is deterministic in equilibrium due to my calibration of the labor income process.

Why should the risky asset earn a 5% risk premium even though its returns are completely uncorrelated with consumption at any horizon? Because the risky asset crashes exactly when expected returns rise, which is when money becomes more valuable. More formally, when expected returns rise, the marginal value of wealth rises because an extra unit of wealth will grow at a faster rate. The risk premium arises because the risky asset crashes exactly when the marginal value of wealth rises.

This example starkly demonstrates that consumption shocks alone, both contemporaneous and long-run, are not sufficient to capture all sources of risk. Expected return shocks are also important and can generate risk premia even when there are no shocks to consumption. While this example is highly stylized and represents a knife-edge result, it makes the general point that both theoretical and empirical work should consider expected return shocks in addition to consumption shocks. Empirical work in Section 5 reinforces this point by showing how expected return shocks contribute to explaining the equity premium and cross section of equity returns.

3 Allowing for general consumption mistakes with power utility

In this section, I derive a risk-return relationship for a consumer with power utility preferences which will robustly hold even if the consumer makes mistakes. In this case, an asset's risk premium will depend on its exposure to news about long-run consumption growth and expected returns. Assets command a higher risk premium if they crash when expected long-run consumption crashes, because money is more valuable when consumption falls. Assets also command a higher risk premium if they crash when long-run expected returns rise, because money is more valuable when expected returns rise since it grows at a faster rate. I call this model the "long-run consumption CAPM (CCAPM)."

The long-run consumption CAPM (CCAPM) emerges from the problem of a benevolent portfolio manager who selects asset allocation on behalf of a consumer with power utility preferences. The manager optimizes as in the standard model, but the consumer may possibly make mistakes in selecting consumption. I model these mistakes by assuming that the consumer has a nearly arbitrary consumption policy which may differ from the optimal policy. Thus my approach does not depend on a particular model of consumer mistakes. Additionally, the long-run CCAPM contains the standard CCAPM as a special case in which the consumer makes no mistakes, that is, when the consumption policy is optimal.

The intuition for this result comes from a wedge between the marginal value of wealth and contemporaneous consumption. The marginal value of wealth reflects how much lifetime utility an extra unit of wealth provides. Assets pay higher risk premia if they crash when the marginal value of wealth rises. When consumers don't make mistakes, as in the standard model, current consumption perfectly reveals the marginal value of wealth. This link between current consumption and the marginal value of wealth breaks when consumers make mistakes. The long-run CCAPM emerges from an alternative representation of the marginal value of wealth based on expectations of long-run consumption and expected returns which does not depend on any behavioral assumptions about the consumer.

3.1 The long-run CCAPM in an infinite horizon economy

3.1.1 Environment

The economy has an infinite number of discrete time periods. The state of the economy is characterized by the level of wealth and a vector of variables X_t which follow some exogenous process. Returns and labor income are exogenous and depend on X_t. $R_{f,t}$ is the gross risk-free rate from time t to $t+1$, $R^e_{i,t+1}$ is the excess return on asset i from t to $t+1$, and Y_t is labor income paid at time t. I assume limited liability, that is, $R_{i,t+1} = R^e_{i,t+1} + R_{f,t} > 0$.

The consumer begins with exogenous financial wealth W_1. Financial wealth evolves according to

$$W_{t+1} = \underbrace{\left(R_{f,t} + \sum_i \theta_{i,t} R^e_{i,t+1}\right)}_{R_{p,t+1}} (W_t - C_t + Y_t) \tag{18}$$

where C_t is consumption and $\theta_{i,t}$ is the portfolio weight on asset i. Financial wealth W_t reflects beginning-of-period wealth, before labor income is paid.

3.1.2 Consumer

The consumer has power utility preferences with risk aversion γ and discount factor β, but does not necessarily follow the optimal consumption policy. I do not explicitly model the link between preferences and the consumption policy, however there are a wide range of possibilities. The consumer may act subject to holding biased beliefs, optimize subject to attention costs (as in Gabaix (2014) or Sims (2003)), or even optimize perfectly with respect to a smaller information set than the portfolio manager.

Instead of modeling the link between preferences and consumption, I focus on the resulting consumption policy

$$C_t = C_t(X_t, W_t) \tag{19}$$

The consumption policy depends on the current level of wealth and exogenous state variables, but does not explicitly depend on the portfolio manager's choice of portfolio weights. [7]

To ensure budget feasibility, I assume that there exists some portfolio process $R_{p,t}$ such that the consumption policy satisfies the no-Ponzi-game condition

$$\lim_{t\to\infty} \frac{W_t}{R_{p,1\to t}} \geq 0 \tag{20}$$

almost surely. The no-Ponzi condition rules out consumption policies that accumulate debt which cannot be repaid. It is sufficient (although stronger than necessary) to assume the consumer does not borrow and that labor income is weakly positive, which means $C(W_t, X_t) \in [0, W_t + Y_t]$, and $Y_t \geq 0$.

3.1.3 Portfolio manager

The consumer delegates management of savings to a benevolent portfolio manager and instructs the manager to maximize expected utility under power utility preferences. The manager has rational expectations and knows the consumption policy, but the manager's information set does not necessarily include all of the state variables X_t.[8] I do not impose any leverage or short sale constraints on the portfolio manager beyond those required to avoid bankruptcy.[9]

[7] In a broader model, a consumer's decision may depend on the portfolio manager's policy function. For example, the consumer may decide how much to save based upon her portfolio's expected return, which will depend on how the portfolio manager decides to invest. I interpret this exogenous consumption policy as representing the Nash equilibrium consumption policy that emerges from that broader model, similarly to how endowment economy models of asset pricing can be interpreted as reflecting the equilibrium consumption outcomes of a richer model with production. Since my aim is to understand equilibrium risk-return relationships rather than how the consumption policy arises, I take consumption as exogenous.

[8] If the manager does not have rational expectations or knowledge of the consumption policy, then these results hold under the manager's subjective expectations or information set.

[9] The lack of portfolio constraints is not a key assumption. The model could be solved including portfolio constraints, similar to He and Modest (1995).

At time t, the portfolio manager's problem is to select portfolio weights $\theta_{i,t}$ for each asset i to maximize the consumer's utility. I write the optimization problem in recursive form as

$$V_t(W_t, X_t) = \max_{\{\theta_{i,t}\}} \frac{C_t(W_t, X_t)^{1-\gamma} - 1}{1-\gamma} + \beta E_t[V_{t+1}(W_{t+1}, X_{t+1})] \tag{21}$$

subject to the budget constraints and consumption policies described in Sections 3.1.1 and 3.1.2. I retain the t subscripts on the consumption policy and value function since I have not assumed that they are stationary. I add the further constraint that the portfolio choice must also satisfy the no-Ponzi-scheme condition, which is always possible due to my earlier assumption about the consumption policy. The no-Ponzi condition rules out the possibility of the portfolio manager taking highly levered positions which could bankrupt the consumer.

Proposition 1. *The portfolio manager's first-order condition for asset i is*

$$E_t[V_{w,t+1} R^e_{i,t+1}] = 0 \tag{22}$$

$$\Longrightarrow E_t[R^e_{i,t+1}] = -Cov_t\left(\frac{V_{w,t+1}}{E_t[V_{w,t+1}]}, R^e_{i,t+1}\right) \tag{23}$$

where $V_{w,t+1} = \frac{\partial V_{t+1}}{\partial W_{t+1}}(W_{t+1}, X_{t+1})$. It is possible to substitute out $V_{w,t+1}$ using

$$\frac{V_{w,t+1}}{C_t^{-\gamma}} = E_{t+1}\left[\sum_{j=0}^{\infty} \beta^j \left(\frac{C_{t+1+j}}{C_t}\right)^{-\gamma} R_{p,t+1\to t+1+j} C_{w,t+1+j} \left(\prod_{k=0}^{j-1}(1 - C_{w,t+1+k})\right)\right] \tag{24}$$

where $R_{p,t+1\to t+1+j}$ is the portfolio return from $t+1$ to $t+1+j$ and $C_{w,t} = \frac{\partial C_t}{\partial W_t}(W_t, X_t)$ is the marginal propensity to consume out of financial wealth (excluding human capital).

Proof. See Appendix B.1. □

As in standard models, equilibrium expected returns are related to covariance with the marginal value of wealth, $V_{w,t+1}$. In the case of optimal consumption, the consumer optimizes such that the marginal utility of consumption equals the marginal value of wealth, that is, $C_{t+1}^{-\gamma} = V_{w,t+1}$. But this equality no longer holds when consumption may be non-optimal. I address this problem by expressing the marginal value of wealth in a form which holds robustly regardless of the consumption policy.

The alternative expression of the marginal value of wealth reflects that wealth is only valuable insofar as it affects future consumption. If saving rates are not too volatile, the key factors driving the marginal value of wealth are expectations of long-run consumption growth and expected portfolio returns. Holding everything else constant, wealth is valuable when future consumption is low, because marginal utility is high, or when expected returns are high, because wealth grows at a faster rate.

While consumption and asset returns are observable, the marginal propensity to consume out of financial wealth is more challenging to pin down. Since the consumption policy does not offer any further guidance, I approximate the marginal propensities to consume out of financial wealth.

Corollary 1. *Using the approximation* $C_{w,t+j} \cdot \prod_{k=0}^{j-1}(1-C_{w,t+k}) \approx \bar{C}_w(1-\bar{C}_w)^j = (1-\phi)\phi^j$ *yields*

$$\frac{V_{w,t+1}}{C_t^{-\gamma}} \approx E_{t+1}\left[\sum_{j=0}^{\infty}(1-\phi)\phi^j\beta^j\left(\frac{C_{t+1+j}}{C_t}\right)^{-\gamma} R_{p,t+1\to t+1+j}\right] \tag{25}$$

where ϕ *represents the average marginal propensity to save financial wealth.*

This approximation works well when the marginal propensity to save financial wealth does not vary much over time and is small relative to movements in consumption growth.

In the special case in which the consumption policy is optimal, proposition 2 shows that this expression will hold *exactly* for all $\phi \in [0, 1)$. The approximation only becomes approximate when consumption is not chosen optimally. But in that case, the standard CCAPM may not hold.

Finally, I provide a log-linear approximation which I use for empirical work.

Corollary 2. *The long-run CCAPM can also be approximately expressed in logarithmic* *form as*

$$\tilde{m}_{t+1}^{long} = -\gamma \sum_{j=0}^{\infty} \phi^j \Delta \tilde{c}_{t+1+j} + \sum_{j=1}^{\infty} \phi^j \tilde{r}_{p,t+1+j} \quad (26)$$

where $\tilde{x} = (E_{t+1} - E_t)x$ *and lowercase letters indicate logarithms.*

Proof. See Appendix B.2. □

3.1.4 Interpretation of the portfolio manager

So far I have focused on interpreting the portfolio manager in the context of delegated management. This model maps to a real-life person who decides how much to save, but hires a financial planner to manage her savings, either directly or indirectly through investing in a managed fund.[10]

However, the model does not literally require delegation. For example, the consumer may be managing her own portfolio following investment strategies which she has read about, but may not fully understand. She may be following a portfolio rule of thumb that is equivalent to portfolio optimization subject to a much smaller information set. Static portfolio weights may correspond to portfolio optimization subject to an empty information set.

Moving further away from the interpretation of delegated management, the model can also capture consumption adjustment costs, which will in general yield a different consumption policy than the usual frictionless benchmark.[11] Focusing on the portfolio allocation problem, taking the consumption policy as given, will give the correct equilibrium relationship between consumption and asset returns. In this setting, consumption is optimal *given the adjustment costs*, but differs from optimal consumption under the standard frictionless model.

Ultimately, the exact assumptions about the portfolio manager are secondary to the model. The key force driving these results is a wedge between contemporaneous con-

[10] The delegation could also apply in an institutional context, where an institution decides how much of its endowment to spend and hires portfolio managers to decide how to invest the endowment.

[11] Adjustment costs can be added to the model by interpreting labor income as net labor income, after subtracting any adjustment costs the consumer pays.

sumption and the marginal value of wealth. In the standard CCAPM, the consumer equates the marginal utility of consumption and the marginal value of wealth. Allowing for consumption mistakes breaks this link. The standard CCAPM fails because contemporaneous consumption does not reflect the marginal value of wealth. The long-run CCAPM fixes this problem by expressing the marginal value of wealth in a way that is robust to allowing for consumption mistakes. Modifying the model by introducing delegation frictions, portfolio constraints, or other features should not change this basic logic.

3.2 Equivalence of the long-run and standard CCAPM when consumption is optimal

When consumption is chosen optimally, the long-run and standard CCAPM have identical pricing implications. The long-run CCAPM will work even when consumption is chosen optimally, although it will have less statistical power than the standard CCAPM.

Proposition 2. *Denote innovations to the SDF from the standard CCAPM as*

$$\tilde{M}_{t+1}^{std} = (E_{t+1} - E_t)\beta \left(\frac{C_{t+1}}{C_t}\right)^{-\gamma} \tag{27}$$

and innovations to the SDF from the long-run CCAPM as

$$\tilde{M}_{t+1}^{long} = (E_{t+1} - E_t)\left[\sum_{j=0}^{\infty}(1-\phi)\phi^j \beta^{j+1}\left(\frac{C_{t+1+j}}{C_t}\right)^{-\gamma} R_{p,t+1\to t+1+j}\right] \tag{28}$$

If consumption is chosen optimally, then $\tilde{M}_{t+1}^{std} = \tilde{M}_{t+1}^{long}$ for all $\phi \in [0,1)$.

Proof.

$$E_{t+1}\left[(1-\phi)\sum_{j=0}^{\infty}(1-\phi)\phi^j\beta^{j+1}\left(\frac{C_{t+1+j}}{C_t}\right)^{-\gamma}R_{p,t+1\to t+1+j}\right] \tag{29}$$

$$=\beta\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}\sum_{j=0}^{\infty}(1-\phi)\phi^j\underbrace{E_{t+1}\left[\beta^j\left(\frac{C_{t+1+j}}{C_{t+1}}\right)^{-\gamma}R_{p,t+1\to t+1+j}\right]}_{=1} \tag{30}$$

$$=\beta\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}\underbrace{\sum_{j=0}^{\infty}(1-\phi)\phi^j}_{=1} \tag{31}$$

from which the result follows by taking innovations of both sides. □

The result is stronger than saying that the standard and long-run CCAPM both price assets. It says that the stochastic discount factors implied by each model are the same random variable. When consumption is optimal, single-period consumption growth is related to news about future consumption growth and expected returns.

Optimal *portfolio* choice is not required for this result. Suppose portfolio weights differ from the optimal weights, but the consumer still chooses consumption optimally subject to these suboptimal weights. In that case, the standard and long-run CCAPM will still have identical pricing implications.

3.3 Special case: Constant propensity to save financial wealth

Consider the same environment as the general case, except now the consumer follows a rule of thumb of consuming

$$C_t = Y_t + (1-\phi)W_t \tag{32}$$

which means that the consumer consumes all labor income plus a fraction of financial wealth. This yields the simple expression for the accumulation of financial wealth

$$W_{t+1} = \phi R_{p,t+1} W_t \tag{33}$$

which leads to the exact expression

$$\frac{V_{w,t+1}}{C_t^{-\gamma}} = E_{t+1}\left[\sum_{j=0}^{\infty}(1-\phi)\phi^j\beta^j\left(\frac{C_{t+1+j}}{C_t}\right)^{-\gamma} R_{p,t+1\to t+1+j}\right] \tag{34}$$

In general, risk premia depend on exposure to long-run consumption and expected return shocks. In the special case with financial income only, where $C_t = (1-\phi)W_t$, it is possible to write consumption growth in terms of portfolio returns.

Corollary 3. *In the special case in which there is only financial wealth, so that labor income $Y_t = 0$, then*

$$\frac{V_{w,t+1}}{C_t^{-\gamma}} \propto R_{p,t+1}^{-\gamma}\cdot E_{t+1}\left[\sum_{j=0}^{\infty}(1-\phi)(\phi^{1-\gamma})^j\beta^j R_{p,t+1\to t+1+j}^{-(\gamma-1)}\right] \tag{35}$$

Proof. Use the substitution $C_{t+j}/C_t = W_{t+j}/W_t = \phi^j R_{p,t\to t+j}$ and simplify. □

This result is essentially identical to Campbell (1993), who builds an intertemporal CAPM in which exposure to the return on wealth carries a price of γ and exposure to discount rate shocks carries a price of $\gamma - 1$.[12]

The $R_{p,t+1}^{-\gamma}$ term reflects sensitivity to contemporaneous shocks to financial wealth (which in this case is all wealth). The $R_{p,t+1\to t+j}^{-(\gamma-1)}$ terms reflect exposure to expected return shocks. If $\gamma = 1$, the case of log utility, they do not matter. If $\gamma > 1$, they command a positive price since high expected returns depress the marginal utility of consumption faster than they increase the growth rate of consumption. If $\gamma < 1$, the opposite is true.

Corollary 4. *In the case in which $C_t = Y_t + (1-\phi)W_t$, it follows that*

$$\frac{V_{w,t+1}}{C_t^{-\gamma}} = E_{t+1}\left[\sum_{j=0}^{\infty}(1-\phi)\phi^j\beta^j\left(\lambda_{y,t}\frac{Y_{t+1+j}}{Y_t} + (1-\lambda_{y,t})\phi^{j+1}R_{p,t\to t+1+j}\right)^{-\gamma} R_{p,t+1\to t+1+j}\right] \tag{36}$$

[12] The result is the same to a first-order approximation, but there are slight differences in higher-order terms.

where

$$\lambda_{y,t} = \frac{Y_t}{Y_t + (1-\phi)W_t} = \frac{Y_t}{C_t} \tag{37}$$

Now the marginal value of wealth depends on both long-run labor income and expected returns. Before, in the case with financial wealth only, an expected return shock both raised the growth rate of financial wealth and reduced the marginal utility of consumption. The overall effect on the marginal value of wealth was ambiguous, depending upon γ.

In this case, an exogenous increase in expected returns has two effects on the marginal value of wealth. First, a faster growth rate of financial wealth raises the marginal value of wealth. Second, higher expected returns raise future consumption, lower marginal utility, and therefore reduce the marginal value of wealth. The magnitude of the second effect depends on the size of financial investments relative to labor income. If financial investments are much smaller than labor income, then labor income shocks will summarize the consumption shocks. This scenario yields an approximate two factor model consisting of shocks to future labor income and expected returns.[13]

4 Long-run models with general preferences

I generalize the long-run CCAPM to preferences beyond power utility. Just as single-period consumption growth may not price assets when consumption is not selected optimally, single-period marginal utility growth may not price assets either. I derive long-run models for general preferences and find that in general shocks to long-run marginal utility growth and expected returns are priced.

I pay particular attention to Epstein-Zin preferences and show that, to a first approximation, Epstein-Zin preferences and the long-run CCAPM have identical implications for the price of exposure to long-run consumption and expected return shocks.

[13] Since the portfolio manager can scale up zero-cost long-short positions without constraint, this result remains approximate even as the level of financial wealth goes to zero.

4.1 Equivalence of long-run and standard models under optimal consumption choice

I start by establishing the more general analog of Proposition 2, which showed that the long-run and standard CCAPM are equivalent if consumption is chosen optimally. In general, standard models use single-period marginal utility growth, which I denote M_{t+1}, as a stochastic discount factor (SDF). M_{t+1} is a valid SDF with optimal consumption choice, but may not be valid if consumption is not optimal. Analogous to the long-run CCAPM, I define a long-run SDF, which depends on news about long-run marginal utility growth and expected returns. I show below that the long-run SDF is equivalent to the standard SDF.

Proposition 3. *If M_{t+1} is a valid stochastic discount factor, then it can be equivalently expressed in long-run form as*

$$M_{t+1}^{long} = E_{t+1}\left[\sum_{j=0}^{\infty}(1-\phi)\phi^j M_{t\to t+1+j} R_{p,t+1\to t+1+j}\right] \tag{38}$$

for all $\phi \in (0,1)$, where $R_{p,t+1\to t+1+j}$ is the gross return on the agent's portfolio from $t+1$ to $t+1+j$ and $M_{t\to t+j} = \prod_{k=1}^{j} M_{t+k}$.

Proof.

$$\begin{aligned}
M_{t+1}^{long} &= E_{t+1}\left[\sum_{j=0}^{\infty}(1-\phi)\phi^j M_{t\to t+1+j} R_{p,t+1\to t+1+j}\right] \\
&= \sum_{j=0}^{\infty}(1-\phi)\phi^j M_{t+1} \underbrace{E_{t+1}[M_{t+1\to t+1+j} R_{p,t+1\to t+1+j}]}_{=1} \\
&= \underbrace{\sum_{j=1}^{\infty}(1-\phi)\phi^{j-1}}_{=1} M_{t+1} \\
&= M_{t+1}
\end{aligned}$$

□

As before, not only do M_{t+1} and M_{t+1}^{long} both price assets, but they are also the same random variable. Since it is frequently convenient to work with a log SDF, I also derive a logarithmic approximation.

Proposition 4. *If m_{t+1} is a log stochastic discount factor, then innovations to m_{t+1}, $\tilde{m}_{t+1} = (E_{t+1} - E_t)m_{t+1}$, can be expressed, to a first-order approximation, in long-run form as*

$$\tilde{m}_{t+1} \approx \sum_{j=0}^{\infty} \phi^j \tilde{m}_{t+1+j} + \sum_{j=1}^{\infty} \phi^j \tilde{r}_{p,t+1+j}$$

for all $\phi \in (0,1)$, where lowercase letters indicate logarithms. The approximation is exact if m_{t+1} and $r_{p,t+1}$ are jointly normal and homoskedastic.

Proof. See Appendix B.3. □

Shocks to the log SDF are a combination of shocks to future values of the SDF and expected returns. Shocks to the log SDF thus implicitly reflect substantial news about the future.

In a consumption-based model with optimal consumption choice, m_{t+1} is single-period log marginal utility growth. Shocks to m_{t+1} reflect shocks to long-run marginal utility growth and long-run expected returns. Optimal consumption choice ensures that shocks to future marginal utility growth and expected returns cancel out. Specifically, separating the contemporaneous marginal utility shock yields

$$\tilde{m}_{t+1} \approx \tilde{m}_{t+1} + \underbrace{\sum_{j=1}^{\infty} \phi^j \tilde{m}_{t+1+j} + \sum_{j=1}^{\infty} \phi^j \tilde{r}_{p,t+1+j}}_{=0} \tag{39}$$

which only leaves the contemporaneous marginal utility shock, $\tilde{m}_{t+1}$. Shocks to single-period marginal utility growth proxy for both the long-run marginal utility and expected return shocks.

However, as in the previous case, this equivalence will not hold if consumption is non-optimal. Next, I show that the long-run representation still prices assets when consumption choice is non-optimal.

4.2 Long-run versions of more general models

The argument for why the long-run CCAPM prices assets even when consumption is not optimal generalizes to more complicated preferences in essentially the same way. If a portfolio manager selects portfolio weights on behalf of a consumer with some single-period marginal utility growth process M_{t+1}, then the resulting risk factors will be long-run marginal utility growth and expected returns.

I retain the same setting as in the case of the long-run CCAPM outlined in Section 3.1. The only difference is that now the consumer has an arbitrary utility function U that is a function of the entire stream of state-contingent consumption. I assume that U is differentiable and strictly concave. The portfolio manager now optimizes

$$\max_{\{\theta_{i,t}\}} U(\{c_t\}) \tag{40}$$

taking the consumption policy as given.

Proposition 5. *In this environment, the portfolio manager's first-order condition for asset i is*

$$\sum_{j=1}^{\infty} E_t \left[M_{t \to t+j} \cdot C_{w,t+j} \cdot \left(\prod_{k=0}^{j-1} (1 - C_{w,t+k}) \right) \cdot R_{p,t+1 \to t+j} \cdot R^e_{i,t+1} \right] = 0 \tag{41}$$

where $C_{w,t} = \frac{\partial C_t}{\partial W_t}(W_t, X_t)$ is the marginal propensity to consume out of financial wealth. I define multi-period marginal utility growth as

$$M_{t \to t+j}(s) = \frac{\partial U / \partial C_{t+j}(s)}{\partial U / \partial C_t} \frac{1}{\pi(s)} \tag{42}$$

where s indicates a state of the world and $\pi(s)$ indicates its probability.

Proof. See Appendix B.4. □

The general long-run model matches the long-run CCAPM, except that marginal utility growth $M_{t \to t+j}$ replaces consumption growth. Generalizing beyond power utility, long-run marginal utility shocks replace long-run consumption shocks.

4.3 Epstein-Zin preferences and the long-run CCAPM

To a first-order approximation, both the long-run CCAPM and Epstein-Zin preferences price exposure to long-run consumption and expected return shocks the same. Thus the long-run CCAPM provides a different microfoundation for why long-run shocks are priced while short-horizon shocks are not. While the two models are similar to a first approximation, they are not exactly the same and differ to higher order approximations.

Proposition 6. *Innovations to the Epstein-Zin SDF*

$$\tilde{m}_{t+1}^{EZ} = -\frac{\theta}{\psi}\Delta\tilde{c}_{t+1} - (1-\theta)\tilde{r}_{w,t+1} \tag{43}$$

can be approximately expressed as

$$\tilde{m}_{t+1}^{EZ,long} = -\gamma\sum_{j=0}^{\infty}\rho^j\Delta\tilde{c}_{t+1+j} + \sum_{j=1}^{\infty}\rho^j\tilde{r}_{w,t+1+j} \tag{44}$$

where $\tilde{x} = (E_{t+1} - E_t)x$, $r_{w,t+1}$ *is the return on wealth, also including the return to human capital,* $\theta = (1-\gamma)/(1-1/\psi)$, *and* ρ *is a parameter from log linearization.*

Proof. Expressing the Epstein-Zin SDF in the long-run representation yields

$$\tilde{m}_{t+1}^{EZ,long} = -\frac{\theta}{\psi}\sum_{j=0}^{\infty}\phi^j\Delta\tilde{c}_{t+1} - (1-\theta)\sum_{j=0}^{\infty}\phi^j\tilde{r}_{w,t+1+j} + \sum_{j=1}^{\infty}\phi^j\tilde{r}_{w,t+1+j} \tag{45}$$

$$= -\frac{\theta}{\psi}\sum_{j=0}^{\infty}\phi^j\Delta\tilde{c}_{t+1} + \theta\sum_{j=1}^{\infty}\phi^j\tilde{r}_{w,t+1+j} - (1-\theta)\tilde{r}_{w,t+1} \tag{46}$$

which holds approximately for all $\phi \in (0,1)$ if Epstein-Zin preferences represent the true model. Wealth is the asset that pays consumption as its dividend, so applying the Campbell-Shiller decomposition yields

$$\tilde{r}_{w,t+1} = \sum_{j=0}^{\infty}\rho^j\Delta\tilde{c}_{t+1+j} - \sum_{j=1}^{\infty}\rho^j\tilde{r}_{w,t+1+j} \tag{47}$$

for a log-linearization parameter ρ close to 1. I set $\phi = \rho$ and substitute $\tilde{r}_{w,t+1}$ which yields

$$\tilde{m}_{t+1}^{EZ,long} = -\left(\frac{\theta}{\psi} + (1-\theta)\right)\sum_{j=0}^{\infty}\rho^j \Delta\tilde{c}_{t+1+j} + \sum_{j=1}^{\infty}\rho^j \tilde{r}_{w,t+1+j} \tag{48}$$

$$= -\gamma \sum_{j=0}^{\infty}\rho^j \Delta\tilde{c}_{t+1+j} + \sum_{j=1}^{\infty}\rho^j \tilde{r}_{w,t+1+j} \tag{49}$$

as desired. □

To a first approximation, Epstein-Zin preferences and the long-run CCAPM have the same implications for the price of exposure to long-run consumption and expected return shocks. There is a subtle difference in that expected return shocks in the long-run CCAPM correspond to returns on *financial* wealth whereas those for Epstein-Zin preferences correspond to returns on *aggregate* wealth, including human capital. This point aside, the two models are similar.[14]

Since this expression of the Epstein-Zin SDF is not standard, I will relate it to expressions which only use long-run consumption and to the intertemporal CAPM frameworks of Campbell (1993) and Campbell and Vuolteenaho (2004). These frameworks provide alternative ways of understanding the pricing implications of Epstein-Zin preferences by rewriting them to focus on different types of shocks.

Corollary 5. *Innovations to the Epstein-Zin SDF can be written in terms of long-run consumption only as*

$$\tilde{m}_{t+1}^{EZ} = -\gamma\Delta\tilde{c}_{t+1} - \left(\gamma - \frac{1}{\psi}\right)\sum_{j=1}^{\infty}\rho^j \Delta\tilde{c}_{t+1+j} \tag{50}$$

by substituting out expected return shocks using the fact that an Epstein-Zin agent endogenously adjusts consumption such that

$$\sum_{j=1}^{\infty}\rho^j \tilde{r}_{w,t+1+j} = \frac{1}{\psi}\sum_{j=1}^{\infty}\rho^j \Delta\tilde{c}_{t+1+j} \tag{51}$$

[14]This distinction may not be important for certain model specifications. For example, in a homoskedastic model, expected return shocks are identical across assets.

Corollary 6. *Innovations to the Epstein-Zin SDF can be written in the form of the intertemporal CAPM of Campbell (1993) as*

$$\tilde{m}_{t+1}^{EZ} = -\gamma \tilde{r}_{w,t+1} - (\gamma - 1) \sum_{j=1}^{\infty} \rho^j \tilde{r}_{w,t+1+j} \tag{52}$$

by substituting out consumption using the accounting identity

$$\sum_{j=0}^{\infty} \rho^j \Delta \tilde{c}_{t+1+j} = \tilde{r}_{w,t+1} + \sum_{j=1}^{\infty} \rho^j \tilde{r}_{w,t+1+j} \tag{53}$$

Corollary 7. *If there is no labor income, then innovations to the Epstein-Zin SDF can be written in the form of the intertemporal CAPM of Campbell and Vuolteenaho (2004) as*

$$\tilde{m}_{t+1}^{EZ} = -\gamma \sum_{j=0}^{\infty} \rho^j \Delta \tilde{d}_{t+1+j} + \sum_{j=1}^{\infty} \rho^j \tilde{r}_{m,t+1+j} \tag{54}$$

where Δd_t and $r_{m,t}$ are dividend growth and returns of the aggregate market. The result follows since $\Delta d_t = \Delta c_t$ and $r_{w,t} = r_{m,t}$ when there is no labor income.

5 Empirical evaluation of the long-run consumption CAPM

I empirically evaluate the long-run CCAPM in the context of a representative agent using aggregate consumption data. I test the unconditional implications of the model, which relate average returns to unconditional covariances with long-run consumption growth and expected returns on the stock market. I focus my analysis on equity strategies, represented by the Fama French 5 factors (market, value, size, profitability, and investment), momentum, and a broad set of 34 equity anomalies from Cho (2018). Since previous empirical work in the long-run risk literature has documented the importance of long-run consumption shocks, I particularly focus on long-run expected return shocks.

I present two novel findings. First, I document that expected return shocks raise the equity premium by 1.3 percentage points. Second, I show that exposure to expected return shocks is negatively priced in the cross-section of equity returns, as the theory predicts.

Additionally, and consistent with previous findings in the literature, I find that the long-run CCAPM improves upon the standard CCAPM. The use of long-run consumption growth reduces required risk aversion as documented by Gabaix and Laibson (2001) and Parker (2003). It also reduces pricing errors in the cross-section for size and value strategies as documented by Parker and Julliard (2005). However, I also find that more recent profitability- and investment-based strategies do not appear to be related to substantial consumption or expected return shocks, deepening the puzzle of why they earn high average returns.

5.1 Data

5.1.1 Consumption and market returns

My measure of aggregate consumption is real per capita nondurable consumption from the Bureau of Economic Analysis.

For the market return, I use the value-weighted aggregate stock market return from CRSP.[15] I convert it into real market returns using the PCE deflator from the Bureau of Economic Analysis. The exception is one analysis using stock returns from 1927Q1-2017Q4. The PCE deflator is not available in the early part of the sample, so I use the urban consumer price index from the Bureau of Labor Statistics instead. The two measures of real stock returns are almost identical (correlation of 0.99) in the sample over which they overlap. Quarterly correlations between the two inflation measures are 0.79 in the sample over which they overlap and annual correlations are 0.97.

My measure of the nominal 3 month risk-free rate comes from Kenneth French's website.[16] For analyses which involve the real risk-free rate, I similarly adjust using the

[15]Center for Research in Security Prices, CRSP 1925 US Stock Database, Wharton Research Data Services, `http://www.whartonwrds.com/datasets/crsp/`.

[16]The data library on Kenneth French's website can be accessed at `http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html`.

PCE deflator.

5.1.2 Financial factors

The financial factors that I use are the Fama French 5 factor model plus momentum. (Fama and French, 2015; Jegadeesh and Titman, 1993) I use these as a parsimonious representation of the cross-section of stock returns.

My main specification uses the financial factors themselves, which are

- Market: Aggregate stock market minus the 3 month Treasury rate
- Size: Small firms minus big firms
- Value: High book-to-market (value) firms minus low book-to-market (growth) firms
- Profitability: High operating profitability firms minus low operating probability firms
- Investment: Low asset growth firms minus high asset growth firms
- Momentum: Stocks with high prior returns minus stocks with low prior returns

Alternative specifications use 10 portfolios sorted on these characteristics or 25 portfolios double sorted on size and the characteristic. In the alternative specifications, I represent the market factor with decile portfolios sorted on prior CAPM beta. I obtain all data on returns from Kenneth French's website.

5.1.3 Equity anomaly portfolios

I test the predictions of the long-run CCAPM on a set of 34 equity anomalies from Cho (2018). The sample is from 1973Q1 to 2017Q4. The anomalies reflect trading strategies based on: Beta arbitrage, Ohlson's O-score, size, post-earnings announcement drift (SUE and CAR3), value, 36-month momentum, long-run reversals, short-term reversals, momentum, annual sales growth, industry-adjusted change in employees, accruals, industry-adjusted book-to-market, industry momentum, industry-adjusted firm size, industry-adjusted cash flow-to-price ratio, Piotroski's F-score, idiosyncratic volatility, price delay, failure probability, asset growth, net issuance, seasonality, industry-adjusted change in profit margin, industry-adjusted change in asset turnover, investment, return on market equity, return on book equity, return on assets, asset turnover, gross margins, gross profitability, and industry-adjusted reversals.

5.2 Making the long-run CCAPM testable

The log-linearized version of the long-run CCAPM implies that, in equilibrium, expected returns approximately follow

$$E_t[R^e_{i,t+1}] = \gamma Cov_t\left(\sum_{j=0}^{\infty}\phi^j \Delta c_{t+1+j}, R^e_{i,t+1}\right) - Cov_t\left(\sum_{j=1}^{\infty}\phi^j r_{p,t+1+j}, R^e_{i,t+1}\right) \tag{55}$$

where $R^e_{i,t+1}$ is an excess return i. There are some empirical obstacles that need to be overcome before I can empirically test the long-run CCAPM.

First, the long-run CCAPM includes infinite sums which I cannot directly observe in practice. I follow an approach similar to Malloy et al. (2009) and truncate the sums after three years. In practice, the covariances are negligible beyond that horizon, so results are similar using longer horizons. Truncation may underestimate long-run covariances in the presence of small, but persistent, responses of consumption or expected returns to realized returns. In that case, the effects that I report may be larger than they appear.

Estimating truncated covariances contrasts with the empirical approach of estimating a vector autoregression (VAR) used by the intertemporal CAPM (Campbell, 1993; Campbell and Vuolteenaho, 2004; Campbell et al., 2018) and the long-run risk (Hansen et al., 2008; Bansal et al., 2009) literatures. VARs are better able to capture long-run impacts, but also require auxiliary assumptions on which variables to include. In contrast, the truncated covariances do not require any similar assumptions.

Second, I need to pick a value for ϕ. In theory, ϕ should be the propensity to save financial wealth. Equivalently, $1-\phi$ reflects the rate at which a consumer withdraws from their brokerage account. In an institutional context, $1-\phi$ corresponds to the spending rate out of an endowment. I pick an annualized $\phi = 0.95$ as an a priori reasonable value, which corresponds to a withdrawal rate of 5%, but note that the main empirical results are not sensitive to the exact choice of ϕ.

Third, the current model is conditional. I address this issue by taking unconditional expectations of the non-linear model and log-linearizing the unconditional model. The conditional non-linear model is

$$E_t \left[E_{t+1} \left[\sum_{j=0}^{\infty} (1-\phi)\phi^j \beta^j \left(\frac{C_{t+1+j}}{C_t} \right)^{-\gamma} R_{p,t+1\to t+1+j} \right] R^e_{i,t+1} \right] = 0 \tag{56}$$

which, after applying the law of iterated expectations and taking an unconditional expectation, yields

$$E \left[\left(\sum_{j=0}^{\infty} (1-\phi)\phi^j \beta^j \left(\frac{C_{t+1+j}}{C_t} \right)^{-\gamma} R_{p,t+1\to t+1+j} \right) R^e_{i,t+1} \right] = 0 \tag{57}$$

which I then log-linearize to obtain a linear relationship between unconditional returns and covariances.[17]

Finally, expected return shocks in the model correspond to the portfolio return. I use the return on the aggregate stock market in its place.

My main empirical specification is

$$E_T[R^e_{i,t+1}] = \gamma Cov_T \left(\sum_{j=0}^{12} \phi^j \Delta c_{t+1+j}, R^e_{i,t+1} \right) - Cov_T \left(\sum_{j=1}^{12} \phi^j r_{m,t+1+j}, R^e_{i,t+1} \right) \tag{58}$$

where E_T and Cov_T represent the sample mean and covariances, t indexes quarters, and $\phi = 0.95^{1/4}$.

5.3 The equity premium and expected return shocks

It is a well-known puzzle that the equity premium is substantially higher than the standard CCAPM with reasonable risk aversion predicts. (Mehra and Prescott, 1985) In the long-run CCAPM, the equity premium depends on both the stock market's exposure to

[17] If $E[R^e_t \exp(m_t)] = 0$, then log-linearizing $\exp(m) \approx \exp(E[m_t])(1 + (m_t - E[m]))$ yields $E[R^e_t] \approx -Cov(R^e_t, m_t)$

long-run consumption and expected return shocks. While many papers have argued that looking at longer horizons of consumption growth can help to explain the equity premium (Gabaix and Laibson, 2001; Parker, 2001; Bansal and Yaron, 2004), none have also examined the importance of expected return shocks.[18] Here I make a novel contribution in showing that equities are exposed to an additional risk, expected return shocks, and that exposure to this risk can explain 1.3 percentage points of the equity premium (with a 95% confidence interval ranging from 0.4 to 3.2 percentage points).

In theory, the standard CCAPM implies that the equity premium is approximately

$$E[R^e_{m,t}] = \gamma Cov(\Delta c_t, R^e_{m,t}) \tag{59}$$

which only depends on exposure to single-period consumption growth. The long-run CCAPM predicts that the equity premium is

$$E[R^e_{m,t}] = \gamma Cov\left(\sum_{j=0}^{\infty} \phi^j \Delta c_{t+j}, R^e_{m,t}\right) - Cov\left(\sum_{j=1}^{\infty} \phi^j r_{m,t+j}, R^e_{m,t}\right) \tag{60}$$

which depends on exposure to long-run consumption and expected returns. The long-run risks literature has focused on the ability of the long-run consumption factor to explain the equity premium, but has not paid attention to expected return shocks. Controlling for exposure to consumption risk, assets which crash when expected returns rise should command a higher premium.

It may seem counterintuitive that the stock market's exposure to expected return shocks should raise the equity premium. If the stock market crashes when expected returns rise, then the stock market mean reverts. At first glance, it may be unclear why mean reversion leads to a higher equity premium.

However, I am considering exposure to expected return shocks *controlling for consumption exposure*. Higher expected returns will, all else equal, also translate into higher expected consumption growth. Mean reversion reduces risk by reducing covariance with

[18] Certain specifications in the long-run risks literature implicitly include the effect of expected return shocks by imposing the restriction $\sum_{j=1}^{\infty} \phi^j \tilde{r}_{m,t+j} = \frac{1}{\psi} \sum_{j=1}^{\infty} \phi^j \Delta \tilde{c}_{t+j}$, which reduces the price of exposure to long-run consumption risk by the inverse of the elasticity of intertemporal substitution, $\frac{1}{\psi}$.

consumption. When the stock market crashes, expected returns rise which, all else equal, raises expected consumption growth. But financial wealth is not the only component of consumption. Expected labor income also falls when the stock market crashes, which leads to a negative covariance of stock returns with consumption growth.

In order to estimate expected return shocks more precisely, I use data on stock returns from 1927Q1-2017Q4. Figure 3 plots the cumulative covariance of the market excess return with future market returns up to a five year horizon. The covariance becomes gradually more negative over time and the difference becomes statistically significant a little before five years.

The 5-year (20-quarter) covariance is

$$Cov_T\left(\sum_{j=1}^{20}\phi^j r_{m,t+j}, R^e_{m,t}\right) = -1.3$$

where $R^e_{m,t}$ is the excess return on the stock market and $r_{m,t+j}$ is the log real market return (not the excess return). The estimate has a bootstrapped 95% confidence interval of [-0.4, -3.2]. Since the price of exposure to expected return shocks is negative, this covariance indicates that the equity premium should be 1.3 percentage points higher due to the market's exposure to expected return shocks.

5.4 Equity anomalies and expected return shocks

The long-run CCAPM predicts that exposure to long-run expected return shocks should be negatively priced, controlling for exposure to long-run consumption shocks. I confirm this prediction of the theory across a broad set of equity anomaly portfolios, which are known to be difficult to price using consumption.

Specifically, I test the model on 34 equity market anomaly portfolios from Cho (2018). The sample runs from 1973Q1 to 2017Q4. I describe the anomalies in Section 5.1.3.

The key question is whether expected return shocks are negatively priced in addition to consumption shocks. I run the unrestricted cross-sectional regression

$$E_T[R^e_{i,t}] = a + \gamma Cov_T\left(\sum_{j=0}^{12}\phi^j \Delta c_{t+j}, R^e_{i,t}\right) + \lambda Cov_T\left(\sum_{j=1}^{12}\phi^j r_{m,t+j}, R^e_{i,t}\right) \tag{61}$$

where the theory predicts $a = 0$ and $\lambda = -1$. I use weighted least squares using the inverse of each asset's variance as a weight.[19] I compute 95% confidence intervals using a stationary block bootstrap.[20]

The cross-sectional regression qualitatively supports the predictions of the long-run CCAPM. Table 1 shows the results of the regression. The point estimate of $\lambda = -2.7$ is statistically significant and indicates that long-run expected return shocks are negatively priced. The negative sign is consistent with the theory. The magnitude is larger than the prediction of $\lambda = -1$, although -1 lies within the 95% confidence interval.

The intercept is positive and statistically different than zero, so the theory does not explain the average level of anomaly returns. Instead, it indicates that as one moves from a portfolio with a low exposure to expected return shocks to one with a high exposure, the average returns on the portfolio change as well.

5.5 Fama French 5 factor model and momentum

Finally, I examine the extent to which the long-run CCAPM can explain the returns on the Fama French 5 factors of the market, size, value, profitability, and investment along with the momentum factor.

5.5.1 Understanding risk exposures

Consistent with earlier literature, I show that exposure to long-run consumption growth is typically larger than exposure to contemporaneous consumption growth. However, I also document exposure to expected return shocks has a non-trivial effect on expected returns.

[19]The estimator is identical to linear GMM using a weighting matrix which perfectly matches the estimated covariances and otherwise uses an inverse variance weighting matrix.

[20]For each iteration of the bootstrap, I sample observations in blocks of random size. The block sizes follow a geometric distribution with a mean of 6 years. Within each bootstrapped sample, I compute the statistics of interest. I report the 95% confidence intervals based on the bias-corrected distribution of statistics.

I also show that the momentum factor is particularly exposed to expected return shocks. While the exposure is not large enough to quantitatively explain the average return of the momentum factor, it provides a way of relating momentum to a consumption-based asset pricing model.

Risk in the long-run CCAPM is driven by covariances with news about long-run consumption growth and expected returns. Table 2 shows covariances with contemporaneous consumption as well 3-year-ahead consumption and stock market returns (excluding the contemporaneous return to reflect expected return shocks). Figure 4 graphically plots these covariances for easier comparison.

The first takeaway is that long-run consumption covariances are uniformly larger in magnitude than contemporaneous consumption covariances. This result is consistent with previous empirical work, including Gabaix and Laibson (2001) and Parker and Julliard (2005). In some cases, the contemporaneous covariance merely understates the long-run covariance, such as for the market. In other cases, such as for value, there is almost no link with contemporaneous consumption growth, but there is a link with consumption growth over longer horizons.

The second takeaway is that expected return shocks seem to matter for certain strategies, although many results are not statistically significant at the 95% level due to limited statistical power. As shown before, exposure to expected return shocks pushes the equity premium upward, although the point estimate becomes smaller and statistically insignificant when the sample is limited to after 1963Q3. Interestingly, the momentum factor is particularly exposed to expected return shocks. Quantitatively, expected return shocks should raise average returns to momentum by 75 basis points. This effect is not large enough to explain the approximately 8% per year return to momentum strategies, but it does suggest a source of risk to which momentum strategies are exposed.

The final takeaway is that the high returns to profitability and investment strategies are even more puzzling than would appear at first glance. Both strategies have limited exposure to consumption growth over short or long horizons. And the point estimates indicate that they are positively related to expected return shocks, which would lead to a *lower* risk premium.

These estimated risk exposures are not sensitive to truncation at a three-year horizon. To show this, I plot cumulative covariances of each factor with long-run consumption growth and stock market returns. For a given factor excess return R_t^e and consumption Δc_t, I plot $Cov(R_t^e, \sum_{j=0}^{h} \phi^j \Delta c_{t+j})$ as the horizon h varies. For stock returns, I exclude the contemporaneous variable and plot $Cov(f_t, \sum_{j=1}^{h} \phi^j r_{m,t+j})$ in order to reflect shocks to expected returns. Figure 6 shows that cumulative consumption covariances typically plateau after around three years, with the exception of momentum which reverses over longer horizons. Visually, it is apparent that the cumulative covariances are not too sensitive to the choice of a three year horizon. Figure 7 similarly shows that the exact choice of horizon is not an important driver for the qualitative results on expected return shocks.

5.5.2 Accounting for persistence

Here I consider an alternative specification which can handle persistence processes better. I estimate the consumption and expected return covariances as

$$\frac{1}{1-\lambda_c \phi} Cov_T \left(\Delta c_t + \sum_{j=1}^{12} \phi^j u_{c,t+j}, R_{i,t}^e \right) \tag{62}$$

$$\frac{1}{1-\lambda_m \phi} Cov_T \left(\phi r_{m,t+1} + \sum_{j=2}^{12} \phi^j u_{m,t+j}, R_{i,t}^e \right) \tag{63}$$

where λ_c and λ_m are estimated coefficients from an AR(1) model for consumption growth and market returns respectively and $u_{c,t+j}$ and $u_{m,t+j}$ are the respective forecast errors relative to an AR(1) model. Appendix C.1 relates this alternative specification to the main specification. It shows that the two are equivalent when the horizon goes to zero, but that this alternative specification may be preferable in the presence of a persistent variable.

I also consider a specification which further breaks down the real market return into

$$r_{m,t} = r_{f,t} + \underbrace{(r_{m,t} - r_{f,t})}_{r^e_{m,t}} \tag{64}$$

where $r_{f,t}$ is the log real risk-free rate from time $t-1$ to t and $r^e_{m,t}$ is the log market excess return over the same time period. This approach has the benefit of better accounting for the persistence of the risk-free rate. I then separately estimate covariance of returns with the risk-free rate and market excess returns. I estimate a separate persistence parameter λ and forecast errors u_t for each and truncate at the same three-year horizon.

Table 3 shows the covariance estimate from this procedure. The estimates are broadly in line with truncating the covariances over three years, so I continue to use the truncated covariances for the cross-sectional regressions.

5.5.3 Fully restricted cross-sectional regressions

I first compare the fully restricted versions of the standard and long-run CCAPM. I do not include an intercept and restrict the price of exposure to expected return shocks as implied by the theory. Therefore both models have only one degree of freedom: The estimate of γ. For the standard CCAPM, I estimate

$$E_T[R^e_{i,t+1}] = \gamma Cov_T(\Delta c_{t+1}, R^e_{i,t+1}) \tag{65}$$

and for the long-run CCAPM, I estimate

$$E_T[R^e_{i,t+1}] = \gamma Cov_T\left(\sum_{j=0}^{12} \phi^j \Delta c_{t+1+j}, R^e_{i,t+1}\right) - Cov_T\left(\sum_{j=1}^{12} \phi^j r_{m,t+1+j}, R^e_{i,t+1}\right) \tag{66}$$

I separately estimate cross-sectional regressions for the Fama-French 3 factor model, 5 factor model, and the 5 factor model plus momentum. I run the cross-sectional regression in the same manner as I do for the equity anomaly portfolios.

Table 4 shows the results of these cross-sectional regressions. The long-run CCAPM sharply reduces the level of risk aversion required to match the data and also substantially reduces the size of the pricing errors.

5.5.4 Unrestricted cross-sectional regressions

I further consider a cross-sectional regression specification which takes the theory less literally. I use the same long-run consumption and expected return risk exposures, but now include an intercept term and allow a free parameter for estimating the price of exposure to expected return shocks. The unrestricted estimates are not far from the predictions of the theory.

For the CCAPM, I estimate

$$E_T[R^e_{i,t}] = a + \gamma Cov_T(\Delta c_t, R^e_{i,t}) \tag{67}$$

and for the long-run CCAPM I estimate

$$E_T[R^e_{i,t}] = a + \gamma Cov_T\left(\sum_{j=0}^{12} \phi^j \Delta c_{t+j}, R^e_{i,t}\right) + \lambda Cov_T\left(\sum_{j=1}^{12} \phi^j r_{m,t+j}, R^e_{i,t}\right) \tag{68}$$

where a is the intercept and λ is the price of exposure to the expected return shock. The theory implies that $a = 0$ and $\lambda = -1$.

Instead of using the financial factors as my test assets, I use decile portfolios constructed from the same characteristics. For example, instead of using the momentum factor, I use decile portfolios sorted based on prior returns. The characteristics associated with each financial factor are straightforward, except for the market factor, which I map to decile portfolios sorted on prior CAPM beta. I run the cross-sectional regressions separately for each group of portfolio and pooled together.

Following Campbell et al. (2018), I use excess returns over the aggregate stock market. This addresses concerns that short-horizon risk-free rates may be driven by liquidity premia. (Krishnamurthy and Vissing-Jorgensen, 2012; Nagel, 2016)

Table 5 shows the result of the unrestricted cross-sectional regressions for the standard and long-run CCAPM. The long-run CCAPM typically has smaller estimated intercepts, risk aversion parameters, and pricing errors compared to the standard CCAPM. Focusing on the long-run CCAPM, the estimates are broadly in line with the theory.

The price of exposure to expected return shocks, λ, is negative across all portfolios, with the exception of investment portfolios. Due to the noisiness of the data, the estimates of λ are only statistically significant for momentum portfolios and for all of the portfolios pooled together.

The momentum regression suggests that expected return shocks may provide a way of relating momentum strategies to a consumption-based asset pricing model. The intercept is zero, so the long-run CCAPM does not fully account for the average level of these momentum returns relative to the stock market. Instead, these results show that greater exposure to long-run expected return shocks is associated with higher average returns. For intuition why this may be the case, consider the perspective of an arbitrageur betting on momentum who is worried about expected return shocks. The arbitrageur will require higher average returns for investing in strategies which are more exposed to expected return shocks, but may not bring the overall alpha across all momentum strategies to zero. This situation corresponds to a positive intercept and priced exposure to expected return shocks.

Estimates of γ are positive, except for momentum and profitability. However Table 2 indicates that both momentum and probability are the only two factors with negative exposures to long-run consumption. The negative estimate for risk aversion comes from seeing higher average returns associated with less exposure to long-run consumption risk. Plausible estimation error of the consumption covariances could easily flip the sign.

Overall, the unrestricted cross-sectional regressions are qualitatively consistent with the predictions of the long-run CCAPM.

6 Conclusion

I address the question of which risk-return relationship we should expect to see in consumption-based asset pricing models when allowing consumers to make mistakes. I build a model which separates consumption and portfolio choice. A benevolent portfolio manager optimally selects portfolio weights on behalf of a consumer with power utility preferences who may make mistakes, which I model by allowing for an arbitrary consumption policy. The model contains the standard model, in which consumers spend optimally, as a special case.

When consumers are allowed to make mistakes, expected returns may not depend on an asset's exposure to single-period consumption growth. However, I show that expected returns will robustly depend on exposure to long-run consumption and expected return shocks. I thus call this model the "long-run CCAPM."

I show that the long-run CCAPM and Epstein-Zin preferences have, to a first-order approximation, equivalent implications for the price of exposure to these two long-run shocks. Thus consumer mistakes are an alternative microfoundation for why long-run risks are priced.

I then empirically evaluate the long-run CCAPM with a particular focus on showing the importance of expected return shocks. I provide novel evidence that expected return shocks can account for 1.3 percentage points of the equity premium. I also show that these shocks are negatively priced in the cross-section of equity anomaly returns, which have been difficult to explain with existing consumption-based models.

For future work, the framework in this paper can be extended in many ways. One extension is to develop general equilibrium models which separate the consumer and the portfolio manager. In these models, the consumer's behavior determines the level of expected returns while the portfolio manager's behavior determines risk premia. This framework aligns with a view of the real world in which borrowing and saving by large groups of potentially less-informed consumers determine the level of average returns while arbitrage by sophisticated fund managers eliminates differences in returns which are not due to risk. This separation could provide another way of addressing puzzles in the asset pricing literature, such as the low volatility of risk-free rates in spite of high implied variation in the marginal value of wealth.

Another extension is to study second-order shocks, i.e., volatility shocks, in addition to first-order shocks. Preliminary results indicate qualitatively important differences in how the long-run CCAPM and Epstein-Zin preferences price volatility which may help to resolve empirical puzzles. Additionally, this approach may address problems with the existence of the Epstein-Zin value function when volatility is stochastic, as highlighted by Campbell et al. (2018).

This paper is not meant to challenge alternate models of expected returns. The long-run CCAPM is simply a different way of reflecting the first-order conditions of a consumer. In general equilibrium, the first-order conditions of firms (Cochrane, 1991; Zhang, 2017), financial intermediaries (Adrian et al., 2014; He et al., 2017; He and Krishnamurthy, 2013), and other market participants will hold simultaneously. Multiple models based on the first-order conditions of different market participants should simultaneously explain asset returns. This paper's technique of iterating the marginal value of wealth forward can potentially be applied to constructing long-run versions of non-consumption-based models as well.

More broadly, the main research direction in consumption-based asset pricing has addressed failures of simple models such as the CCAPM by building more complicated models of preferences and environments while retaining the assumption that consumers respond to shocks instantly and optimally (Cochrane, 2017). Here, I offer an additional option of allowing for consumers to make mistakes, which can help consumption-based models to link asset returns to macroeconomic risk in a realistic way.

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Table 1: Cross-sectional regression for equity anomaly portfolios

Cross-sectional regressions using 34 equity anomaly portfolios from Cho (2018). "CCAPM" reports the results of the cross-sectional regression

$$E_T[R^e_{i,t}] = a + \gamma Cov_T\left(\Delta c_t, R^e_{i,t}\right)$$

"Long-run Con" reports the results of the cross-sectional regression

$$E_T[R^e_{i,t}] = a + \gamma Cov_T\left(\sum_{j=0}^{12} \phi^j \Delta c_{t+j}, R^e_{i,t}\right)$$

and "Long-run CCAPM" reports the results of the cross-sectional regression

$$E_T[R^e_{i,t}] = a + \gamma Cov_T\left(\sum_{j=0}^{12} \phi^j \Delta c_{t+j}, R^e_{i,t}\right) + \lambda Cov_T\left(\sum_{j=1}^{12} \phi^j r_{m,t+j}, R^e_{i,t}\right)$$

The long-run CCAPM predicts that $\lambda = -1$. Bootstrapped 95% confidence intervals are underneath all estimates. Observations are quarterly. The sample is 1973Q1 to 2017Q4.

	$\hat{a}$	$\hat{\gamma}$	$\hat{\lambda}$	MAPE
CCAPM	3.50	-42.61		3.41
	[2.45, 4.48]	[-146.86, 61.40]		[1.95, 4.05]
Long-run Con	3.65	-5.38		3.42
	[2.87, 4.65]	[-23.54, 22.72]		[2.01, 4.07]
Long-run CCAPM	3.77	-7.86	-2.73	3.11
	[2.95, 4.95]	[-33.37, 12.02]	[-6.41, -0.25]	[1.64, 3.88]

Table 2: Consumption and expected return risk exposures

The standard CCAPM predicts that the risk premium is

$$E[R_t^e] = \gamma Cov(\Delta c_t, R_t^e)$$

and the long-run CCAPM predicts

$$E[R_t^e] = \gamma Cov\left(\sum_{j=0}^{\infty} \phi^j c_{t+j}, R_t^e\right) - Cov\left(\sum_{j=1}^{\infty} \phi^j r_{m,t+j}, R_t^e\right)$$

where R_t^e is an excess return, Δc_t is the change in log consumption, and $r_{m,t}$ is the log market return. This table shows related sample analogs of these quantities which truncate the infinite sums at three years. Observations are quarterly. I multiply returns by 4 so they are approximately annualized percentages. Bootstrapped 95% confidence intervals are underneath each estimate. The sample period is 1963Q3 to 2017Q4.

Factor	$E_T[R_t^e]$	$Cov_T(\Delta c_t, R_t^e)$	$Cov_T(\sum_{j=0}^{12} \phi^j \Delta c_{t+j}, R_t^e)$	$Cov_T(\sum_{j=1}^{12} \phi^j r_{m,t+j}, R_t^e)$
Market	6.65	0.04	0.18	-0.34
	[3.32, 10.05]	[0.01, 0.08]	[0.06, 0.31]	[-1.17, 0.50]
Size	3.07	0.02	0.07	-0.65
	[-0.11, 6.23]	[-0.01, 0.04]	[-0.04, 0.20]	[-1.59, 0.14]
Value	4.33	0.00	0.07	0.11
	[1.92, 6.74]	[-0.02, 0.02]	[0.01, 0.19]	[-0.48, 0.87]
Profitability	3.05	-0.01	-0.03	0.36
	[0.95, 5.16]	[-0.03, 0.00]	[-0.09, 0.03]	[0.00, 1.04]
Investment	3.53	-0.01	0.02	0.19
	[1.49, 5.56]	[-0.03, 0.01]	[-0.03, 0.11]	[-0.29, 0.85]
Momentum	8.01	0.00	-0.03	-0.76
	[4.43, 11.57]	[-0.03, 0.03]	[-0.12, 0.09]	[-1.80, 0.20]

Table 3: Alternative consumption and expected return risk exposures

This table shows estimates for long-run consumption and expected return covariance using an alternative method which accounts for long-run persistence. Section 5.5.2 outlines the method. The "3Y Con" and "3Y Mkt" columns show estimates for the consumption covariance $Cov(\sum_{j=0}^{\infty} \phi^j \Delta c_{t+j}, R^e_{i,t})$ and expected return $Cov(\sum_{j=1}^{\infty} \phi^j r_{m,t+j}, R^e_{i,t})$ for each financial factor. The "3Y Mkt (sep)" column shows an alternative expected return estimate based on separating real market returns into the excess market return and the real risk-free rate. The "3Y Mkt Excess" and "3Y Real Rf" columns show covariances with each component of the market return. "3Y Mkt (sep)" is the sum of "3Y Mkt Excess" and "3Y Real Rf." Observations are quarterly. The excess returns $R^e_{i,t}$ are multiplied by 4. The sample period is 1963Q3 to 2017Q4.

Factor	3Y Con	3Y Mkt	3Y Mkt (sep)	3Y Mkt Excess	3Y Real Rf
Market	0.23	-0.33	-0.25	-0.47	0.22
	[0.07, 0.41]	[-1.27, 0.44]	[-1.16, 0.58]	[-1.45, 0.23]	[0.07, 0.57]
Size	0.11	-1.05	-1.03	-0.98	-0.05
	[0.00, 0.27]	[-1.96, -0.32]	[-1.95, -0.27]	[-1.80, -0.28]	[-0.27, 0.14]
Value	0.09	-0.08	-0.03	-0.11	0.08
	[0.03, 0.23]	[-0.72, 0.79]	[-0.65, 0.85]	[-0.79, 0.76]	[-0.11, 0.34]
Profitability	-0.04	0.30	0.28	0.29	-0.01
	[-0.11, 0.03]	[-0.08, 1.06]	[-0.11, 1.03]	[-0.10, 1.06]	[-0.17, 0.12]
Investment	0.02	0.11	0.11	0.14	-0.03
	[-0.03, 0.12]	[-0.47, 0.96]	[-0.48, 0.99]	[-0.40, 0.97]	[-0.17, 0.11]
Momentum	-0.06	-0.93	-0.93	-1.09	0.16
	[-0.20, 0.07]	[-2.06, 0.11]	[-2.05, 0.12]	[-2.36, -0.01]	[-0.02, 0.46]

Table 4: Restricted cross-sectional regressions for Fama French 5 factors and momentum "CCAPM" reports the results of the cross-sectional regression for the standard consumption CAPM

$$E_T[R^e_{i,t}] = \gamma Cov_T(\Delta c_t, R^e_{i,t})$$

and "LR-CCAPM" reports the results of the cross-sectional regression for the long-run CCAPM

$$E_T[R^e_{i,t}] = \gamma Cov_T\left(\sum_{j=0}^{12} \phi^j \Delta c_{t+j}, R^e_{i,t}\right) - Cov_T\left(\sum_{j=1}^{12} \phi^j r_{m,t+j}, R^e_{i,t}\right)$$

The regressions impose the theoretical restrictions that the intercept is zero and the price of exposure to expected return shocks is -1. Both regressions use weighted least squares with an inverse variance weighting matrix. The table below reports estimates of γ for each regression and the mean absolute pricing error (MAPE). Cross-sectional regressions include the market, size, and value factors (FF3), the FF3 plus profitability and investment (FF5), and the FF5 plus momentum (FF5 + Mom). Bootstrapped 95% confidence intervals are underneath each estimate. Observations are quarterly. The sample period is 1963Q3-2017Q4.

	$\hat{\gamma}$		MAPE	
Assets	CCAPM	L-H CCAPM	CCAPM	L-H CCAPM
FF 3	161.85	37.48	1.69	0.95
	[-77.04, 324.36]	[-5.24, 63.56]	[0.00, 2.69]	[0.00, 1.59]
FF 5	43.50	34.16	3.76	1.91
	[-116.23, 157.44]	[-8.04, 70.94]	[2.30, 4.96]	[0.00, 2.07]
FF 5 + Mom	41.47	29.31	4.49	3.09
	[-86.07, 179.76]	[5.99, 75.58]	[3.14, 5.98]	[0.77, 3.64]

Table 5: Unrestricted cross-sectional regressions for Fama French 5 factors and momentum

"CCAPM" reports the results of cross-sectional regressions of the form

$$E_T[R^e_{i,t}] = a + \gamma Cov_T(\Delta c_t, R^e_{i,t})$$

and "L-H CCAPM" reports the results of

$$E_T[R^e_{i,t}] = a + \gamma Cov_T\left(\sum_{j=0}^{12} \phi^j \Delta c_{t+j}, R^e_{i,t}\right) + \lambda Cov_T\left(\sum_{j=1}^{12} \phi^j r_{m,t+j}, R^e_{i,t}\right)$$

where $R^e_{i,t}$ is a return in excess of the stock market. Observations are quarterly, but I multiply the excess return by 4 to aid in interpreting it in annualized units. Theory implies that $a = 0$ and $\lambda = -1$, but I do not impose these restrictions. MAPE is the mean absolute pricing error or, equivalently, the mean absolute value of the alphas. Under each estimate is a bootstrapped 95% confidence interval. The sample is from 1963Q3 to 2017Q4.

CCAPM	$\hat{a}$	$\hat{\gamma}$		MAPE
CAPM Beta	0.80	18.47		0.53
	[-0.02, 1.44]	[-83.97, 107.63]		[0.00, 0.72]
Value	1.03	123.12		0.77
	[0.16, 2.05]	[81.39, 340.72]		[0.00, 0.92]
Size	0.51	160.03		0.54
	[-0.67, 1.10]	[-84.48, 358.44]		[0.00, 0.73]
Profitability	0.20	-61.49		0.47
	[-0.27, 0.50]	[-163.78, 10.95]		[0.00, 0.59]
Investment	0.87	-8.99		0.97
	[-0.02, 1.53]	[-140.28, 103.73]		[0.09, 1.28]
Momentum	0.67	5.84		2.67
	[-0.41, 1.76]	[-77.00, 151.14]		[1.72, 4.01]
All	0.76	32.03		1.33
	[0.27, 1.26]	[-26.33, 124.49]		[0.64, 1.63]

L-H CCAPM	$\hat{a}$	$\hat{\gamma}$	$\hat{\lambda}$	MAPE
CAPM Beta	0.67	9.04	-1.41	0.30
	[0.05, 1.21]	[-21.74, 29.94]	[-5.54, 0.78]	[0.00, 0.35]
Value	0.77	19.95	-1.69	0.76
	[-0.01, 1.91]	[-11.43, 50.54]	[-6.23, 4.30]	[0.00, 1.06]
Size	0.42	0.84	-4.37	0.59
	[0.08, 0.90]	[-63.04, 44.60]	[-11.43, 1.91]	[0.03, 0.91]
Profitability	0.22	-18.83	-0.84	0.59
	[-0.09, 0.49]	[-58.34, 18.64]	[-3.91, 2.59]	[0.00, 0.84]
Investment	0.73	40.21	3.78	0.41
	[-0.07, 1.25]	[39.72, 106.86]	[1.73, 10.38]	[0.00, 0.24]
Momentum	1.18	-11.58	-4.98	1.58
	[0.38, 2.59]	[-53.52, 26.07]	[-10.68, -1.60]	[0.45, 2.33]
All	0.65	4.12	-2.23	1.12
	[0.36, 1.05]	[-13.75, 25.98]	[-5.14, -0.02]	[0.39, 1.27]

Figure 3: Stock market's exposure to expected return shocks

Under the long-run CCAPM, the part of the equity premium arising from exposure to long-run expected return shocks is $-Cov(\sum_{j=1}^{\infty} \phi^j r_{m,t+j}, R^e_{m,t})$. This figure plots the sample analog, showing covariance of current market excess returns with future market returns, $Cov_T(\sum_{j=1}^{h} \phi^j r_{m,t+j}, R^e_{m,t})$, as the horizon ranges from $h = 1$ quarter ahead to $h = 20$ quarters (5 years) ahead. $R^e_{m,t}$ is the excess return on the aggregate stock market and $r_{m,t}$ is the log real market return (not subtracting the risk-free rate). I multiply $R^e_{m,t}$ by 4 to approximately annualize it. Dotted lines indicate a bootstrapped 95% confidence interval. The sample period is 1927Q1 to 2017Q4.

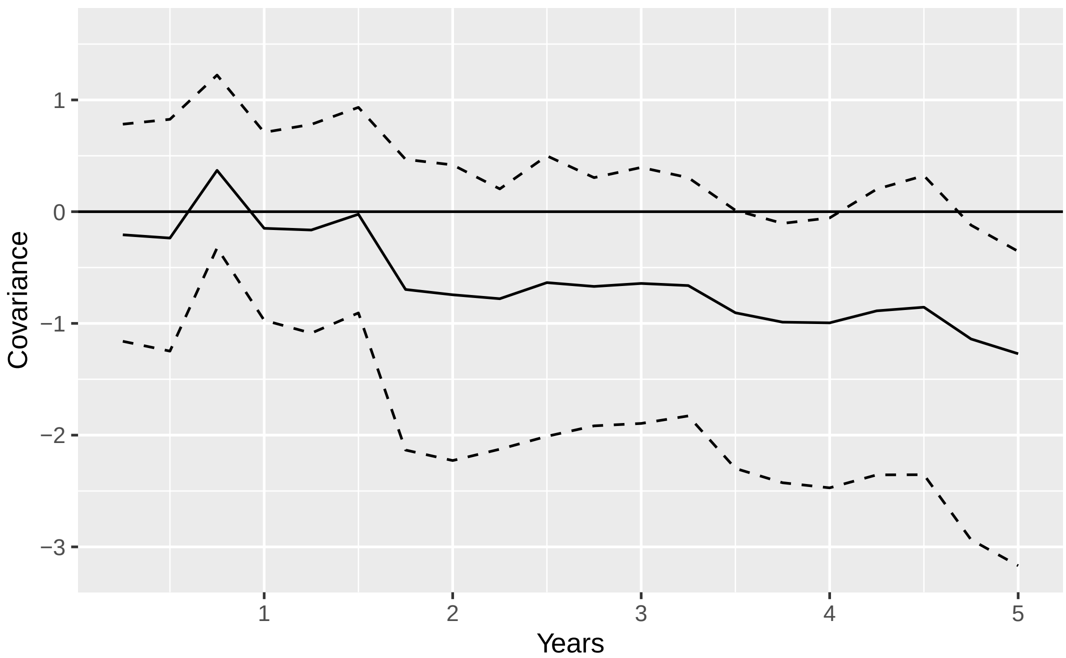


Figure 4: Exposure to consumption and expected return shocks

This figure graphically displays the covariances from Table 2. The standard CCAPM predicts

$$E[R_t^e] = \gamma Cov(\Delta c_t, R_t^e)$$

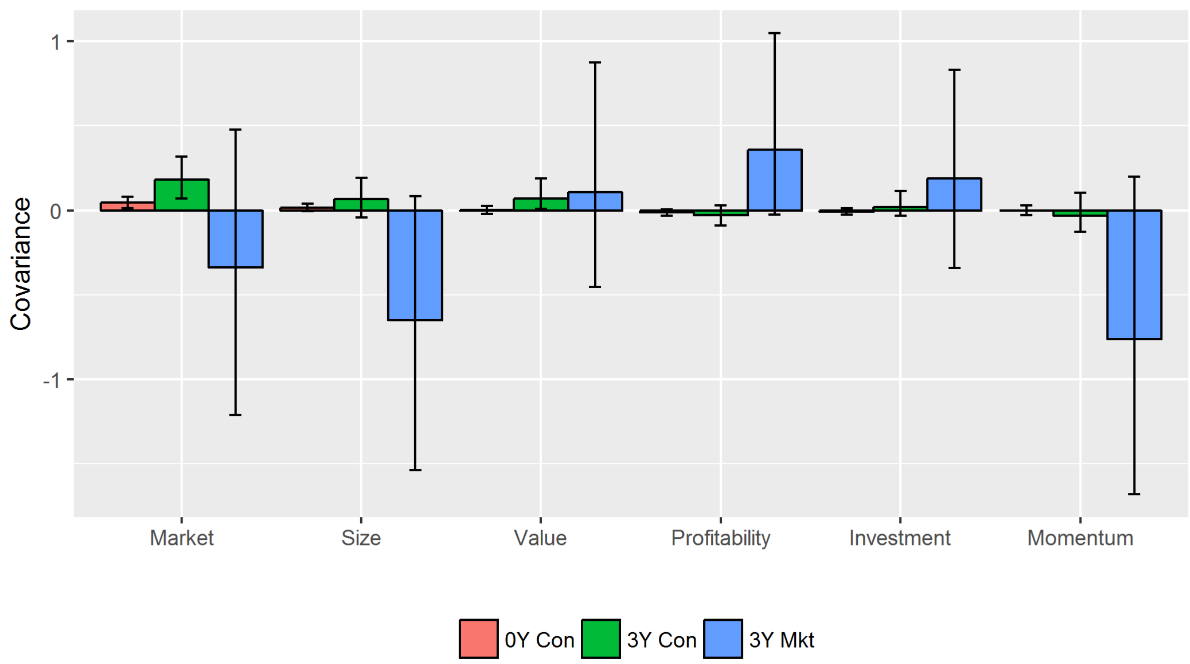
and the long-run CCAPM predicts

$$E[R_t^e] = \gamma Cov\left(\sum_{j=0}^{\infty} \phi^j c_{t+j}, R_t^e\right) - Cov\left(\sum_{j=1}^{\infty} \phi^j r_{m,t+j}, R_t^e\right)$$

The bars labeled 0Y Con, 3Y Con, and 3Y Mkt are sample estimates

$$Cov_T\left(\Delta c_t, R_t^e\right) \qquad Cov_T\left(\sum_{j=0}^{12} \phi^j \Delta c_{t+j}, R_t^e\right) \qquad Cov_T\left(\sum_{j=1}^{12} \phi^j r_{m,t+j}, R_t^e\right)$$

respectively, where R_t^e is an excess return, Δc_t is the change in log consumption, $r_{m,t}$ is the log real market return, and Cov_T is the sample covariance. Errors bars represent a bootstrapped 95% confidence interval. Observations are quarterly. I multiply R_t^e by 4 to approximately annualize the excess returns. The sample is from 1963Q3 to 2017Q4.



A Supplemental figures

Figure 5: Normalized exposure to consumption and expected return shocks
This figure graphically displays the covariances from Table 2 as a percentage of the expected return. The bars labeled "0Y Con" are

$$\frac{Cov_T\left(\Delta c_t, R_t^e\right)}{E_T[R_t^e]} \cdot 100\%$$

the bars labeled "3Y Con" are

$$\frac{Cov_T\left(\sum_{j=0}^{12} \phi^j \Delta c_{t+j}, R_t^e\right)}{E_T[R_t^e]} \cdot 100\%$$

and the bars labeled "3Y Mkt" are

$$\frac{Cov_T\left(\sum_{j=1}^{12} \phi^j r_{m,t+j}, R_t^e\right)}{E_T[R_t^e]} \cdot 100\%$$

where R_t^e is an excess return, Δc_t is the change in log consumption, $r_{m,t}$ is the log real market return, and Cov_T and E_T are the sample covariance and expectation respectively. Each estimate includes a 95% confidence interval for the covariances scaled by the average return. Observations are quarterly. I multiply R_t^e by 4 to approximately annualize the excess returns. The sample is from 1963Q3 to 2017Q4.

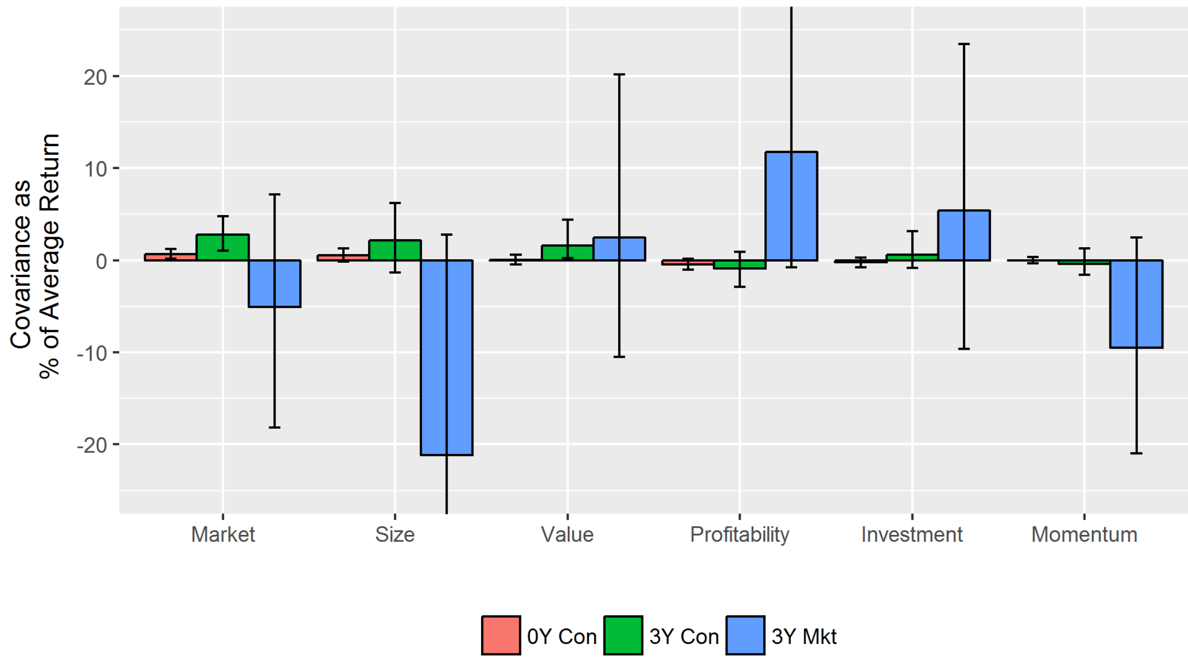


Figure 6: Covariance of Financial Factors with Horizons of Consumption Growth

Plots the cumulative covariance of financial factors against real nondurable consumption growth over varying horizons. For a given factor excess return R_t^e, the graph plots $Cov(R_t^e, \sum_{j=0}^{h} \phi^j \Delta c_{t+j})$ as the quarterly horizon h varies from contemporaneous ($h = 0$) to five years ahead ($h = 20$). Dotted lines indicate a 95% confidence interval computed from bootstrap sampling in six year blocks. Each plot has an identical y-axis, so the magnitudes of the covariances are visually comparable. The sample period is 1963Q3 to 2017Q4.

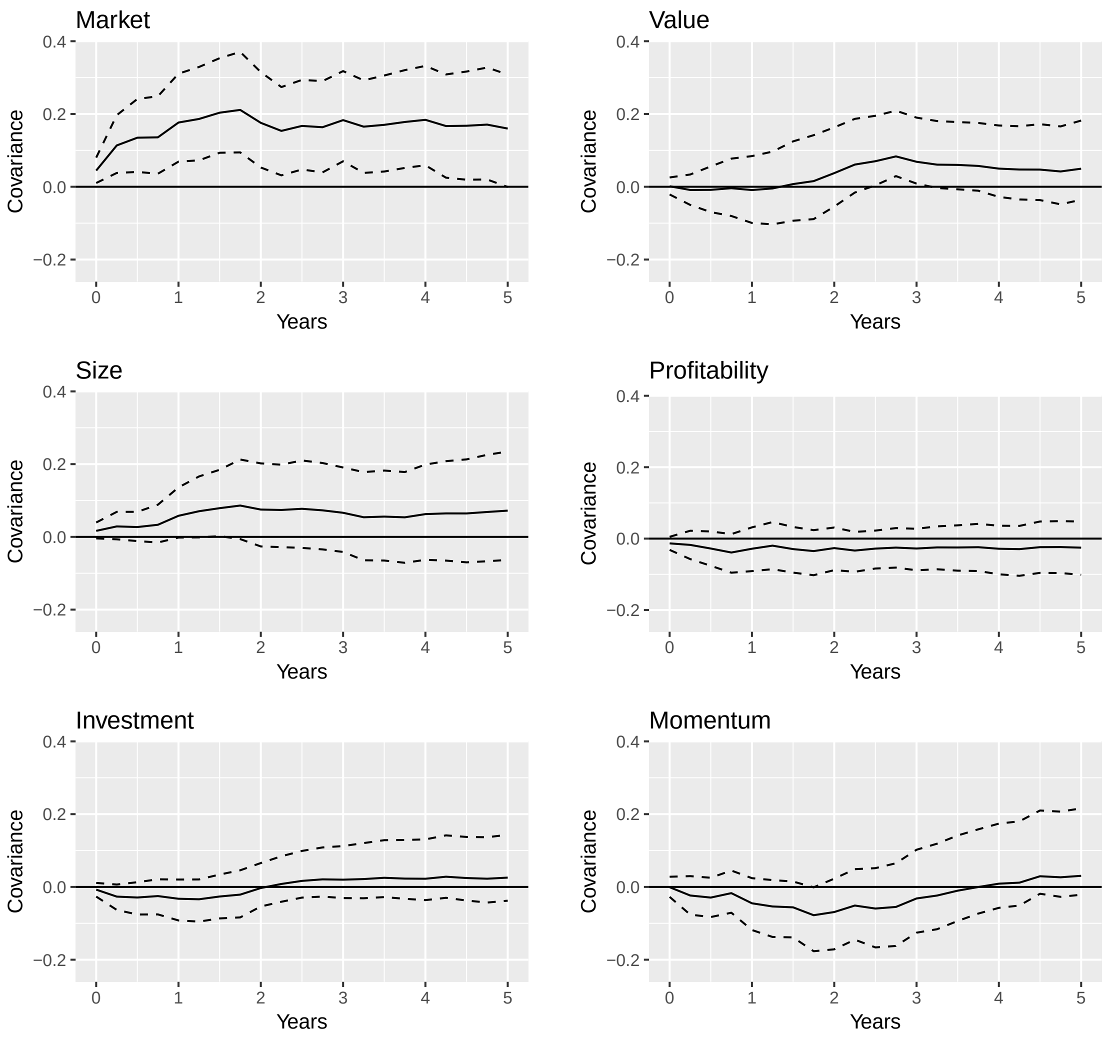


Figure 7: Covariance of Financial Factors with Horizons of Market Returns

Plots the cumulative covariance of financial factors against real stock market returns over varying horizons, excluding the contemporaneous return. For a given factor excess return R_t^e, the graph plots $Cov(R_t^e, \sum_{j=1}^{h} \phi^j r_{m,t+j})$ as the quarterly horizon h varies from one quarter ahead ($h = 1$) to five years ahead ($h = 20$). Dotted lines indicate a 95% confidence interval computed from bootstrap sampling in six year blocks. Each plot has an identical y-axis, so the magnitudes of the covariances are visually comparable. The sample period is 1963Q3 to 2017Q4.

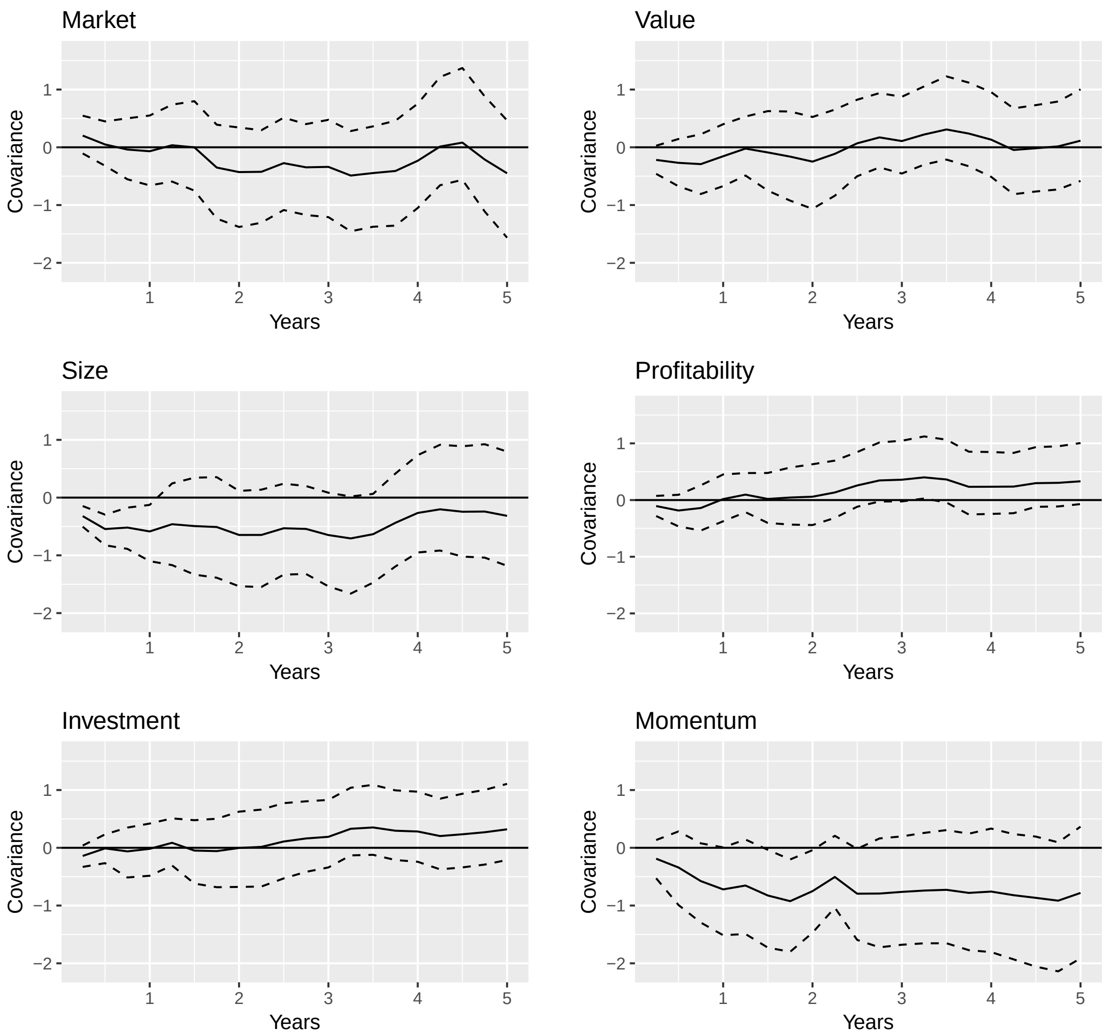


Figure 8: Covariance of Financial Factors with Horizons of Market Excess Returns

Plots the cumulative covariance of financial factors against stock market excess returns over the quarterly risk-free rate over varying horizons, excluding the contemporaneous return. For a given financial factor f_t, plots $Cov(f_t, \sum_{j=1}^{h} \phi^j r^e_{m,t+j})$ as the quarterly horizon h varies from one quarter ahead ($h = 1$) to five years ahead ($h = 20$). Dotted lines indicate a 95% confidence interval computed from bootstrap sampling in six year blocks. Each plot has an identical y-axis, so the magnitudes of the covariances are visually comparable. The sample period is 1963Q3 to 2017Q4.

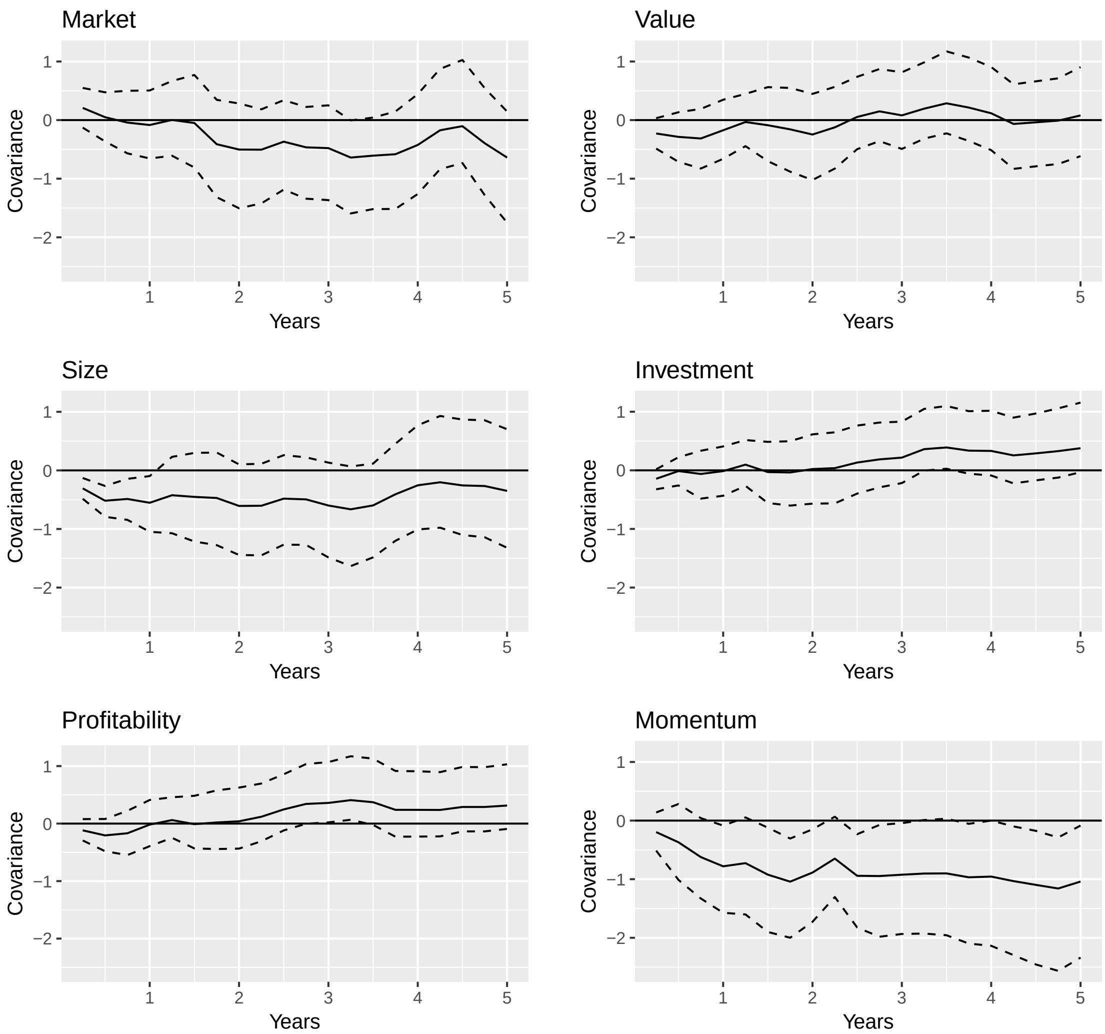
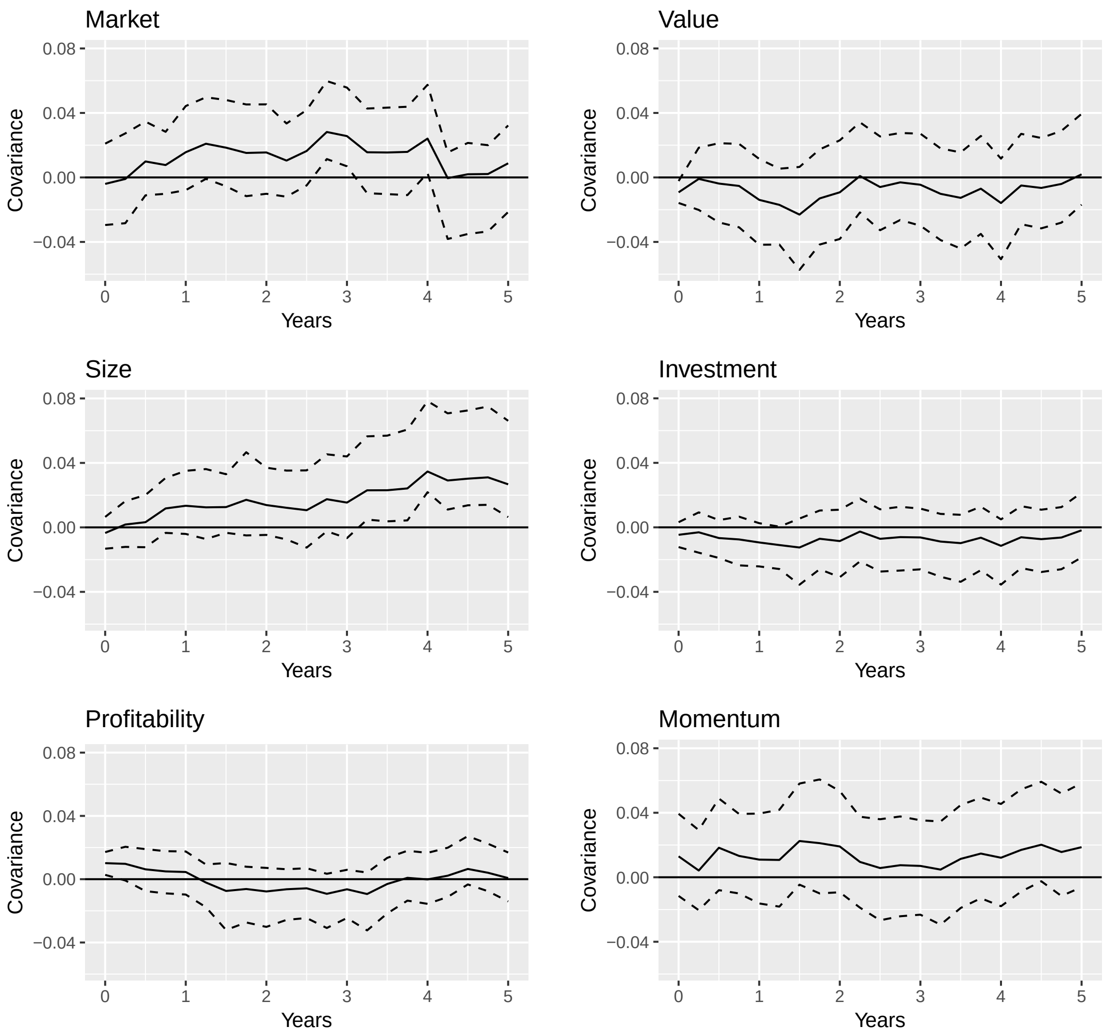


Figure 9: Covariance of Financial Factors with Horizons of Changes in 3 Month Real risk-free Rate

Plots the cumulative covariance of financial factors against quarterly changes in the three month real risk-free rate over varying horizons, excluding the contemporaneous change. For a given financial factor f_t, plots $Cov(f_t, \sum_{j=0}^{h} \phi^j \Delta r_{f,t+j})$ as the quarterly horizon h varies from the contemporaneous change ($h = 0$) to five years ahead ($h = 20$). Dotted lines indicate a 95% confidence interval computed from bootstrap sampling in six year blocks. Each plot has an identical y-axis, so the magnitudes of the covariances are visually comparable. The sample period is 1963Q3 to 2017Q4.



B Proofs for the long-run models

B.1 Portfolio manager first-order conditions for long-run CCAPM

I will proceed to characterize an interior optimum. In that case, the portfolio manager's first-order condition for asset i is

$$E_t[V_{w,t+1}R^e_{i,t+1}] = 0 \tag{69}$$

where $V_{w,t+1} = \frac{\partial V_{t+1}}{\partial W_{t+1}}(W_{t+1}, X_{t+1})$ is the partial derivative of the value function with respect to wealth. I can rewrite the first-order condition using

$$E_t[V_{w,t+1}R^e_{i,t+1}] = E_t[V_{w,t+1}]E_t[R^e_{i,t+1}] + Cov_t(V_{w,t+1}, R^e_{i,t+1}) \tag{70}$$

to obtain

$$E_t[R^e_{i,t+1}] = -Cov_t\left(\frac{V_{w,t+1}}{E_t[V_{w,t+1}]}, R^e_{i,t+1}\right) \tag{71}$$

I next explicitly characterize $V_{w,t}$ by differentiating the value function with respect to wealth and iterating to obtain

$$V_{w,t} = C_t^{-\gamma}C_{w,t} + \beta E[V_{w,t+1}R_{p,t+1}(1 - C_{w,t})] \tag{72}$$

$$= \sum_{j=0}^{\infty} E_t\left[\beta^j C_{t+j}^{-\gamma} R_{p,t\to t+j} C_{w,t+j}\left(\prod_{k=0}^{j-1}(1 - C_{w,t+k})\right)\right] \tag{73}$$

where $C_{w,t} = \frac{\partial C_t}{\partial W_t}(W_t, X_t)$ is the partial derivative of the consumption policy with respect to financial wealth. It reflects the marginal propensity to consume out of *financial* wealth. I next construct $V_{w,t+1}/C_t^{-\gamma}$ to obtain

$$\frac{V_{w,t+1}}{C_t^{-\gamma}} = E_{t+1}\left[\sum_{j=0}^{\infty}\beta^j\left(\frac{C_{t+1+j}}{C_t}\right)^{-\gamma} R_{p,t+1\to t+1+j}C_{w,t+1+j}\left(\prod_{k=0}^{j-1}(1 - C_{w,t+1+k})\right)\right] \tag{74}$$

as desired.

B.2 Log-linear approximation

I start by working with the *realized* value of the overall utility stream to the consumer, that is

$$\begin{aligned} V_t &= \sum_{j=0}^{\infty} \beta^j \frac{C_{t+j}^{1-\gamma} - 1}{1-\gamma} \\ &= \frac{C_t^{1-\gamma} - 1}{1-\gamma} + \beta V_{t+1} \end{aligned}$$

which is not known at time t since it depends on future realizations of consumption. The realized marginal value of wealth is

$$V_{w,t+1} = (1 - \phi_{t+1}) C_{t+1}^{-\gamma} + \phi_{t+1} \beta V_{w,t+2} R_{p,t+2}$$

where $\phi_{t+1} = 1 - C_{w,t+1}$ is the marginal propensity to save financial wealth. Both the realized marginal value of wealth and expectations of it price the excess return

$$E_t[V_{w,t+1} R_{i,t+1}^e] = E_t[E_{t+1}[V_{w,t+1}] R_{i,t+1}^e] = 0$$

It is thus convenient to work with the realized marginal value of wealth. I multiply it by $\beta / C_t^{-\gamma}$ to obtain

$$\begin{aligned} \beta \frac{V_{w,t+1}}{C_t^{-\gamma}} &= (1 - \phi_{t+1}) \beta \left(\frac{C_{t+1}}{C_t} \right)^{-\gamma} + \phi_{t+1} \beta^2 \frac{V_{w,t+2}}{C_{t+1}^{-\gamma}} \left(\frac{C_{t+1}}{C_t} \right)^{-\gamma} R_{p,t+2} \\ \beta \frac{V_{w,t+1}}{C_t^{-\gamma}} &= \beta \left(\frac{C_{t+1}}{C_t} \right)^{-\gamma} \left((1 - \phi_{t+1}) + \phi_{t+1} \beta \frac{V_{w,t+2}}{C_{t+1}^{-\gamma}} R_{p,t+2} \right) \end{aligned}$$

I'll define $M_{t+1}^{long} = \beta V_{w,t+1} / C_t^{-\gamma}$, which yields the recursion

$$M_{t+1}^{long} = \beta \left(\frac{C_{t+1}}{C_t} \right)^{-\gamma} \left((1 - \phi_{t+1}) + \phi_{t+1} M_{t+2}^{long} R_{p,t+2} \right)$$

Taking logarithms yields

$$m_{t+1}^{long} = -\rho - \gamma\Delta c_{t+1} + \log((1-\phi_{t+1}) + \phi_{t+1}\exp(m_{t+2}^{long} + r_{p,t+2}))$$

where $\rho = -\log\beta$. I take a first-order log-linearization of ϕ_{t+1} around its unconditional average ϕ and of $m_{t+2}^{long} + r_{p,t+2}$ around 0 to obtain

$$\begin{aligned}\log((1-\phi_t) + \phi_t\exp(m_{t+2}^{long} + r_{p,t+2})) &\approx \log((1-\phi) + \phi\exp(0)) \\ &+ \underbrace{\frac{\phi\exp(0)}{1-\phi+\phi\exp(0)}}_{\phi}(m_{t+2}^{long} + r_{p,t+2}) + \underbrace{\frac{-1+\exp(0)}{1-\phi+\phi\exp(0)}}_{0}(\phi_t - \phi)\end{aligned}$$

which yields

$$\begin{aligned}m_{t+1}^{long} &\approx -\rho - \gamma\Delta c_{t+1} + \phi r_{p,t+2} + \phi m_{t+2}^{long} \\ &= -\frac{\rho}{1-\phi} - \gamma\sum_{j=0}^{\infty}\phi^j\Delta c_{t+1+j} + \sum_{j=1}^{\infty}\phi^j r_{p,t+1+j}\end{aligned}$$

Risk premia depend on $(E_{t+1} - E_t)m_{t+1}^{long} = \tilde{m}_{t+1}^{long}$, which is

$$\tilde{m}_{t+1}^{long} \approx -\gamma\sum_{j=0}^{\infty}\phi^j\Delta\tilde{c}_{t+1+j} + \sum_{j=1}^{\infty}\phi^j\tilde{r}_{p,t+1+j}$$

B.3 Long-run representation of general log SDF

For all horizons $h \geq 1$, it follows that

$$\begin{aligned}M_{t+1} &= E_{t+1}[M_{t\to t+h}R_{p,t+1\to t+h}] &(75)\\ &= M_{t+1}\cdot\underbrace{E_{t+1}[M_{t+1\to t+h}R_{p,t+1\to t+h}]}_{=1} &(76)\end{aligned}$$

Taking logarithms yields

$$m_{t+1} = \log E_{t+1}\exp\left(\sum_{j=0}^{h-1}m_{t+1+j} + \sum_{j=1}^{h-1}r_{p,t+1+j}\right) \tag{77}$$

where lowercase letters indicate logarithms. To a first-order approximation, which holds exactly if m_{t+1} and $r_{p,t+1}$ are jointly normal and homoskedastic,

$$\tilde{m}_{t+1} = \sum_{j=0}^{h-1} \tilde{m}_{t+1+j} + \sum_{j=1}^{h-1} \tilde{r}_{p,t+1+j}$$

where $\tilde{x} = (E_{t+1} - E_t)x$. Averaging these equations over each horizon using the exponential weights $(1-\phi)\phi^{h-1}$ yields

$$\begin{aligned} \tilde{m}_{t+1} &= \sum_{h=1}^{\infty} (1-\phi)\phi^{h-1} \left(\sum_{j=0}^{h-1} \tilde{m}_{t+1+j} + \sum_{j=1}^{h-1} \tilde{r}_{p,t+1+j} \right) \\ &= \sum_{j=0}^{\infty} \phi^j \tilde{m}_{t+1+j} + \sum_{j=1}^{\infty} \phi^j \tilde{r}_{p,t+1+j} \end{aligned}$$

as desired.

B.4 Portfolio manager's first-order condition for general long-run model

The first-order condition for asset i is

$$\int_s \sum_{j=1}^{\infty} \frac{\partial U}{\partial C_{t+j}(s)} \cdot C_{w,t+j}(s) \cdot \left(\prod_{k=0}^{j-1} (1 - C_{w,t+k}(s)) \right) \cdot R_{p,t+1 \to t+j}(s) \cdot R^e_{i,t+1}(s) ds = 0 \quad (78)$$

where s is the state characterizing the full infinite stream of consumption. Suppose that the current marginal utility of consumption $\partial U / \partial C_t$ is known at time t. I define the standard multi-period SDF as

$$M_{t \to t+j}(s) = \frac{\partial U / \partial C_{t+j}(s)}{\partial U / \partial C_t} \frac{1}{\pi(s)} \quad (79)$$

For example, suppose that the consumer has expected utility preferences with flow utility $u(C)$ and discount factor β. Then

$$M_{t \to t+j}(s) = \frac{\partial U / \partial C_{t+j}(s)}{\partial U / \partial C_t} \frac{1}{\pi(s)} = \frac{\beta^j u'(C_{t+j}(s)) \pi(s)}{u'(C_t)} \frac{1}{\pi(s)} = \beta^j \frac{u'(C_{t+j}(s))}{u'(C_t)} \quad (80)$$

I divide the manager's first-order condition by $\partial U/\partial C_t$, multiply by $\pi(s)/\pi(s)$, and rearrange to obtain

$$\sum_{j=1}^{\infty} E_t \left[M_{t\to t+j} \cdot C_{w,t+j} \cdot \left(\prod_{k=0}^{j-1} (1 - C_{w,t+k}) \right) \cdot R_{p,t+1\to t+j} \cdot R^e_{i,t+1} \right] = 0 \tag{81}$$

where E_t represents an integral with weights $\pi(s)$. This expression justifies the use of the long-run SDF for a wider range of preferences than power utility.

C Empirical work

C.1 Alternative specification accounting for persistent effects

Truncation is a conservative approach which may underestimate the magnitude of long-run covariances, especially if there are persistent effects. I additionally use another specification which can better account for persistent effects that is still in the model-free spirit of using truncated covariances.

Suppose my goal is to estimate a covariance with the infinite sum

$$\sum_{j=0}^{\infty} \phi^j x_{t+j} \tag{82}$$

where for simplicity I will assume that x_t has been demeaned. I can mechanically decompose

$$x_{t+j} = \lambda x_{t+j-1} + \underbrace{(x_{t+j} - \lambda x_{t+j-1})}_{u_{t+j}} = \lambda^j x_t + \sum_{k=0}^{j-1} \lambda^k u_{t+j-k} \tag{83}$$

Even though the decomposition is suggestive of an AR(1) process, it is simply an accounting identity which assumes nothing about the distribution of x_t. I then rewrite the infinite sum as

$$\sum_{j=0}^{\infty} \phi^j x_{t+j} = \frac{1}{1-\lambda\phi} \left(x_t + \sum_{j=1}^{\infty} \phi^j u_{t+j} \right) \tag{84}$$

I then truncate the infinite sum of the u_{t+j} terms to obtain an alternative estimator for the infinite-horizon covariance

$$\frac{1}{1-\lambda\phi} Cov_T\left(x_t + \sum_{j=1}^{h} \phi^j u_{t+j}, R^e_{i,t}\right) \tag{85}$$

The first term with x_t captures the covariance which would be implied if x_t followed an AR(1) process and $R^e_{i,t}$ were only associated with contemporaneous shocks to x_t. The second term with an infinite sum of u_{t+j} captures whether returns forecast deviations from this AR(1) model. If the true model is an AR(1), then the u_{t+j} terms are unforecastable and the second term equals zero. If the true model deviates from an AR(1), then the second term will not equal zero in general and will reflect the extent of the deviations from an AR(1).

If the $Cov(R_{i,t}, u_{t+j})$ terms go to zero faster than $Cov(R^e_{i,t}, x_{t+j})$ terms (which may be the case if x_t is persistent, for example), then it will be preferable to work with the u_{t+j} representation.

While I will not use it in my empirical work, there is also a straightforward generalization to multivariable processes. Suppose that x_t is now a k by 1 vector of variables and Γ is a k by k matrix. Following a similar logic as the univariate case, I write

$$Cov\left(R^e_{i,t}, \sum_{j=0}^{\infty} \phi^j x_{t+j}\right) = (I - \phi\Gamma)^{-1}\left(Cov(R^e_{i,t}, x_t) + Cov\left(R^e_{i,t}, \sum_{j=1}^{\infty} \phi^j u_{t+j}\right)\right) \tag{86}$$

where $u_{t+j} = x_{t+j} - \Gamma x_{t+j-1}$. As before, this expression is merely an accounting decomposition which is true regardless of which process x_t follows. It is then possible to use an estimator which truncates the infinite sum of u_{t+j} at a finite point.