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The Factor Structure of Disagreement

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Abstract

We estimate a Bayesian three-dimensional dynamic factor model on the individual forecasts in the Survey of Professional Forecasters. The factors extract the most important dimensions along which disagreement comoves across variables. We interpret our results through a general semi-structural dispersed information model. The two most important factors in the data describe disagreement about aggregate supply and demand, respectively. Up until the Great Moderation, supply disagreement was dominant, while in recent decades and particularly during the Great Recession, demand disagreement was most important. By contrast, disagreement about monetary policy shocks seems to play a minor role in the data. Our findings can serve to discipline structural models of heterogeneous expectations.

Keywords: Disagreement, Forecast Dispersion, Heterogeneous Expectations, Noisy Information, Dynamic Factor Model.

JEL: C33, C38, E37

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1 Introduction

There is hardly an aspect of the future that people don't disagree about. Survey measures of macroeconomic expectations reveal substantial heterogeneity of expectations. This heterogeneity matters in many ways: It can lead to inertia in price dynamics (Woodford, 2002; Mackowiak and Wiederholt, 2009), non-fundamental driven business cycle fluctuations (Lorenzoni, 2009; Angeletos and La'O, 2013; Ilut and Schneider, 2014), as well as speculative dynamics and booms and busts in asset prices (Scheinkman and Xiong, 2003; Barillas and Nimark, 2013; Burnside, Eichenbaum, and Rebelo, 2016).

The datasets commonly used to study heterogeneity of macroeconomic expectations are panel datasets that contain forecasts about multiple variables of several individuals made at different points in time. Yet the literature has largely ignored the multivariate nature of the data. Most studies of disagreement analyze only one variable at a time, with a particular focus on inflation (e.g. Mankiw, Reis, and Wolfers, 2003). Even where the multivariate structure of forecasts is explicitly modeled, empirical measures of disagreement are still based on univariate forecast dispersion such as the cross-sectional standard deviation (e.g. Capistran and Timmermann, 2009; Dovern, Fritsche, and Slacalek, 2012; Andrade, Crump, Eusepi, and Moench, 2016). This approach has advanced our understanding of the heterogeneity of expectations considerably. But the data contain much more information about the structure of disagreement. In particular, cross-sectional covariances of disagreement across macroeconomic variables can be highly informative. Macroeconomic models of heterogeneous expectations usually imply strong predictions for these covariances. However, the literature has thus far fallen short on a systematical estimation of multivariate comovement in individual expectations.

In this paper, we take a first step towards filling this gap in the literature. We estimate a probabilistic model of multivariate heterogeneous expectations. We directly estimate the model on individual-level forecasts in the well-studied Philadelphia Fed's Survey of Professional Forecasters (SPF). The model is a Bayesian three-dimensional dynamic factor model that is structured in a particular way: Each forecaster is endowed with a small number of factors that describe the deviation of their predictions from the consensus forecast. The factors are assumed to be independent across forecasters, but the loadings on forecast variables are common for all forecasters. Almost all existing structural models of heterogeneous expectations in macroeconomics imply a reduced-form factor structure of multivariate expectations with these features. The estimated factor loadings in our model represent the most important cross-sectional comovement dimensions across forecast variables. Because of the assumed independence of latent factors across forecasters, the estimation procedure can also be parallelized efficiently. The Bayesian framework also handles the well-known problem of missing data in survey data without difficulty.

While our dynamic factor model is a reduced-form model, we offer an interpretation through a general semi-structural model of dispersed or "noisy" information. Almost all existing structural macroeconomic models with heterogeneous expectations (including Mackowiak and Wiederholt, 2009; Woodford, 2002; Lorenzoni, 2009; Angeletos, Collard, and Dellas, 2018, to name just a few prominent contributions) can be subsumed into this semi-structural model, in which agents predict a dynamic multivariate data-generating process by filtering noisy signals about the aggregate state, and differences in expectations arise because some of the signals contain idiosyncratic noise. We map the expectations generated by this model into our reduced form factor model. When agents only receive signals about the exogenous shocks of the aggregate model, we can further identify the factor loadings of the reduced form with the impulse responses of the aggregate shocks agents disagree about.

We use our estimated model to uncover new facts about disagreement. First, we find that the comovement of disagreement can be described by two factors that we label "demand" and "supply" disagreement, respectively. Innovations to supply disagreement increase a forecaster's output expectations but decrease their inflation expectations. They also increase expectations of longer-run GDP growth, productivity and the natural rate of unemployment. Innovations to demand disagreement increase inflation, output and interest rate expectations. Second, demand disagreement is substantially more persistent than supply disagreement. Third, the relative importance of the two factors in the data varies over time. Demand disagreement has become more important over time and has been particularly important during the Great Recession. Supply disagreement has mattered most during the 1970s and again during the recent COVID crisis. Fourth, demand disagreement is not driven by disagreement about the course of monetary policy.

Our findings can serve to discipline structural models of heterogeneous expectations. For example, signals about total factor productivity are the most important driver of disagreement in many noisy information models (e.g. Lorenzoni, 2009; Nimark, 2014). According to our results, this type of disagreement only fits the data well before the Great Moderation. The model of Melosi (2014) adds idiosyncratic signals about monetary policy shocks. Our estimation finds little evidence of this type of disagreement in the data. By contrast, a model in which there is disagreement about technology and discount factor shocks, as in Benhima and Poilly (2020), is more consistent with our results. We hope that future research will make use of our methodology to align structural models more closely with the factor structure of disagreement in the data.

The cross-sectional dispersion in our estimated factors can also be used as a new set of measures of macroeconomic disagreement. These measures are more comprehensive than univariate forecast dispersion because the factors efficiently summarize disagreement across a wide range of variables. We show that dispersion in the factors exhibits salient features previously documented in the literature, yet carries additional information compared to univariate dispersion measures.

The only data we use in the paper are survey forecasts. We do not use realizations of the forecast variables, either to estimate the model or to analyze forecast errors. The immediate benefit is that we avoid common problems in evaluating survey forecasts such as data definitions and revisions. But it also means that our analysis remains silent on issues such as forecast accuracy or systematic biases in expectations. Instead, our focus is to shed light on previously undocumented facts about the cross-sectional comovement of expectations, without judgment about the accuracy or rationality of these expectations.

We evaluate a particular factor structure in which loadings are common across forecasters and the factors themselves are forecaster-specific, implicitly assuming that everyone uses the same forecasting model conditional on their factors. In practice, disagreement may also arise from heterogeneous forecasting models. However, modeling heterogeneous beliefs in this way quickly becomes intractable even in simple setups. To the extent that factor loadings are in fact forecaster-specific, our estimation procedure will reveal an average forecasting model which still improves our understanding of forecasters' expectations formation.

Our paper relates to a large literature that uses survey data to inform models of expectations. Most of this literature either uses consensus forecasts (e.g. Coibion and Gorodnichenko, 2015) or use dispersion statistics like the cross-sectional standard deviation to summarize disagreement (e.g. Andrade, Crump, Eusepi, and Moench, 2016). A few studies go beyond dispersion statistics. For example, Patton and Timmermann (2010) study the persistence of forecaster disagreement over time. Rich and Tracy (2017) study how the extent of individual disagreement predicts forecast revisions and forecast accuracy, and relate disagreement to individual uncertainty measured in density forecasts. Bordalo, Gennaioli, Ma, and Shleifer (2018) use individual forecasts to study the predictability of forecast errors by forecast revisions. All of these studies focus on one variable at a time. Our paper is the first to estimate a full probabilistic model directly on multivariate, individual forecasts.

The direct precursor to our analysis is Dovern (2015). In addition to studying time-varying univariate forecast dispersion, Dovern also examines the cross-sectional correlations between individual forecasts of output growth, unemployment and inflation. He concludes that these correlations are not particularly strong in the data, in contrast to what is predicted by most theoretical models of forecast disagreement. Consistent with his findings, the idiosyncratic components in our estimated model explain about two thirds of the variance of disagreement. But to us, the glass is a third full rather than two thirds empty: The comovement we estimate is still informative for inferring structural sources of disagreement. Moreover, the factor model we estimate can—and does—uncover strong conditional comovement, highlighting the limitations of unconditional cross-sectional correlations.

Our paper also relates to a small literature that examines "theory-consistency" of forecasts. Carvalho and Nechio (2014) and Dräger, Lamla, and Pfajfar (2016) compute the percentage of individual forecasts in the Michigan survey of consumers that conform to a Fisher equation, a Taylor rule or a Phillips curve. Kuang, Tang, Zhang, and Zhang (2020) test whether forecasts are consistent with cointegrating relationships implied by theory. In this paper, we do not test for specific comovement relations, but let the data "speak freely" about the most important comovement relations.

The remainder of this paper is structured as follows. Section 2 describes our dynamic factor model and offers a semi-structural interpretation through a model of dispersed information. Section 3 describes the data and the estimation procedure. Section 4 presents the main estimation results. In Section 5, we look at the dispersion of the factors and how they relate to univariate disagreement. Section 6 contains a number of robustness checks and additional estimation results. Section 7 concludes.

2 A factor model of multivariate disagreement

We observe a panel of forecasts $\hat{y}_{jt+h|it}$, each made by an individual i at a time t about a variable j, concerning the realization y_{jt+h} at a future time $t+h$. The forecast horizon $h = 0$ corresponds to the current-quarter nowcast, $h = 1$ corresponds to the one quarter-ahead forecast and so on. Let M be the total number of forecasters covered in the data set, T the number of time periods, and $\mathcal{J}$ the set of variables j and forecast horizons h considered. Then the panel has three dimensions: Time $t \in (1, \ldots, T)$, individual $i \in (1, \ldots, M)$, and forecast variable/horizon $(j, h) \in \mathcal{J}$.[1] Define the *consensus* forecast as the cross-sectional average forecast:

$$\bar{y}_{jt+h|t} = \frac{1}{\mid \mathcal{I}_t \mid} \sum_{i \in \mathcal{I}_t} \hat{y}_{jt+h|it} \tag{1}$$

where $\mathcal{I}_t \subset \{1, \ldots, M\}$ is the subset of forecasters who respond to the forecast of variable j at horizon h in period t. We define *disagreement* as the deviation of an individual's forecast from the consensus. An individual's disagreement, denoted y_{jhit}, is simply the cross-sectionally

[1] One could of course consider the data a four-dimensional panel by separating out the variables and forecast horizons. We discuss this issue in Section 6, but here we group forecast variables at different horizons into one dimension.

demeaned forecast:

$$y_{jhit} = \hat{y}_{jt+h|it} - \bar{y}_{jt+h|t}. \tag{2}$$

2.1 Model

We now construct a time series model for disagreement. We assume that the random variables driving disagreement are identically distributed and independent across individuals. Thus, it is sufficient to describe the probability model for an arbitrary individual i's disagreement. The aggregate model is merely the collection of individual models sharing common parameters. Recall from (2) that at time t, an individual's i disagreement for the h-step forecast of variable j is denoted by y_{jhit}. We can write individual i's disagreement at time t as a vector of length $N = |\mathcal{J}|$,

$$y_{it} = [y_{jhit}]_{(j,h)\in\mathcal{J}}.$$

For each individual i, we assume the elements of this vector disagreement can be decomposed into a systematic component which represents the part of disagreement that is common across variables and an orthogonal, idiosyncratic component. Thus, disagreement can be written as a factor model,

$$y_{it} = \Lambda f_{it} + \xi_{it}. \tag{3}$$

Here Λ is an $N \times p$ matrix of factor loadings, f_{it} is a vector of p common factors, and ξ_{it} is vector of N idiosyncratic errors. The p factors each follow independent autoregressive processes (AR) of order 1. Let ϕ be the $p \times 1$ vector corresponding to the autoregressive coefficients associated with these processes and $\Phi = \mathrm{diag}(\phi)$, where the $\mathrm{diag}(\cdot)$ operator places the vector ϕ on the diagonal of $p \times p$ matrix whose other elements are zero. Then we can write the dynamics of the factors as a vector autoregression (VAR):

$$f_{it} = \Phi f_{it-1} + u_{it}, \quad u_{it} \overset{\text{iid}}{\sim} \mathcal{N}(0, I_p). \tag{4}$$

Finally, the idiosyncratic components also follow independent AR processes of order 1. Let ρ be the $N \times 1$ vector of autoregressive coefficients of these processes and $P = \mathrm{diag}(\rho)$. Similarly, let σ^2 be the vector of variances of their innovations and $\Sigma = \mathrm{diag}(\sigma^2)$. We can write the VAR for the idiosyncratic term as:

$$\xi_{it} = P\xi_{it-1} + v_{it}, \quad v_{it} \overset{\text{iid}}{\sim} \mathcal{N}(0, \Sigma). \tag{5}$$

Equations (3), (4), and (5) form a dynamic factor model for individual i's disagreement.[2] It's worth mentioning a few comments on the characteristics of such a model. First, for almost all Λ, the factors are already identified up to sign and label by the assumption that they are independent from one another. This identifying assumption makes interpretation of the factors easier. Moreover, as shown in section 2.2, it is consistent with popular models of heterogeneous information. Second, we again emphasize that each forecaster i is described by the same econometric model, with differences only occurring in the realization of the factors and idiosyncratic terms. That is, the parameters $\theta = (\Lambda, \phi, \rho, \sigma^2)$ are identical across forecasters. Let $Y_i = [y_{i1}, \ldots, y_{iT}]'$ be the matrix of the time series of disagreement for individual i. Given a vector of parameters θ, the likelihood function for individual i, $p(Y_i|\theta)$, can be evaluated using the Kalman filter using the state space representation implied by (3), (4), and (5).[3] This approach has the advantage of efficiently handling missing observations.

A different way of writing the model is to recast the model (3)–(5) in a standard two-dimensional structure. We can stack the disagreement of all forecasters $i = 1, \ldots, M$ in one large vector and write:

$$\begin{pmatrix} y_{1t} \\ y_{2t} \\ \vdots \\ y_{Mt} \end{pmatrix} = \begin{pmatrix} \Lambda & 0 & \cdots & 0 \\ 0 & \Lambda & & 0 \\ \vdots & \vdots & & \ddots \\ 0 & 0 & \cdots & \Lambda \end{pmatrix} \begin{pmatrix} f_{1t} \\ f_{2t} \\ \vdots \\ f_{Mt} \end{pmatrix} + \begin{pmatrix} \xi_{1t} \\ \xi_{2t} \\ \vdots \\ \xi_{Mt} \end{pmatrix} \tag{6}$$

$$\begin{pmatrix} f_{1t} \\ f_{2t} \\ \vdots \\ f_{Mt} \end{pmatrix} = \begin{pmatrix} \Phi & 0 & \cdots & 0 \\ 0 & \Phi & & 0 \\ \vdots & & \ddots & \vdots \\ 0 & 0 & \cdots & \Phi \end{pmatrix} \begin{pmatrix} f_{1t-1} \\ f_{2t-1} \\ \vdots \\ f_{Mt-1} \end{pmatrix} + \begin{pmatrix} u_{1t} \\ u_{2t} \\ \vdots \\ u_{Mt} \end{pmatrix}, \; u_t \sim \mathcal{N}\left(0, I_M \otimes I_p\right) \tag{7}$$

$$\begin{pmatrix} \xi_{1t} \\ \xi_{2t} \\ \vdots \\ \xi_{Mt} \end{pmatrix} = \begin{pmatrix} P & 0 & \cdots & 0 \\ 0 & P & & 0 \\ \vdots & & \ddots & \vdots \\ 0 & 0 & \cdots & P \end{pmatrix} \begin{pmatrix} \xi_{1t-1} \\ \xi_{2t-1} \\ \vdots \\ \xi_{Mt-1} \end{pmatrix} + \begin{pmatrix} v_{1t} \\ v_{2t} \\ \vdots \\ v_{Mt} \end{pmatrix}, \; v_t \sim \mathcal{N}\left(0, I_M \otimes \Sigma\right). \tag{8}$$

Expressed in this form, we can interpret our model of individual disagreement as a standard dynamic factor model with pM factors.[4] We impose restrictions on the factor loadings of

[2] Both f_{it} and ξ_{it} are initialized at time $t = 0$ as $f_{i0} \sim \mathcal{N}(0, I_p)$ and $\xi_{i0} \sim \mathcal{N}(0, I_N)$. This simplifies the computation , with minor effects on the results. Note that in the empirical implementation the timing is forecaster specific, with $t = 1$ referring to the first period a given forecaster is in the sample and $t = T$ the last period. We omit this detail in the exposition for simplicity.

[3] In a slight abuse of notation, we use p to denote the number of factors and a generic probability density function. The meaning should be clear from context.

[4] One can also interpret the consensus forecast as an additional common factor in our model, which relates

the remaining pM factors that assign each set of p factors to one particular forecaster.

2.2 A semi-structural interpretation

Our factor model is a reduced-form description of disagreement, but the choice of this structure is motivated by the theoretical literature on heterogeneous expectations. Almost all structural macroeconomic models with heterogeneous expectations imply a factor structure of disagreement that of the same form as our reduced form factor model, as we will now show.

In these models, which are often called dispersed information or noisy information models, a continuum of agents, indexed by $i \in [0, 1]$, forms expectations about aggregate macroeconomic variables:

Assumption 1. *All aggregate endogenous variables y_t are linear functions of a finite set of state variables x_t which follow linear stochastic processes with normally distributed shocks:*

$$y_t = Cx_t \tag{9}$$

$$x_t = Ax_{t-1} + B\varepsilon_t, \qquad \varepsilon_t \overset{iid}{\sim} \mathcal{N}\left(0, I_{n_x}\right). \tag{10}$$

Agents in the model do not directly observe y_t or the state variables x_t, but the model structure and parameters are common knowledge. Instead, the information set of each agent is commonly described by a set of signals s_{it} that the agent receives each period. We allow for a very general signal structure:

Assumption 2. *Agents' information sets are given by a set of Gaussian signals $s_{it} = Hx_t + Q_e(L)\, e_t + Q_u(L)\, u_{it}$. The common noise shocks $e_t \sim \mathcal{N}\left(0, I_{n_e}\right)$ are the same for all agents, while the idiosyncratic noise shocks $u_{it} \sim \mathcal{N}\left(0, I_p\right)$ are independently distributed across agents. All shocks are independent across time. The distributed lags are such that $Q_e(L)\, e_t$ and $Q_u(L)\, u_{it}$ are stationary.*

Our final assumption relates agents' expectations to the forecasts recorded in survey data, by including measurement error:

Assumption 3. *Agents' observed forecasts are given by*

$$\hat{y}_{jh+t|it} = \mathbb{E}_{it}\left[y_{jt+h}\right] + \xi_{jhit} \tag{11}$$

where the measurement errors ξ_{jhit} are stationary processes that are independent across forecasters i and variables j.

forecasts to disagreement through $\hat{y}_{it} = \bar{y}_t + y_{it}$.

Together, our assumptions cover most models of heterogeneous expectations that have been put forward in the literature. First, the law of motion for y_t in Assumption 1 may be an exogenous data-generating process, like in much of the existing literature that fits dispersed information models to survey data, including Coibion and Gorodnichenko (2015), Andrade, Crump, Eusepi, and Moench (2016), and others. It may also be the potentially complex equilibrium outcome of a model in which agents' signal extraction problem influences aggregate outcomes, like in Lorenzoni (2009), Angeletos and La'O (2013) and related studies, or of a rational inattention model where the signals are chosen endogenously (see Mackowiak, Matejka, and Wiederholt, 2018 for a survey). Second, common noise shocks that affect aggregate dynamics can be accommodated by including them in ε_t instead of e_t. Third, signals can be arbitrarily correlated across agents by rewriting the common shocks e_t and idiosyncratic shocks u_{it} accordingly. Fourth, by extending the state vector appropriately, the signals can be made to reveal parts of the state x_t with finite lags or leads ("news shocks"). Finally, the state x_t need not be stationary, so that disagreement about the long run ("fundamental disagreement" in the language of Andrade, Crump, Eusepi, and Moench, 2016) is possible.

Our formulation also has limitations. It does not accommodate non-linearities or non-Gaussian shocks (e.g. Nimark, 2014). We also do not allow for infrequent updating of information sets like in the sticky information models of Mankiw and Reis (2002). Most importantly, our assumptions do not accommodate heterogeneity of beliefs in dimensions other than the realizations of the idiosyncratic shocks u_{it}, including heterogeneous precision of signals, disagreement about the aggregate process in Assumption 1, or any form of heterogeneous priors. Any heterogeneity in forecasting models is likely to be an important source of disagreement that manifests itself in survey data on expectations (Patton and Timmermann, 2010). However, modeling such heterogeneity becomes difficult even in the very stylized environment discussed here. Indeed, we are not aware of a structural macroeconomic model in the literature with such a form of heterogeneity in beliefs.

We can now express disagreement in the semi-structural model as follows:

Proposition 1. *Under Assumptions 1–3, disagreement has the form:*

$$y_{jhit} = \sum_{s=0}^{\infty} \Psi_{jhs} u_{it-s} + \xi_{jhit}. \tag{12}$$

Proof. See Appendix B. □

The representation in (12) is a generalized dynamic factor model (Forni, Hallin, Lippi, and Reichlin, 2000) that captures the essential restrictions of our estimated reduced-form model: Each forecaster i has its own set of factors, but the loadings of the factors on the deviation from the average forecast have identical loadings for all forecasters.[5] The factors in our reduced form model thus correspond to the idiosyncratic shocks u_{it} to agents' signals, and the factor loadings correspond to the effects of these shocks on agents' model-based expectations.

In order to characterize the factor loadings further, one now has to become more specific about the signal structure and the underlying model. In much of the literature on dispersed information (e.g. Woodford, 2002; Melosi, 2014; Angeletos, Collard, and Dellas, 2018), agents are assumed to only receive signals about the exogenous driving processes ε_t. Benhima and Poilly (2020) additionally assume that agents learn the state of the economy perfectly after a finite number of periods. While these assumptions are made for tractability of solving these models, they also lead to a reduced-form factor structure that is easier to interpret. Consider the following special case of Assumption 2.

Assumption 2A. *The signals s_{it} that describe agents' information sets have the form:*

$$s_{it} = \begin{pmatrix} \varepsilon_t + \eta_{it} \\ \varepsilon_{t-1} \end{pmatrix} \tag{13}$$

$$\begin{aligned} \eta_{kit} =& \rho_k \eta_{kit-1} + e_{kt} + u_{kit} + \vartheta_k \left(e_{kt-1} + u_{kit-1} \right), \\ & e_{kt} \overset{iid}{\sim} \mathcal{N}\left(0, \sigma^2_{ek}\right), \, u_{kit} \overset{iid}{\sim} \mathcal{N}\left(0, \sigma^2_{uk}\right) \end{aligned} \tag{14}$$

where $\rho_k \in (0,1)$, e_{kt} are iid across k and t, and u_{kit} are iid across i, k and t. Additionally, $\sigma_{uk} = 0$ for $k > p$.

The assumption states that agents receive signals about the structural shocks and that only the first p signals contain an idiosyncratic noise. The state of the economy is perfectly revealed with a one-period lag. Further, the noise component of the signal follows an ARMA(1,1) process, which is the simplest process for which the reduced-form factors also display an autoregressive component.[6]

With this assumption, the factor representation of disagreement in the model is simplified further. We are able to represent the reduced-form factors as AR(1) processes, just like in

[5] In addition, our reduced form imposes specific dynamics on the factors and idiosyncratic components, namely that $\Psi_{jh}(L) u_{it} = \Lambda f_t$ with $f_t = \Phi f_{t-1} + u_{it}$, and $\xi_{it} = P\xi_{it-1} + v_{it}$. We impose these additional restrictions in order to keep the number of estimated parameters small without needing to impose a particular structural model, which would also come with its own set of restrictions to $\Psi_{jh}(L)$ and ξ_{it}.

[6] Note that we also implicitly assume that the common noise shocks e_t are independent of the structural shocks ε_t, i.e. they do not affect aggregate outcomes. Allowing for such aggregate noise shocks would render our mapping to the reduced form less sharp, as an idiosyncratic shocks u_{kit} would now carry information about both the "true" structural shock and of the aggregate noise shock, and the corresponding reduced-form factor loadings would be weighted averages of the aggregate effects of ε_{kt} and e_{kt}.

our reduced-form model. More importantly, the factor loadings have a precise interpretation.

Proposition 2. *Under assumptions 1, 2A and 3, disagreement has the form:*

$$y_{jhit} = \Lambda_{jh} f_{it} + \xi_{jhit} \tag{15}$$

$$f_{kit} = a_k f_{kit-1} + u_{kit}. \tag{16}$$

where $a_k \in (0,1)$, $k = 1, \ldots, p$. Moreover, there exist constants c_k such that the loadings of the kth factor are proportional to the impulse response function to ε_{kt}:

$$\Lambda_{kjh} = c_k C_{j\cdot} A^h B_{\cdot k}. \tag{17}$$

Proof. See Appendix B. □

The factor loadings are identified, up to a scalar, with the impulse response functions to the shocks that agents disagree about. Thus, if agents' disagreement stems from idiosyncratic signals about, say, a technology shock, then the loadings on one of the factors in the reduced form will trace out the impulse responses of the economy to this shock. Under this interpretation of our reduced form factor model, we can then apply standard sign restrictions on impulse responses used in the literature on VARs to our estimated factor loadings in order to characterize the shocks about which agents disagree most about.

3 Data and estimation

3.1 Data

We use individual responses in the SPF. The survey is the longest-running quarterly survey of macroeconomic forecasts in the United States and has been well studied. The sample starts in 1968:Q4 and we include data through 2020:Q4. Since 1990, the survey is run by the Philadelphia Fed. In the middle of each quarter, participants are asked to forecast a wide range of variables for the current quarter and each of the following quarters, up to four quarters out. In addition, they are also asked to report a number of longer-run forecasts, as well as recession probabilities.[7] The full set of variables and additional summary statistics are documented in Appendix C.

The SPF is a panel data set in which each forecaster can be tracked over time. However, the panel contains a large amount of missing data for a number of reasons. First, forecast

[7] The survey also includes forecasts at annual horizons, as well as density forecasts. We do not consider these additional forecasts here.

variables have been introduced into the survey at various points in time. While the initial 1968 survey only asked about eight variables, that number has since grown to 27. Second, many forecasters enter and exit the sample. Over the sample, 446 forecasters have participated in the survey, but the average number of respondents in any given quarter is only 36 on average. Third, even forecasters that do participate do not always respond to all questions in the survey in a given quarter. While the more prominent forecasts such as real GDP are almost always filled in, some less prominent ones like the natural rate of unemployment are only filled in by about half of respondents, on average.

The complex missing data patterns complicates inference about individual responses, especially when studying multivariate forecasts. The Kalman filter we use to estimate our factor model is ideally suited to handle missing data, an advantage that has been previously exploited in the literature on mixed-frequency data and nowcasting (e.g. Banbura, Giannone, Modugno, and Reichlin, 2013).

In this paper, we focus on the horizon $h = 4$ together with long-run forecasts.[8] We omit nominal GDP forecasts, as they are a linear combination of real GDP and GDP deflator forecasts, as well as PCE and core PCE inflation forecasts which are very highly correlated with their CPI counterparts. This leaves our baseline sample with 28 observables. These include GDP and its components, three other indicators of real activity, four measures of inflation, two labor market indicators, and four components of interest rates, in addition to recession probability. We include 6 long-run ($h = 40$) average forecasts for real GDP, CPI, the Treasury bill, productivity growth, and stock prices. Finally, we include disagreement about the natural rate of unemployment. In the SPF, this forecast is not made with reference to a particular horizon; we group it with the long-run variables for presentation purposes. Most variables are forecast in levels. This potential for nonstationarity can be challenging for factor models. Therefore we transform the individual forecasts (and hence the derived disagreement) using the classifications from Stock and Watson (2002), ensuring the measures of disagreement are stationary.[9] Table 1 provides details on transformation, summary statistics for disagreement, and information about the prevalence of the each of . The data we use has 8,457 observations, each representing one forecaster's predictions of the 28 variables at a point in time.

[8]We focus on a particular horizon (along with long-run forecasts) to be consistent with most of the other literature on disagreement. In addition, while including multiple short-run horizons is conceptually straightforward in our framework, disentangling the natural comovements of the same variable forecasted at different horizons from the relationships across variables can be challenging. Jointly estimating a factor model with all horizons included, along the lines of Moench, Ng, and Potter (2013), is left to future research.

[9]In the literature on factor models, observables are often standardized prior to estimation. In order to make interpretation of the coefficients easier, we do not standardize. Results are robust, however, to this kind of data pretreatment.

Table 1: Summary Statistics.

Variable y_{jhit}	format	s.d.	autocorr.	# time periods	average # forecasters
GDP components					
GDP	5	1.10	0.51	204	36.8
Consumption	5	0.88	0.50	158	32.7
Non-res. investment	5	2.56	0.59	158	32.0
Res. investment	5	4.58	0.65	158	31.9
Non-federal gov't	5	1.18	0.6	158	31.1
Federal gov't	5	2.23	0.62	158	31.0
Inventories	5	0.21	0.46	158	31.5
Net exports	5	0.42	0.54	158	31.9
Other real activity					
Industrial production	5	1.80	0.58	204	34.9
Housing starts	4	10.28	0.67	204	35.5
Corporate profits	5	6.02	0.50	204	27.7
Inflation					
GDP deflator	6	0.81	0.53	195	36.6
CPI	6	0.64	0.49	158	33.7
Core CPI	6	0.41	0.62	56	37.3
Core PCE	6	0.36	0.64	56	34.8
Labor market					
Employment	5	0.85	0.50	69	35.2
Unemployment rate	1	0.46	0.52	204	37.4
Natural rate of unemp.	1	0.42	0.92	25	17.5
Interest rates					
3-month T-bill rate	2	0.63	0.64	158	33.0
10-year term spread	1	0.43	0.54	116	34.9
Aaa spread	1	0.32	0.47	116	29.7
Baa-Aaa spread	1	0.24	0.24	44	26.2
Recession prob.					
Recession prob.	1	12.75	0.60	205	36.2
Long-term forecasts					
Real GDP (10y avg.)	5	0.35	0.86	29	33.4
CPI (10y avg.)	6	0.47	0.78	117	33.6
T-bill rate (10y avg.)	2	0.81	0.78	29	28.5
Prod. growth (10y avg.)	4	0.46	0.87	29	29.6
Equity return (10y avg.)	4	2.00	0.82	29	24.3

Note: Sample runs from 1968Q4 through 2020Q4. "s.d." is the pooled standard deviation of $\tilde{y}_{jhit}$ and "autocorr." is its average first autocorrelation. "# time periods" describes the number of time periods for which the variable could be constructed from available responses from at least one forecaster. "average # forecasters" describes the average number of forecasters per time period for which the variable could be constructed from available responses. Transformations: 1 is plain levels. 2 is the difference of the time t-forecast level for $t+h$ minus the "forecast" level for $t-1$ (realizations at $t-1$ are known to survey participants at t). 4 is log levels. 5 is the time t-forecast annualized log growth rate between $t+h$ and $t-1$. 6 is the difference of the time t-forecast of the annualized log growth rate between $t-1$ and the "forecast" annualized log growth rate between $t-1$ and $t-2$. Transformations are applied to individual forecasts before differencing out the consensus. Forecast horizons are four quarters except for UBAR which does not have a horizon and long-term forecasts for which the horizon is ten years.

3.2 Estimation

As mentioned above, aggregation is simple because the factors and idiosyncratic terms are independent across individuals. Let $Y = [Y_1', \dots, Y_M']'$. Then the likelihood function for the entire set of disagreement is given by:

$$p(Y|\theta) = \prod_{i=1}^{M} p(Y_i|\theta). \tag{18}$$

Given this likelihood function, one could estimate the model via MLE as in (Stock and Watson, 1989). Instead, we follow a Bayesian approach.[10] The central object of Bayesian inference in the posterior distribution, $p(\theta|Y)$, which is the combination of the likelihood and a prior distribution, $p(\theta)$, specifying initial beliefs about θ, using Bayes rule:

$$p(\theta|Y) = \frac{p(Y|\theta)p(\theta)}{p(Y)}. \tag{19}$$

In the Bayesian approach, the calculus of probability characterizes how the state of knowledge or degree of beliefs about some object (for example the parameters θ) changes in light of the data. On advantage in this application is that, the posterior distribution completely characterizes the uncertainty about an object of interest, without reference to potentially inaccurate asymptotic approximations or tedious bootstrapping. In what follows, for ease of notation we drop references to horizon h and only index variables using $j = 1, \dots, N$.

Prior Distribution. Bayesian inference requires a prior distribution over the parameters θ. We set the priors as follows. For each $\sigma_j^2, j = 1, \dots, N$, we use an inverse gamma distribution. We derive a prior distribution for this variance by specifying a prior over its standard deviation. We follow the formulation used in Zellner (1971), parameterized by degrees of freedom, ν_j, and scaling parameter, s_j.[11] For each j we set

$$\nu_j = 10 \text{ and } s_j = \text{std}(Y_j)/2, \quad i = 1, \dots, N, \tag{20}$$

where Y_n denotes the stacked (across forecasters and time) vector of disagreements of the jth variable. This means that, at the prior mode, the factors explain almost half of the standard

[10] We note that frequentist estimation of factor models in three-dimensional panels is also an active area of research (Lu and Su, 2018).

[11] The pdf of this inverse gamma random variable σ is given by

$$p(\sigma|\nu, s) = \frac{2}{\Gamma(\nu/2)} \left(\frac{\nu s^2}{2}\right)^{\nu/2} \frac{1}{\sigma^{\nu+1}} e^{-\nu s^2/(2\sigma^2)}.$$

deviation of the observed disagreements. This reflects the view that a factor structure is likely to be present in the data.

The prior distribution for Λ is as follows. The priors for each row of Λ, corresponding to the loading on the jth variable, are independent of one another; each is conditional their respective of σ_j^2. The first row, Λ_1, follows a truncated random normal distribution,

$$p(\Lambda_1) \propto \mathbf{1}_{\left\{\Lambda_{1,1}\geq 0, \Lambda_{1,k}>\Lambda_{1,k-1}\right\}} \times \mathcal{N}(0, \sigma_1^2 V_\Lambda), \tag{21}$$

where $N\left(0, \sigma_1^2 V_\Lambda\right)$ denotes the multivariate normal pdf with mean 0 and variance $\sigma_1^2 V_\Lambda$. The function $\mathbf{1}$ denotes the indicator which takes the value 1 if the conditions in brackets are satisfied and zero otherwise. This truncation ensures that: (1) the factors are sign normalized and (2) that each factor is different. In practice, the sign restrictions do not bind when eliciting the posterior distribution using the Gibbs sampler. The second normalization is not necessarily innocuous. We are assuming that each factor loads on the first variable—in all of our applications the disagreement for real GDP growth. Estimation results without this restriction imposed, however, deliver identical posteriors once the inequality restriction is imposed in post-processing of the draws. The prior of the remaining rows are simply given by unrestricted multivariate normal distributions,

$$\Lambda_j|\sigma_j^2 \sim \mathcal{N}\left(0, \sigma_j^2 V_\Lambda\right), \quad j = 2, \ldots, N. \tag{22}$$

The variance for each row is given by $\sigma_j^2 V_\Lambda$, where:

$$V_\Lambda = 4 \times I_p.$$

Note that this means–leaving aside the truncation–that we can write the prior distribution over Λ as matrix normal with mean zero and covariance matrices $\Sigma = \text{diag}(\sigma^2)$ (an $N \times N$ matrix) and V_Λ (a $p \times p$ matrix). The $vec(\Lambda)$ given Σ follows a multivariate normal distribution with mean zero and variance $V_\Lambda \otimes \Sigma$.

The priors for the autoregressive parameters for the factors and idiosyncratic errors are,

$$\phi_k \propto \mathbf{1}_{\{0\leq\phi_k<1\}}\mathcal{N}(\mu_\phi, V_\phi), \quad k = 1, \ldots, p \tag{23}$$

$$\text{and } \rho_j \propto \mathbf{1}_{\{-1\leq\rho_j<1\}}\mathcal{N}(\mu_\rho, V_\phi), \quad j = 1, \ldots, N. \tag{24}$$

That is, we require ϕ_k and ρ_n to lie in the unit interval. The values for the prior hyperparameters are:

$$\mu_\phi = \mu_\rho = 0.5, \text{ and } V_\phi = V_\rho = 0.15^2.$$

Estimation. The posterior distribution of the parameters is not available in closed form and thus we must rely on simulation methods to estimate the model. We follow the literature on Bayesian dynamic factor models—e.g., (Geweke and Zhou, 1996) and (Otrok and Whiteman, 1998)—and use a Gibbs sampler to elicit draws from the posterior. The sampler iterates over a sequence of conditional distributions, ultimately producing a set of (correlated) draws from the posterior distribution of interest. The Gibbs sampler, its validation, and convergence diagnostics are detailed in Appendix A.

4 Results

In this section, we present the parameter estimates of our baseline estimation and give an economic interpretation to these estimates. We also perform several variance decompositions to show which aspects of disagreement in the data are picked up by the estimated factors.

4.1 Parameter estimates

Table 2 shows the posterior mean, fifth, and ninety-fifth percentiles for Λ associated with the baseline model. Both factors load on real GDP substantially. A unit increase in the first factor increases an individual's forecast about real GDP by about 0.75 percent relative to the consensus forecast at the posterior mean. For the second factor, the increase is about 0.4, about half the magnitude of the associated coefficient for the first factor.

An increase in the first factor is associated with an unambiguous increase of individual forecasts of the components of real GDP and other real activity indicators. Employment forecasts increase, those of the unemployment rate, an inversely related labor market indicator, decrease. Similarly, estimates of recession probabilities also decrease. Most strikingly, while positive movements in the first factor tend to increase expectations of real activity, inflation expectations fall strongly. For this reason, we label the first factor *supply disagreement*, as it moves real activity and inflation expectations in opposite directions. The loadings on long-term real GDP and productivity forecasts are positive, and expectations of the natural rate of unemployment fall. Viewed through the lens of the semi-structural interpretation in Section 2.2, these patterns are consistent with disagreement about permanent productivity shocks.

A unit increase in the second factor is also associated with an increase of individual forecasts of the majority of real GDP components and other real activity indicators. In contrast to the first factor, an increase in the second factor is associated with an increase in inflation expectations. This positive comovement between inflation and real activity forecasts leads us to label the second factor as *demand disagreement*. In the long-term forecasts, an increase in the second factor leads an increase in long-run inflation expectations, and a smaller increase of productivity and real GDP forecasts. Interestingly, forecasts of federal government spending decrease in response to increases in the second factor. Through the lens of our semi-structural model, we can then rule out disagreement about fiscal shocks (at least at the federal level) as the source of disagreement represented by this factor. Likewise, short-term interest rate forecasts increase moderately in response to increases in the second factor, thus ruling out disagreement about monetary policy shocks.[12]

In fact, neither of the two factors are associated with meaningful movements in forecasts of short-term interest rates or spreads. Thus, the the factor reveals that disagreement about financial variables, though important in magnitude, is not strongly related to disagreement about real activity or prices. Of course, such a conclusion may be influenced by the assumptions underpinning the factor model, namely the independence of the factors and the number of factors. However, it is consistent with the simple fact that the unconditional correlations of interest rate disagreement with real GDP and CPI disagreement are only 0.09 and 0.17, respectively. In Section 6, we document that with four factors instead of two, some of the factors start loading on interest rates. Even then, patterns representing disagreement about monetary policy shocks do not emerge.

Because our factor model is dynamic, we can also use it to document the persistence of supply and demand disagreement. The bottom row of Table 2 displays the estimates of the autoregressive coefficients associated with the dynamics of each of the factors. The second factor representing demand disagreement, whose associated ϕ parameter has a posterior mean of 0.76, is substantially more persistent than first factor representing supply disagreement. In fact, its persistence is higher than that of disagreement in any of the medium-term forecasts in Table 1, except for the longer-run forecasts.

4.2 Variance decompositions

The estimates of Λ can already reveal something about the degree to which heterogeneity in expectations of different variables can be explained by the factors. To get a better sense of

[12]Our results do not preclude that forecasters may disagree about the *effects* of monetary policy, insofar as innovations to the second factor could stem from disagreement about the strength of aggregate demand that is induced by monetary policy surprises.

Table 2: Posterior of Λ and ϕ

Variable	$\Lambda_{\cdot 1}$ Mean	$\Lambda_{\cdot 1}$ [5, 95]	$\Lambda_{\cdot 2}$ Mean	$\Lambda_{\cdot 2}$ [5, 95]
GDP Components				
GDP	0.78 (0.00)	[0.75, 0.82]	0.41 (0.01)	[0.35, 0.47]
Consumption	0.66 (0.00)	[0.62, 0.70]	0.36 (0.01)	[0.31, 0.42]
Non-res. investment	1.14 (0.01)	[1.03, 1.24]	0.76 (0.01)	[0.66, 0.86]
Res. investment	1.65 (0.01)	[1.48, 1.82]	1.08 (0.02)	[0.92, 1.24]
Non-federal gov't	0.28 (0.00)	[0.24, 0.32]	0.14 (0.00)	[0.11, 0.17]
Federal gov't	0.20 (0.00)	[0.12, 0.29]	-0.06 (0.00)	[-0.11, -0.00]
Inventories	0.04 (0.00)	[0.03, 0.05]	0.03 (0.00)	[0.03, 0.04]
Net exports	0.05 (0.00)	[0.03, 0.07]	-0.01 (0.00)	[-0.02, 0.00]
Other Real Activity				
Housing starts	0.80 (0.02)	[0.59, 1.01]	1.36 (0.01)	[1.15, 1.55]
Industrial production	0.30 (0.01)	[0.25, 0.36]	0.52 (0.00)	[0.48, 0.57]
Corporate profits	0.63 (0.01)	[0.48, 0.78]	1.06 (0.01)	[0.92, 1.21]
Labor Market				
Unemployment rate	-0.06 (0.00)	[-0.07, -0.05]	-0.12 (0.00)	[-0.13, -0.11]
Employment	0.23 (0.00)	[0.19, 0.28]	0.20 (0.00)	[0.17, 0.24]
Inflation				
GDP deflator	-0.39 (0.00)	[-0.41, -0.37]	0.19 (0.00)	[0.15, 0.22]
CPI	-0.34 (0.00)	[-0.38, -0.31]	0.30 (0.00)	[0.27, 0.33]
Core CPI	-0.24 (0.00)	[-0.27, -0.22]	0.21 (0.00)	[0.19, 0.23]
PCE	-0.29 (0.00)	[-0.33, -0.26]	0.26 (0.00)	[0.24, 0.29]
Core PCE	-0.21 (0.00)	[-0.24, -0.19]	0.19 (0.00)	[0.17, 0.21]
Interest rates				
3m T-bill rate	-0.01 (0.00)	[-0.03, 0.01]	0.07 (0.00)	[0.05, 0.08]
10y term spread	0.01 (0.00)	[-0.01, 0.03]	-0.00 (0.00)	[-0.02, 0.01]
Aaa spread	-0.03 (0.00)	[-0.04, -0.01]	-0.03 (0.00)	[-0.04, -0.02]
Baa-Aaa spread	-0.02 (0.00)	[-0.03, -0.00]	-0.02 (0.00)	[-0.03, -0.01]
Recession prob.				
Recession prob.	-1.22 (0.02)	[-1.47, -0.96]	-1.51 (0.02)	[-1.77, -1.25]
Long-term forecasts				
Natural rate of unemp.	-0.08 (0.00)	[-0.13, -0.04]	-0.04 (0.00)	[-0.07, -0.01]
Stock Prices	0.10 (0.00)	[-0.10, 0.31]	0.13 (0.00)	[0.01, 0.26]
Productivity	0.10 (0.00)	[0.06, 0.15]	0.03 (0.00)	[0.00, 0.06]
Real GDP	0.14 (0.00)	[0.11, 0.17]	0.05 (0.00)	[0.03, 0.07]
Treasury Bill	-0.05 (0.00)	[-0.14, 0.03]	0.16 (0.00)	[0.11, 0.21]
Term Spread	0.02 (0.00)	[-0.07, 0.11]	-0.11 (0.00)	[-0.16, -0.06]
Inflation	-0.10 (0.00)	[-0.12, -0.08]	0.10 (0.00)	[0.09, 0.11]
autoregressive coef. (ϕ)	0.52 (0.00)	[0.49, 0.55]	0.75 (0.00)	[0.73, 0.77]

Note: Parameter estimates are computed using pooled draws from 10 MCMC chains each of length 40,000 after a 10,000 period burn-in. Standard deviation of the posterior mean across chains in parentheses. See the Appendix for details.

where the factors matter, we next decompose the variance of the observables into portions attributable to each of the factors and the idiosyncratic term. We do this from two temporal perspectives: the unconditional variance, and the conditional variation at each time t.

The unconditional variance of any given observable y_{jit} conditional on the parameters θ can be written as the sum of the variation due to each of the factors and the idiosyncratic component, since these are all assumed to be independent. Thus:

$$\mathbb{V}[y_{jit}|\theta] = \sum_{k=1}^{p} \Lambda_{jk}^2/(1-\phi_k^2) + \sigma_j^2/(1-\rho_j^2). \tag{25}$$

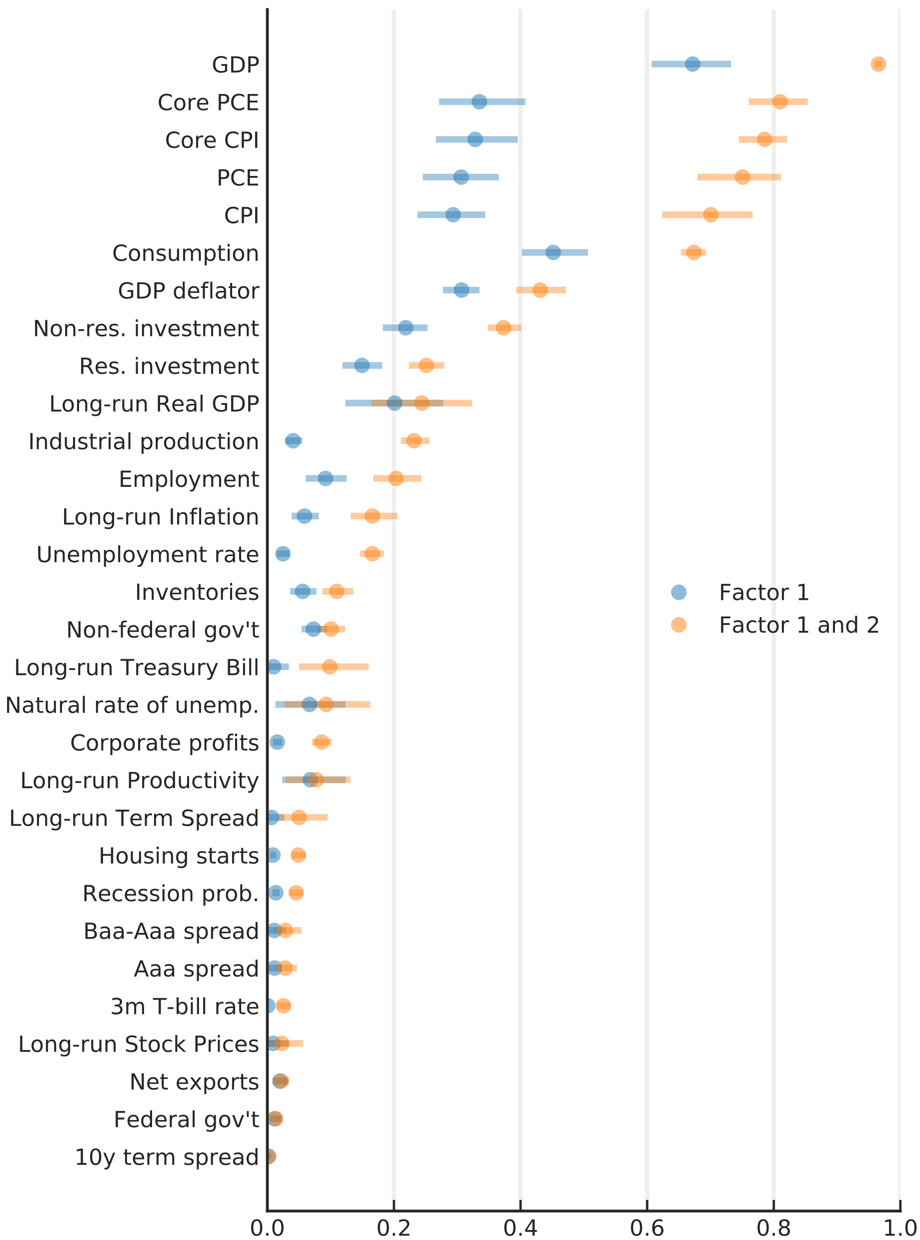
The share of this variance due to factor k is given by:

$$\text{share of variance of } j \text{ due to factor } k = \frac{\Lambda_{jk}^2/(1-\phi_k^2)}{\mathbb{V}[y_{jit}|\theta]}.$$

We use draws from the posterior to compute the distribution of this measure for each observable and factor. Figure 1 plots the variance decomposition for each of the 28 observables in our baseline model. The dots indicate the mean estimate, while the bars span the fifth to ninety-fifth percentile of this statistic. The figure plots the cumulative variance shares, so that the distance between the blue and yellow circles represents the share of variation due to the second factor. The observables in Figure 1 are sorted from most to least amount of variation explained by the factors. On average, the two factors explain about 25 percent of the variation in the observables. This means that most much of the variation in forecast disagreement in the SPF is uncorrelated across variables, consistent with Dovern (2015). The factors explain almost all of the movements in GDP, core PCE, and core CPI, with the factors contributing in roughly equal measure. The factors explain little about the disagreement in financial variables as well as federal government spending and net exports. The disagreement about these variables is largely disconnected from variables like GDP and core PCE, even though the factor model allows for flexible modeling of these correlations.

Next, we analyze the relative role of the factors in explaining disagreement at a particular point in time in the data. This decomposition is not straightforward because of the presence of multiple forecasters in our sample, and because the disagreement, averaged over these forecasters, is zero. Therefore, to analyze the relative roles of the factors and idiosyncratic shocks , we examine the cross-sectional covariance of disagreement with each model-implied component. Let $\hat{\mathbb{V}}_i$ and $\widehat{\mathbb{C}ov}_i$ be the empirical cross-sectional variance and

Figure 1: Unconditional Variance Decompositions



Note: The figure plots the cumulative share of the unconditional variance for the observables attributable to the first factor (blue) and the sum of the two factors (orange) according to Equation (25). The dots indicate the mean estimate, while the bars span the fifth to ninety-fight percentile of the statistic. Based on 500 draws from the posterior.

covariance, respectively. For example:

$$\widehat{\mathbb{C}ov}_i\left(y_{jit}, \mathbb{E}\left[\xi_{jit} \mid \theta, Y\right]\right) = \frac{1}{|\mathcal{I}_t|}\sum_{i\in\mathcal{I}_t} y_{jit}\left(\mathbb{E}\left[\xi_{jit}|\theta, Y\right] - \frac{1}{|\mathcal{I}_t|}\sum_{j\in\mathcal{I}_t}\mathbb{E}\left[\xi_{jit}|\theta, Y\right]\right).$$

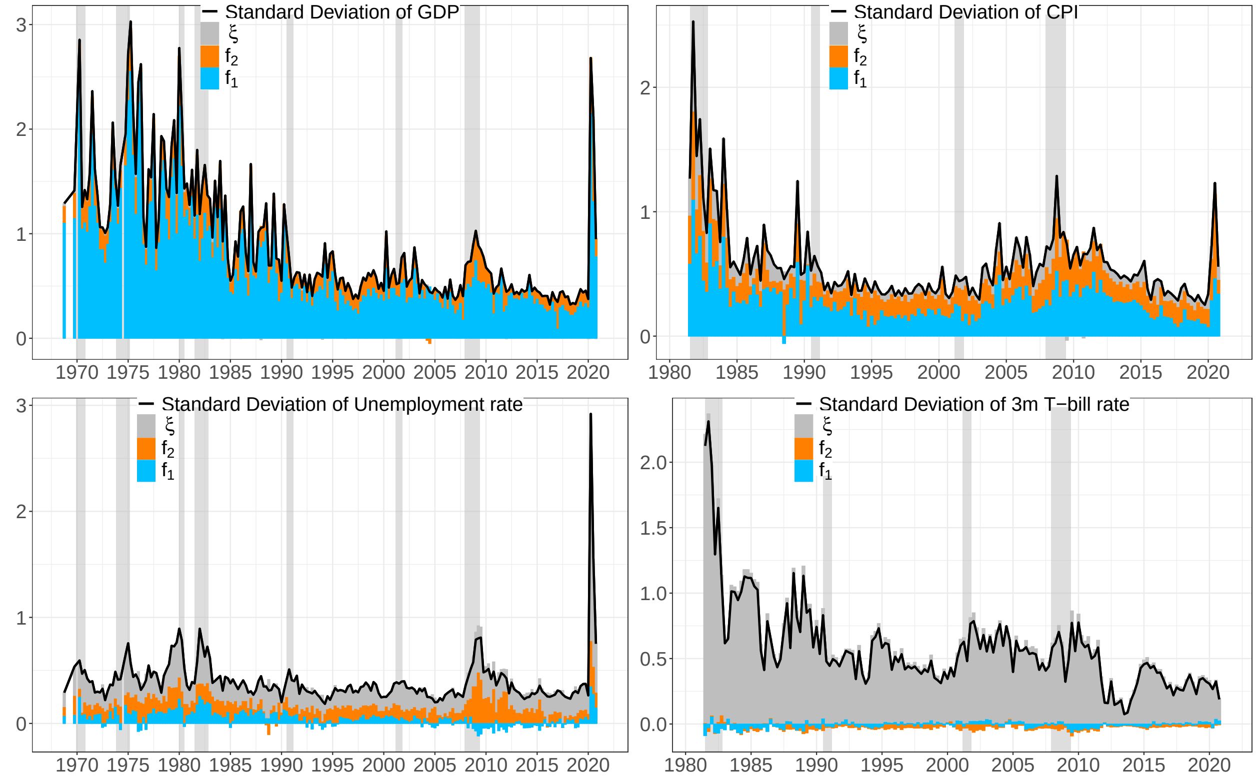
We can decompose the variance of disagreement $\mathbb{V}_i\left(y_{jit}\right)$ at any point in time into components based on the estimated latent factors. Given the data Y and a set of parameters θ:

$$\mathbb{V}_i\left(y_{jit}\right) = \sum_{k=1}^{p}\Lambda_{jk}\widehat{\mathbb{C}ov}_i\left(y_{jit}, \mathbb{E}\left[f_{jit} \mid \theta, Y\right]\right) + \mathbb{C}ov_i\left(y_{jit}, \mathbb{E}\left[\xi_{jit} \mid \theta, Y\right]\right). \tag{26}$$

In Figure 2, we plot this decomposition for GDP and the CPI at the posterior mean parameter values. For better readability of the figure, we take the (signed) square root, so that we decompose the standard deviation instead of the variance.

The black lines correspond to the cross-sectional forecast dispersion in the data, while the blue, orange, and gray regions correspond to the contributions from the first factor, second factor, and idiosyncratic term according to the above decomposition. The top left panel shows that forecast dispersion in real GDP traces the decline of disagreement since the start of the Great Moderation. Disagreement increases during the Great Recession and spikes to levels not seen since the 1970s during the Covid-19 crisis. The two factors explain nearly all of the variation in real GDP disagreement over the whole sample, consistent with the earlier unconditional decomposition. The figure also reveals that the decline in disagreement over the sample is attributed mainly to a decline in the contribution of the first factor, while the contribution of the second factor is more stable over time. As seen in the top-right panel, similar patterns hold for CPI inflation, a series that starts in the early 1980s, although a noticeable portion of dispersion in that variable is left unexplained by the factors. The bottom-left panel illustrates that dispersion in unemployment rate forecasts is explained less well by the factors. The exception is the Great Recession and its aftermath. During this time, the second factor representing demand disagreement explains a large portion of the dispersion in unemployment rate forecasts. Finally, the bottom-right panel illustrates that interest rate forecast dispersion is not being picked up much at all by the factors, again consistent with the earlier unconditional decomposition. If anything, it is the demand factor that can explain some of this dispersion. Interestingly, the contribution of this factor is negative during the Volcker disinflation of 1981 and 1982. This negative contribution indicates that the cross-sectional covariance of the second factor and interest rate disagreement was negative during this time, while the interest rate loading is positive. This is the only period of time in our sample when disagreement about the course of monetary

Figure 2: Decomposition of Dispersion Over Time.



Note: Decomposition of the sample variance of disagreement across forecasters into sample covariances with both factors and error terms as in Equation (26). Variables are real GDP, CPI inflation, the unemployment rate and the 3-month T-bill rate. Factors and error terms are smoothed mean values at the posterior mean parameters. Variances and covariances are transformed by taking the signed squared root. Black line is the sample cross-sectional standard deviation.

policy—i.e., high individual forecasts of real activity and inflation forecasts being associated with low interest rate forecasts—seems to play a role in the data.

5 Factors as a measure of disagreement

Most of the existing literature on disagreement compresses the distribution of cross-sectional forecasts of a particular variable into a summary statistic measuring dispersion of expectations, such as the standard deviation. In our model, we can compute new measures of the dispersion of expectations by computing the cross-sectional standard deviation of our estimated factors. The measures of dispersion thus obtained are more comprehensive than those based on a single variables because they aggregate disagreement about many variables. Moreover, factor dispersion represents common components of heterogeneity in the set of forecast variables and can be given a structural interpretation.

We evaluate the dispersion of the mean smoothed factors:

$$\text{Factor Dispersion}_{kt}(\theta) = \sqrt{\hat{\mathbb{V}}_i\left(\mathbb{E}[f_{kit}|Y,\theta]\right)}. \tag{27}$$

Figure 3 plots the time series of factor dispersion at the posterior mean parameter values. The figure shows that our model can capture time variation in dispersion ex post, even though our model assumes homoskedasticity ex ante.

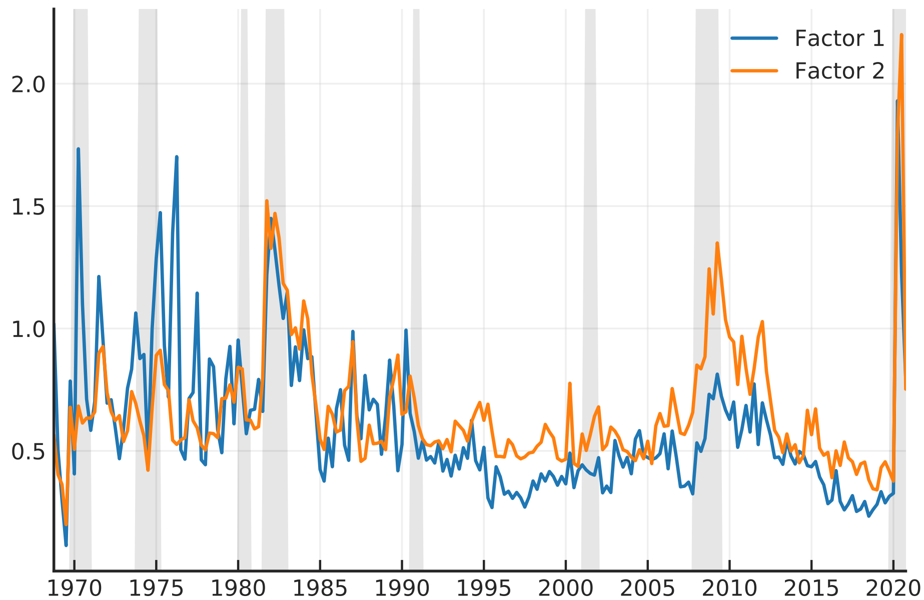
Prior to the COVID pandemic, dispersion for the first factor, which we interpret as supply disagreement, was highest in the early part of the sample. Supply disagreement declined at the start of the Great Moderation and has hovered at low levels until 2020, interspersed only with a moderate peak during the Great Recession. We confirm the fading importance of supply disagreement with split-sample analysis in Section 6.

The dispersion of the second factor, which we interpret as demand disagreement,reached about equal peaks during the Volcker disinflation and the Great Recession. Unlike the first factor, the dispersion of the second factor has not declined over the sample. This implies that the contribution of demand disagreement relative to supply disagreement has become more important since the Great Moderation. Demand disagreement also seems to have been the dominant source of disagreement during the Great Recession.

At the onset of the COVID pandemic, dispersion for both factors increased dramatically, with demand disagreement hitting an unprecedented level, and supply side disagreement reaching levels not seen since the 1970s.

The dispersion of each of the two factors follows distinct patterns. The correlation between of the two time series is only 0.65, as shown in Table 3, such that each factor provides distinct information about the evolution of forecaster disagreement over time. The table also shows that factor dispersion constitutes a measure of disagreement that is distinct from univariate measures. Dispersion of the first factor (supply disagreement) is strongly correlated with both dispersion in real GDP and inflation forecasts, while dispersion of the second factor (demand disagreement) is only weakly correlated with either. None of the factors correlate strongly with dispersion in interest rate forecasts, which is consistent with the factors not exhibiting large loadings on this variable. We can interpret factor dispersion as a new measure of disagreement among forecasters that aggregates information from many variables. At the same time, it shares many characteristics of disagreement that have been documented previously for disagreement about a single variable, most commonly inflation. For example, Mankiw, Reis, and Wolfers (2003) have documented several stylized facts about the dispersion of inflation forecasts, including that it rises with the level of inflation, rises with changes in inflation, and shows no clear relationship with the output gap. We revisit

Figure 3: Cross-Sectional Factor Dispersion Over Time



Note: Figure plots the time series factor dispersion, defined in (27), evaluated at the posterior mean parameter vector. The dispersion for factor 1 is displayed in blue and dispersion for factor 2 is displayed in orange. Shaded regions denote NBER recession dates.

Table 3: Factor Dispersion and Univariate Forecast Dispersion

	Factor 1	Factor 2	GDP deflator	Real GDP	3m T-bill rate
Factor 1	1.00				
Factor 2	0.65	1.00			
GDP deflator	0.79	0.47	1.00		
Real GDP	0.86	0.48	0.77	1.00	
3-month T-bill rate	0.46	0.35	0.60	0.50	1.00

Note: The table reports pairwise correlations of the time series of dispersion (cross-sectional standard deviation) for each factor, as well as for the individual forecasts of the GDP deflator, real GDP, and the 3-month T-bill rate at the four-quarter horizon.

these facts in our extended sample in Table 4, and also examine them for the dispersion in real GDP and interest rate forecasts, as well as for the dispersion in our estimated factors.

The first row of Panel A shows that supply disagreement (first factor) rises when the level of inflation is high, which also holds for dispersion in inflation, real GDP and interest rate forecasts. However, demand disagreement (second factor) is not systematically related to the level of inflation. We can thus already see how the factors provide measures of disagreement that are different from univariate measures. The second row shows that dispersion in both factors rises at times when the level of inflation changes. Again, this is also the case for real GDP and inflation forecast dispersion and to a lesser extent for interest rate forecast dispersion. The third row shows factor dispersion is strongly countercyclical. This is is contrast to univariate dispersion measures and also to the findings of Mankiw, Reis, and Wolfers (2003), underlining that factor dispersion carries additional information about disagreement.

The fourth and fifth row document that disagreement bears little relation to the consensus forecasts of inflation and GDP growth in our data set.[13] There is some statistical significance of the consensus forecast for the change in inflation, but it disappears in the multivariate regressions shown in Panel B. This result suggests that our choice of modeling disagreement as independent of the consensus forecast is supported by the data. Finally, the last row of Panel A documents that heterogeneity of expectations is not related to financial volatility as measured by the VIX index.[14]

[13]Similar results hold for consensus forecasts of other variables.

[14]This variable is not included in Panel B because of its shorter time sample.

Table 4: FACTOR DISPERSION AND ECONOMIC FUNDAMENTALS

	Factor 1	Factor 2	GDP deflator	Real GDP	3m T-bill rate
Panel A: bivariate regressions (each cell represents a separate regression)					
π_t	0.067***	0.017	0.113***	0.183***	0.198***
	(0.016)	(0.062)	(0.012)	(0.025)	(0.033)
$(\Delta\pi_t)^2$	0.554***	0.395	0.609***	0.939***	0.612
	(0.125)	(0.272)	(0.173)	(0.133)	(0.398)
$ygap_t$	-0.062***	-0.103***	-0.044	-0.044	-0.038
	(0.020)	(0.025)	(0.031)	(0.040)	(0.025)
$\hat{\bar{\pi}}_{t+4\|t} - \pi_{t-1}$	-0.029**	-0.007	-0.028**	-0.057**	-0.014
	(0.011)	(0.013)	(0.013)	(0.027)	(0.012)
$\hat{\bar{y}}_{t+4\|t} - y_{t-1}$	-0.001	-0.025	-0.021	-0.006	0.024
	(0.030)	(0.026)	(0.047)	(0.008)	(0.035)
VIX_t	-0.009	-0.011	-0.007	0.005	0.018**
	(0.017)	(0.031)	(0.009)	(0.026)	(0.007)
Panel B: multivariate regressions					
π_t	0.057***	0.013	0.105***	0.169***	0.185***
	(0.016)	(0.019)	(0.012)	(0.030)	(0.026)
$(\Delta\pi_t)^2$	0.256*	0.265**	0.150	0.209	0.208*
	(0.140)	(0.107)	(0.155)	(0.206)	(0.129)
$ygap_t$	-0.052***	-0.086***	-0.047**	-0.036*	-0.010
	(0.010)	(0.022)	(0.014)	(0.022)	(0.019)
$\hat{\bar{\pi}}_{t+4\|t} - \pi_{t-1}$	-0.003	-0.047	-0.007	-0.011	0.036
	(0.028)	(0.040)	(0.017)	(0.049)	(0.033)
$\hat{\bar{y}}_{t+4\|t} - y_{t-1}$	-0.010	-0.003	0.001	-0.014	0.003
	(0.007)	(0.008)	(0.010)	(0.014)	(0.011)
R^2	0.498	0.500	0.546	0.609	0.645

Note: The dependent variables are the time series of dispersion (cross-sectional standard deviation) for each factor, as well as for the individual forecasts of the GDP deflator, real GDP, and the 3-month T-bill rate at the four-quarter horizon. The regressors are the four-quarter change in the GDP deflator, the squared difference between the four-quarter change of the GDP deflator at time t and the corresponding change at time $t-4$, the difference between actual output and potential output as estimated by the CBO, the consensus forecasts for the change in inflation and output growth between $t+4$ and $t-1$,and the quarterly average of the VIX index. Newey-West standard errors in parentheses.

6 Robustness

In this section, we demonstrate the robustness of our results to the set of variables included in the estimation, to different forecast horizons, as well as to the number of factors. We also explore differences in our estimates when applied to subsamples of the data, and discuss the relation of disagreement to consensus forecasts.

6.1 Set of variables

Our results are robust to the precise set of SPF variables that are included in the estimation. Our approach is to include all variables available for our chosen forecast horizons, with the exception of nominal GDP. Because nominal GDP is the sum of real GDP and the GDP deflator in logarithms, including it in the estimation introduces a collinearity that violates the assumption that the idiosyncratic components ξ are uncorrelated across variables. In fact, including nominal GDP does tilt the factor loadings towards explaining this collinearity, but qualitatively our results are little changed. We also verify that our results carry through if we estimate the factor model only on the six variables (again omitting nominal GDP) that have been present since the start of the survey in 1968: Real GDP, housing starts, industrial production, corporate profits, the unemployment rate, and the GDP deflator. The results of this estimation are contained in Appendix D.1.

6.2 Forecast horizons

In our baseline specification, we include forecasts at the four-quarter horizon as well as longer-run forecasts. Appendix D.2 shows estimation results for the model using either nowcasts, one-quarter, two-quarter, or three-quarter ahead forecasts. These estimations confirm that our results are robust to the choice of the forecast horizon. One difference that materializes across forecast horizons is that the factors are able to cover a larger share of the variance in the data as the forecast horizon increases. For nowcasts, the factors still explain a large share of the cross-sectional variance in GDP and inflation predictions, but only a small share of the variance in the remaining variables. This share then increases gradually as the forecast horizon increases. This finding could indicate that heterogeneity in very short-run forecasts may be driven more by disagreement about measurement that is specific to individual variables, while heterogeneity of forecasts at medium- to longer-term horizons may reflect disagreement about more structural economic forces.

6.3 Number of factors

Determining the appropriate number of factors p is challenging. It is natural, given our Bayesian model, to use Bayesian model selection techniques. Unfortunately, this requires a dependable estimate of the marginal data density $p(Y) = \int p(Y|\theta)p(\theta)d\theta$. In our setting, this can only estimated via Monte Carlo. Given the size of the parameter space, particularly as the number of factors grows, the variance of this estimator is too large for reliable model comparison. Instead, we use an informal metric. Table 5 documents how much the share of the cross-sectional variance of forecasts explained by the factors increases with p. The table shows that going from zero factors to one factor, and again from one factor to two factors, our factor model is able to explain a large additional share of disagreement in a number of variables. The model with $p = 1$ explains almost all of real GDP disagreement as well as disagreement about a range of other real activity variables. With $p = 2$, the model can additionally explain disagreement in the inflation variables.

However, adding more factors beyond $p = 2$ only leads to minor improvements in the explained variance shares. If anything, the model with $p = 3$ is able to pick up some additional disagreement in industrial production and the unemployment rate. With $p = 4$, the factors are able to explain forecasts of the 3-month T-bill rate and the 10-year term spread, picking up a strong negative correlation of disagreement in the data (the sample correlation is -0.48). But apart from these two variables, the ability of the model to explain the data is unchanged. We document the results from these estimations in detail in Appendix D.3. We therefore conclude that the model with two factors offers the best balance between parsimony and an ability to explain the most salient comovement features in the data.

The case of $p = 4$ factors is of some interest because the factors pick up on short-term interest rates in this case. In our baseline estimation, the factors cannot fit disagreement in short-term interest rates well, and our result that disagreement about the course of monetary policy does not play a role in the data might be sensitive to this lack of fit, and in particular to the number of factors. However, this is not the case. Even with $p = 4$, we find no evidence of a factor that describes monetary policy disagreement, in the sense that it has positive loadings on real activity and inflation and negative loadings on interest rates.[15] Rather, the four factors seem to describe two versions of demand disagreement and two versions of supply disagreement, with the second version of each loading more heavily on investment and housing starts than the first.

[15] If anything, the third factor in Table D.11 fits this pattern, but the loading on interest rates is not statistically different from zero.

Table 5: Number of Factors and the Explained Share of Variance.

Variable	$p = 1$	$p = 2$	$p = 3$	$p = 4$
GDP Components				
GDP	0.97	0.00	0.01	0.00
Consumption	0.67	0.00	0.01	0.00
Non-res. investment	0.37	0.00	0.07	-0.01
Res. investment	0.25	0.00	0.09	0.01
Non-federal gov't	0.10	0.00	0.00	0.00
Federal gov't	0.00	0.01	0.02	0.00
Inventories	0.11	0.00	0.04	-0.01
Net exports	0.01	0.01	0.07	0.00
Other Real Activity				
Housing starts	0.03	0.02	0.10	0.01
Industrial production	0.14	0.09	0.22	0.00
Corporate profits	0.05	0.03	0.08	0.00
Labor Market				
Unemployment rate	0.09	0.07	0.17	0.01
Employment	0.20	0.00	0.11	-0.01
Inflation				
GDP deflator	0.11	0.32	0.06	0.01
CPI	0.00	0.70	0.09	0.00
Core CPI	0.01	0.78	0.02	0.00
PCE	0.01	0.74	0.08	0.00
Core PCE	0.01	0.80	0.00	0.00
Interest rates				
3m T-bill rate	0.01	0.02	0.01	0.65
10y term spread	0.00	0.00	0.00	0.73
Aaa spread	0.03	0.00	0.01	0.00
Baa-Aaa spread	0.03	0.00	0.01	0.00
Recession prob.				
Recession prob.	0.03	0.01	0.02	0.00
Long-term forecasts				
Natural rate of unemp.	0.08	0.01	0.00	0.04
Long-run Stock Prices	0.02	0.01	0.01	0.00
Long-run Productivity	0.06	0.02	-0.01	0.00
Long-run Real GDP	0.22	0.03	-0.01	0.00
Long-run Treasury Bill	0.03	0.07	0.22	0.02
Long-run Term Spread	0.02	0.02	0.23	0.03
Long-run Inflation	0.01	0.16	0.07	0.00

Note: Each column contains the combined share of the variance explained by the factors in the model with p factors, minus the corresponding share in the model with $p-1$ factors. The shares are computed using Equation (25).

6.4 Subsamples

Our model assumes that the factor structure of disagreement is time-invariant. This is of course a simplification, since forecast dispersion does vary over time: It was generally higher before the 1980s, and tends to be countercyclical. Here, we examine the stability of our estimates over time with a simple split sample analysis. Table 6 splits the sample in 1990. This is the year in which a lot of forecasters exited the survey and new ones entered, as the Philadelphia Fed took over the SPF from the NBER. It also splits the sample into a pre- and post-moderation period.

Table 6 reveals that the factor structure of disagreement has changed substantially over time. In the second subsample after 1990, the factor loadings look very similar to those in our full sample, with the first factor describing supply disagreement and the second factor describing demand disagreement. In the first subsample before 1990, however, supply disagreement dominates: Both factors describe negative comovement between GDP and inflation forecasts. The first factor is qualitatively similar to its counterpart post-1990. The exception is in housing starts, IP and corporate profits, where the loadings are negative but not statistically different from zero. The second factor pre-1990 also describes supply disagreement, even though the loadings on inflation are small (and insignificant for CPI inflation, a variable that is only included in the survey since 1981). The factor loads more heavily on residential investment and housing than the first factor.

In sum, supply disagreement is more prominent in the data early in the sample, in line with our previous discussion of time-varying factor dispersion in Section 5. Appendix D.4 contains results for additional sample splits, which yield similar results.

6.5 Heterogeneity of model parameters

Our factor model assumes that all forecasters share the same parameters and differ only in their realizations of the factors and idiosyncratic components. In particular, the factors load on variables with the same loadings for all forecasters. This modeling choice aligns with structural models of heterogeneous expectations in the literature (Section 2.2) and allows us to us to extract the most important comovement relationships in disagreement across variables in a parsimonious way. However, it may also mask substantial heterogeneity of forecasters in other dimensions. For example, one may conjecture that, contrary to what is assumed in structural models, forecasters use different models to interpret similar information, violating our assumption of identical loadings.

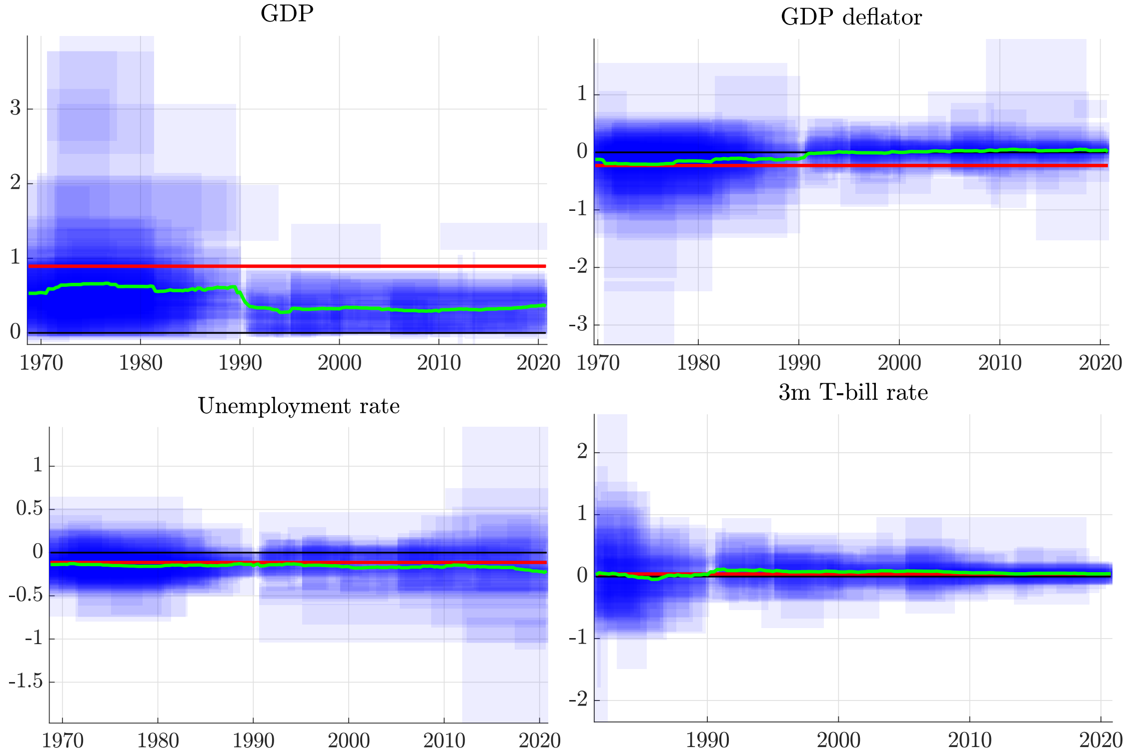
Here, we document that this type of heterogeneity is indeed likely to be present in the data, but that our parameter estimates based on the entire panel are akin to a weighted

Table 6: Posterior of Λ in Split Samples.

Variable	$\Lambda_{\cdot 1}$				$\Lambda_{\cdot 2}$			
	1968Q4–1990Q2		1990Q3–2020Q4		1968Q4–1990Q2		1990Q3–2020Q4	
	Mean	[5, 95]	Mean	[5, 95]	Mean	[5, 95]	Mean	[5, 95]
GDP Components								
GDP	0.97	[0.88, 1.05]	0.43	[0.37, 0.46]	0.69	[0.62, 0.76]	0.19	[0.12, 0.29]
Consumption	0.72	[0.56, 0.86]	0.40	[0.34, 0.43]	0.60	[0.51, 0.68]	0.19	[0.12, 0.28]
Non-res. investment	0.81	[0.52, 1.10]	0.80	[0.68, 0.88]	1.16	[0.99, 1.34]	0.42	[0.28, 0.61]
Res. investment	0.57	[-0.04, 1.21]	1.03	[0.86, 1.16]	2.30	[1.98, 2.62]	0.54	[0.34, 0.79]
Non-federal gov't	0.77	[0.60, 0.93]	0.15	[0.12, 0.18]	0.14	[0.04, 0.25]	0.07	[0.03, 0.11]
Federal gov't	1.11	[0.74, 1.47]	0.13	[0.09, 0.18]	-0.49	[-0.68, -0.30]	0.02	[-0.03, 0.07]
Inventories	0.03	[0.00, 0.06]	0.03	[0.03, 0.04]	0.06	[0.04, 0.07]	0.02	[0.01, 0.03]
Net exports	0.07	[0.01, 0.14]	0.03	[0.02, 0.04]	0.03	[0.00, 0.07]	-0.01	[-0.02, -0.00]
Other Real Activity								
Housing starts	-0.16	[-0.53, 0.21]	1.37	[1.04, 1.64]	2.70	[2.41, 3.01]	0.98	[0.66, 1.36]
Industrial production	-0.10	[-0.20, 0.00]	0.35	[0.28, 0.40]	1.09	[1.03, 1.16]	0.25	[0.18, 0.33]
Corporate profits	-0.23	[-0.53, 0.06]	0.70	[0.53, 0.86]	2.24	[2.06, 2.42]	0.32	[0.12, 0.54]
Labor Market								
Unemployment rate	0.00	[-0.01, 0.02]	-0.09	[-0.11, -0.06]	-0.18	[-0.19, -0.16]	-0.10	[-0.13, -0.08]
Employment			0.15	[0.12, 0.18]			0.11	[0.08, 0.16]
Inflation								
GDP deflator	-0.59	[-0.64, -0.54]	-0.07	[-0.11, -0.03]	-0.05	[-0.09, -0.01]	0.19	[0.17, 0.20]
CPI	-0.11	[-0.23, 0.00]	-0.10	[-0.17, -0.06]	-0.02	[-0.08, 0.04]	0.28	[0.25, 0.30]
Core CPI			-0.09	[-0.13, -0.05]			0.20	[0.18, 0.22]
PCE			-0.12	[-0.18, -0.07]			0.27	[0.24, 0.29]
Core PCE			-0.08	[-0.12, -0.04]			0.18	[0.16, 0.20]
Interest rates								
3m T-bill rate	0.02	[-0.06, 0.10]	0.02	[-0.01, 0.03]	-0.06	[-0.11, 0.00]	0.08	[0.07, 0.10]
10y term spread			0.01	[-0.00, 0.02]			-0.01	[-0.02, 0.00]
Aaa spread			-0.02	[-0.03, -0.01]			-0.02	[-0.02, -0.01]
Baa-Aaa spread			-0.01	[-0.02, -0.01]			-0.01	[-0.02, -0.01]
Recession prob.								
Recession prob.	-0.90	[-1.43, -0.35]	-0.84	[-1.10, -0.57]	-3.17	[-3.60, -2.74]	-0.72	[-1.01, -0.45]
Long-term forecasts								
Natural rate of unemp.			-0.05	[-0.08, -0.03]			-0.01	[-0.04, 0.01]
Stock Prices			0.05	[-0.05, 0.16]			0.09	[-0.01, 0.19]
Productivity			0.05	[0.03, 0.07]			0.01	[-0.01, 0.03]
Real GDP			0.08	[0.06, 0.10]			0.02	[-0.00, 0.04]
Treasury Bill			0.01	[-0.04, 0.05]			0.12	[0.08, 0.16]
Term Spread			-0.03	[-0.07, 0.02]			-0.07	[-0.11, -0.03]
Inflation			-0.03	[-0.06, -0.01]			0.10	[0.08, 0.11]

Note: Parameter estimates are computed using pooled draws from 10 MCMC chains each of length 40,000 after a 10,000 period burn-in. See the Appendix for details.

Figure 4: INDIVIDUAL AND PANEL ESTIMATES OF FACTOR LOADINGS.



Note: Each panel plots the posterior distribution of factor loadings for the model with $p = 1$, estimated either on the entire panel or on each forecaster individually. Red lines represent the panel posterior mean. Blue rectangles represent individual estimates of a single forecaster. The horizontal position indicates the time when that forecaster is present in the sample, and the vertical position indicates the range of values containing 67 percent of the posterior distribution. Green lines denote the average individual posterior mean. Individual estimates are displayed only for forecasters with more than 20 observations.

average of the individual parameters. We estimate our factor model separately for each forecaster. That is, in Equations (3), (4) and (5), we replace the common parameters Λ, Φ, P and Σ with individual parameters Λ_i, Φ_i, P_i and Σ_i with priors that are independent across forecasters. A problem that arises in this context is that the time series for individual forecasters are relatively short, which means that we cannot reliably identify more than one factor per forecaster[16]. We thus restrict ourselves to the one-factor model with $p = 1$.

Figure 4 visualizes the resulting distribution of posterior estimates for the individual factor loadings Λ_{ij} for a few variables (all other variables are documented in Appendix D.5). Each blue rectangle represents the 67 percent range of the posterior distribution for an individual forecaster. It is immediately apparent that the individual loadings display sizable heterogeneity, and are also imprecisely estimated even with only one factor.

[16]Recall that our strategy for identifying multiple factors operates through the restriction that Φ is diagonal, which is only valid when the diagonal elements are distinct. The short time series for individual forecasters leads to weak identification of Φ, rendering this strategy ineffective.

However, the average of the individual loadings (green lines) are close to the loadings of our baseline model estimated on the whole panel (red lines). If anything, the individual average is sometimes smaller in magnitude than the panel estimate. This discrepancy can be explained by the higher weight of the zero-centered prior: While the combined panel has 8,445 observations, the average number of observations for an individual is just 19. Thus, even though some heterogeneity of parameters exists, our model can be used to adequately describe the average behavior of disagreement in the data.

6.6 Relation to consensus forecasts

The data we use to estimate our factor model are deviations of individual forecasts from the consensus (mean) forecast. We thus ignore all forecast movements that affect all forecasters alike. Of course, movements in consensus forecasts are also informative for expectations formation, as shown by many existing studies in the literature. One might worry that our model ignores this valuable information by assuming that the consensus forecast is disconnected from disagreement. Here, we present some evidence in favor of our assumption.

We apply the factor model (3)–(5) on consensus forecasts $\bar{y}_{jt}$ instead of disagreement y_{ijt}, treating the observations as if they were coming from a single forecaster ($M = 1$). The priors are the same as before. The consensus forecasts constitute a standard two-dimensional panel in time and forecast variables. Moreover, the consensus forecasts are closely tied to the realizations of the variables. For these two reasons, the estimation of the factor model on consensus forecasts becomes quite similar to the dynamic factor models on realized data e.g. in Stock and Watson (2002).

Figure 5 displays the variance decomposition according to Equation for the model estimated on consensus forecasts. Additional estimation results are reported in Appendix D.6. Comparing the figure to the corresponding variance decomposition of disagreement in Figure (1), one can immediately see that the consensus estimates are subject to greater uncertainty, a direct result of the fact that the consensus data has many fewer observations (175 quarterly observations, compared to 8,457 in the data on disagreement).

More importantly, the figure reveals that the two factors are essentially identical to the consensus inflation and real GDP forecasts: The first factor explains 95 percent of the consensus CPI forecast, while the second factor explains 90 percent of the consensus real GDP forecast. All variables are explained exclusively by one of the two factors, save for core CPI and PCE inflation for which the available time series are short. This behavior is very different from our baseline estimation on disagreement and indicates that there is no strong relationship between factors driving disagreement and factors driving consensus forecasts.

Figure 5: Unconditional Variance Decompositions in the Consensus Forecast Model.

Note: The figure plots the cumulative share of the unconditional variance for the consensus forecast model attributable to the first factor (blue) and the sum of the two factors (orange) according to Equation (25). The dots indicate the mean estimate, while the bars span the fifth to ninety-fight percentile of the statistic. Based on 500 draws from the posterior.

This conclusion is further supported by the fact documented in Table (4) that the dispersion of factors is not related to the consensus forecasts of either GDP or inflation.

7 Conclusion

In this paper, we have estimated a dynamic factor model of multivariate heterogeneous expectations with full information Bayesian methods on individual responses in the Survey of Professional Forecasters. The factor structure in the model is set up so that disagreement—the deviation of individual predictions from the consensus forecast—is described by factors that are independent across forecasters, but with loadings that are common for all forecasters. This type of factor structure is the reduced form of practically all existing structural macroeconomic models with heterogeneous expectations, and our results can be used to discipline these types of models. The estimated factor loadings can be interpreted as representing the most important comovement relationships between variables in the data.

The model allowed us to describe new facts about forecaster disagreement. First, disagreement can be described by two factors that are consistent with disagreement about aggregate demand and supply shocks, respectively. Second, demand disagreement is much more persistent than supply disagreement. Third, variance decompositions reveal that demand disagreement has become more important over time and has been particularly important during the Great Recession. Fourth, demand disagreement is not driven by disagreement about the course of monetary policy. Our results are robust to a wide range of alternative specifications.

In this paper, all forecaster heterogeneity is assumed to stem from different realizations of stochastic variables with ex-ante identical distributions, but future research could further examine heterogeneity in the expectation formation process as a source of disagreement. Our methodology could also be readily applied to other quantitative surveys of expectations, such as the Blue Chip or Consensus Forecast datasets which, although having shorter overall samples than the SPF, feature monthly surveys and different sets of forecast variables.

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Appendix for

"The Factor Structure of Disagreement"

by Edward Herbst and Fabian Winkler

A The Gibbs Sampler

Our goal is to elicit the posterior distribution,

$$p(\theta|Y) \propto p(Y|\theta)p(\theta).$$

The joint posterior does not have an analytically convenient form. To access the posterior, we follow Otrok and Whiteman (1998) and use a Gibbs sampling algorithm. This section outlines the sampler.

A.1 The Algorithm

Gibbs sampling is a Markov chain Monte Carlo (MCMC) algorithm which, under some regularity conditions, generates a Markov chain that converges in distribution to the posterior of interest. This justifies the use of sample (in the sense of the chain) averages as estimates of posterior expectations. Gibbs sampling works by iteratively sampling from the conditional distributions of the posterior. Deriving these distributions can be tedious and error-prone. In next subsection, we validate the sampler.

To aid in the derivation of conditional posteriors, it is useful in Gibbs sampling to estimate not only the "static" parameters $[\Lambda, \sigma^2, \rho, \phi]$, but the dynamic ones—the factors—as well. In addition, some elements of Y are missing because forecasters are not in our sample for every period, some variables are infrequently surveyed, and not every forecaster gives forecast for every variable. Overall, we have about 79 percent missing observations. We denote this missing data by Y^*, so that given both Y and Y^* we have a completely balanced panel. Thus, the Gibbs sampler jointly estimates $[Y^*, F]$ and $[\Lambda, \sigma^2, \rho, \phi]$. The sequence of conditional posteriors are given below.

1. $Y^*, F|Y, \Lambda, \sigma^2, \rho, \phi$. Conditional on the data and the model parameters, the joint distribution for Y^* and F is normally distributed. This distribution can be sampled from using the simulation smoother of Durbin and Koopman (2000). Note for each

for forecaster $i = 1, \ldots, M$, each pair of variables $[Y_i^*, F_i]$ can be draw independently. Therefore, we parallelize this step (using `OpenMP`) greatly speeding up computation.

2. $\Lambda, \sigma^2 | Y, Y^*, F, \rho, \phi$. For each observable $j = 1, \ldots, N$, quasi-difference equation (3) using ρ_j:

$$\begin{aligned} y^i_{jit} - \rho_j y_{jit-1} = \Lambda_j (f_{it} - \rho_j f_{it-1}) + \xi_{jit} - \rho_j \xi_{jit-1} = \\ \hat{y}_{jit} = \Lambda_j \tilde{f}_{jit} + \epsilon_{jit}. \end{aligned} \tag{28}$$

Notice ϵ_{jit} is i.i.d. Stack the observations, factors, and errors by time and then forecaster:

$$\begin{aligned} \hat{Y}_j &= [\hat{y}_{n12}, \ldots, \hat{y}_{n1T} \ldots, \hat{y}_{nM2}, \ldots \hat{y}_{nMT}]', \\ \tilde{F}_j &= \left[\tilde{f}_{j12}, \ldots, \tilde{f}_{nMT} \ldots, \tilde{f}_{nM2}, \ldots \tilde{f}_{nMT}\right]', \text{ and} \\ \epsilon_j &= [\epsilon_{j12}, \ldots, \epsilon_{jMT} \ldots, \epsilon_{jM2}, \ldots \epsilon_{jMT}]'. \end{aligned}$$

We have:

$$\tilde{Y}_j = \tilde{F}_j \Lambda_j' + \epsilon_j. \tag{29}$$

Define:

$$\tilde{V}_{\Lambda_j} = \left(\tilde{F}_j' \tilde{F}_j + V_\Lambda^{-1}\right)^{-1} \text{ and } \tilde{\mu}_{\Lambda_j} = \tilde{V}_{\Lambda_j} (\tilde{F}_j' \tilde{Y}_j + V_\Lambda^{-1} \mu_\Lambda). \tag{30}$$

It can be shown that the conditional distribution of σ_j^2 is inverse gamma with hyperparameters $\tilde{\alpha}_j$ and $\tilde{\beta}_j$:

$$\tilde{\alpha}_j = M(T-1)/2 + \alpha_j \text{ and } \tilde{\beta}_j = \frac{1}{2}\left(\tilde{Y}_j' \tilde{Y}_j + \mu_\Lambda' V_\Lambda^{-1} \mu_\Lambda - \tilde{\mu}_{\Lambda_j}' V_{\Lambda_j}^{-1} \tilde{\mu}_{\Lambda_j}\right) + \beta_j. \tag{31}$$

Here α_j and β_j are the hyperparameters associated with the prior distribution for σ_j^2. Conditional on σ_j^2 (in addition to the other variables), the distribution of Λ_j is normally distributed,

$$\Lambda_j | Y, Y^*, F, \sigma^2, \phi, \rho \sim \mathcal{N}\left(\tilde{\mu}_{\Lambda_j}, \sigma_j^2 \tilde{V}_{\Lambda_j}\right). \tag{32}$$

For Λ_1, the posterior inherits the truncation from the prior.

$$p(\Lambda_1|Y,Y^*,F,\sigma^2,\phi,\rho) \propto \mathbf{1}_{\left\{\Lambda_{1,1}\geq 0,\Lambda_{1,k}>\Lambda_{1,k-1}\right\}} \times \mathcal{N}\left(\tilde{\mu}_{\Lambda_1},\sigma_1^2\tilde{V}_{\Lambda_1}\right).$$

3. $\rho|Y,Y^*,F,\Lambda,\sigma,\phi$. Define:

$$\hat{\xi}_t^i = y_t^i - \Lambda f_t^i. \tag{33}$$

For each $j = 1,\ldots,N$, we have

$$\hat{\xi}_{jt}^i = \rho_j \hat{\xi}_{jt-1}^i + \upsilon_{jt}^i. \tag{34}$$

Stack the errors by time and then forecaster:

$$\hat{E}_j = \left[\hat{\xi}_{j12},\ldots,\hat{\xi}_{j1T}\ldots,\hat{\xi}_{jM2},\ldots\hat{\xi}_{jMT}\right]',$$
$$\hat{L}_j = \left[\hat{\xi}_{j11},\ldots,\hat{\xi}_{j1T-1}\ldots,\hat{\xi}_{jM1},\ldots\hat{\xi}_{jMT-1}\right]'.$$

Define,

$$\hat{V}_{\rho_j} = \left(\hat{L}_j'\hat{L}_j + V_\rho^{-1}\right)^{-1} \text{ and } \hat{\mu}_{\rho_j} = V_{\rho_j}\left(\hat{L}_j'\hat{E}_j + V_\rho^{-1}\mu_\rho\right). \tag{35}$$

Then the conditional distribution of ρ_j is given by:

$$p(\rho_j|Y,Y^*,F,\sigma^2,\phi) \propto \mathbf{1}_{\{-1<\rho_n<1\}} \times \mathcal{N}\left(\hat{\mu}_{\rho_n},\sigma_j^2\hat{V}_{\rho_n}\right). \tag{36}$$

4. $\phi|Y,Y^*,F,\Lambda,\sigma^2,\rho$. For $k = 1,\ldots,K$, stack the factors by time and then forecaster:

$$F_k = \left[f_{k12},\ldots,f_{k1T}\ldots,f_{kM2},\ldots f_{kMT}\right]',$$
$$H_k = \left[f_{kM1},\ldots,f_{kMT-1}\ldots,f_{kM1},\ldots f_{kMT-1}\right]'.$$

Define

$$\hat{V}_{\phi_k} = \left(H_j'H_j + V_{\phi_k}^{-1}\right)^{-1} \text{ and } \hat{\mu}_{\phi_k} = V_{\phi_k}\left(H_k'F_k + V_\phi^{-1}\mu_\phi\right). \tag{37}$$

From (**??**) it is apparent that F_k the conditional distribution of ϕ_k is given by:

$$p(\phi_k|Y,Y^*,F,\kappa,\phi) \propto \mathbf{1}_{\{0<\phi_k<1\}} \times \mathcal{N}\left(\hat{\mu}_{\phi_k},\hat{V}_{\phi_k}\right). \tag{38}$$

Table A.1: "Getting It Right" Test Output For The Gibbs Sampler

Test function	p-value	Test function	p-value	Test function	p-value
$\Lambda_{1,1}$	0.84	$\Lambda_{1,1} \times \Lambda_{2,1}$	0.89	ϕ	0.96
ρ_1	0.88	ρ_2^2	0.86	σ_2^2	0.79

A.2 Validation of the Sampler

We validate the sampler in the manner suggested by Geweke (2004). Specifically, we augment the algorithm with an additional step which simulates the from the conditional distribution of the data given the parameters. Thus, the augmented Gibbs sampler produces draws from the joint distribution

$$p(Y, \theta).$$

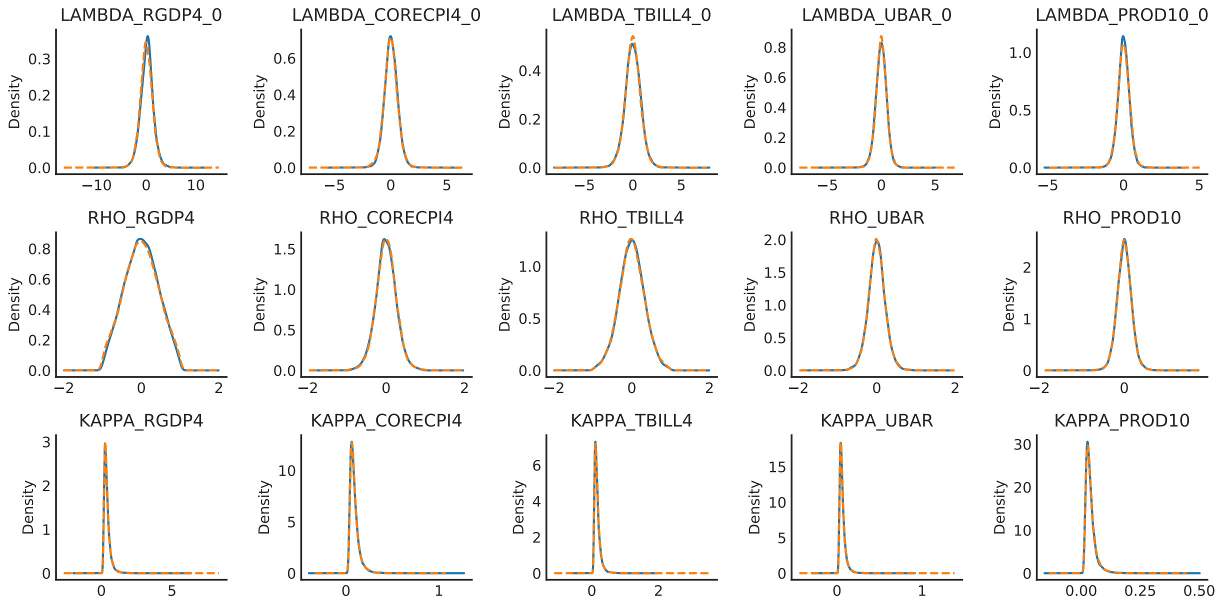
A marginal of this joint distribution is the prior distribution for the parameters. We compare our Gibbs sampler draws for these parameters (called the "successive-conditional (SC)" simulator) with the ones generated directly from the prior (the "marginal-conditional (MC)" simulator).y. In this exercise, we set $T = 30, M = 5$, $I = 1$, and $p = 1$, using the standard deviations from the consensus forecasts of one-, two-, three-, and four-quarter real GDP forecasts as well as long-run unemployment forecasts, though in the presentation below we simply index the corresponding parameters with integers.

We draw 10e3 draws from MC simulator and 10e4 draws from the SC simulator which we thin to 10e3 draws. Following Bognanni and Zito (2020), we postulate six test functions. For each test function, we report the p-value from the hypothesis that the mean of the distribution of test function are equal from SC and MC simulators. Table A.1 reports the results. For each test function, the null hypothesis of equality cannot be rejected. As a supplement, Figure A.1 displays Q-Q plots for the six test functions. The black dots represent percentiles from the samples generated by the two simulators with the 45 degree line represented by a red dashed line. It is clear that the percentiles are very similar for the two samples.

A.3 Convergence Diagnostics

For our baseline model, we run the Gibbs sampler $N_{run} = 10$ times, and pool the draws from the runs, yielding a posterior distribution with 400,000 draws. As measure of accuracy, we report the numerical standard error (NSE) of the posterior mean, computed as the standard deviation of the estimate of the posterior mean across the 10 runs. The square of this measures serves as an estimate of the CLT variance (if one applies) associated with the Gibbs-sampler-based estimate of the sample mean (as the length of the simulation becomes

Figure A.1: Q-Q Plots For SC And MC Simulators



large). In general, the Gibbs-sampler-based estimates of the posterior mean are relatively precise. Corresponding calculations for the models in the robustness section are omitted.

B Proof of Propositions 1 and 2

Proof of Proposition 1. Assumptions 1 and 2 imply that all agents solve the same linear filtering problem up to the realizations of the stochastic disturbances u_{it}, and because $Q_e(L)\,e_t$ and $Q_u(L)\,u_{it}$ are stationary, the difference between forecasts and realizations of the state $\mathbb{E}_{it}x_t - x_t$ is also stationary and admits a moving average representation. Moreover, this difference has to be measurable in the history of shocks $(\varepsilon_{t-s}, e_{t-s}, u_{it-s})_{s\geq 0}$ that enter the filtering problem. Therefore, we can write this difference as

$$\mathbb{E}_{it}x_t - x_t = \Phi_\varepsilon(L)\,\varepsilon_t + \Phi_e(L)\,e_t + \Phi_u(L)\,u_{it}.$$

Expectations of y_{jt+h} are given by $\mathbb{E}_{it}[y_{jt+h}] = C_{j\cdot}A^h\mathbb{E}_{it}x_t$ and so the observed forecasts, as per Assumption 3, are given by

$$\hat{y}_{jh+t|it} = C_{j\cdot}A^h\left(x_t + \Phi_\varepsilon(L)\,\varepsilon_t + \Phi_e(L)\,e_t + \Phi_u(L)\,u_{it}\right) + \xi_{jhit}. \tag{39}$$

Taking the average across forecasters, we can use the fact that u_{it} and ξ_{jhit} are independent across i to write:

$$\bar{y}_{jh+t|t} = \int_0^1 \hat{y}_{jh+t|it} di$$
$$= C_{j\cdot} A^h \left(x_t + \Phi_\varepsilon(L)\varepsilon_t + \Phi_e(L) e_t\right) \tag{40}$$

Substracting (40) from (39), the proposition follows now with $\Psi_{jhs} = C_{j\cdot} A^h \Phi_{us}$. □

Proof of Proposition 2. We are interested in finding expressions for the expectations of agent i for the shock ε_t conditional on the agent's history of signals $s_{it}, s_{it-1}, \ldots$. These conditional expectations are denoted $\mathbb{E}_{it}[\cdot]$. Because of the independence of signals across shocks k, we can just focus on the expectations for the kth shock ε_{kt} in isolation. We proceed in two steps: First, we find expectations conditional only on $\varepsilon_{kt-1}, \varepsilon_{kt-2}, \ldots$. Second, we take these expectations as a prior and add information from $\varepsilon_{kt} + \eta_{kit}$.

In the first step, we observe that the conditional distribution of ε_t given $\varepsilon_{t-1}, \varepsilon_{t-2}, \ldots$ equals the unconditional distribution. We now want to solve the filtering problem for the MA component of η_{kit}, which we denote $\nu_{kit} = e_{kt} + u_{kit}$. Noting that knowledge of ε_{t-1} and s_{it-1} implies knowledge of η_{kit-1}, the agent is left with filtering ν_{kit}. The conditional distribution of ν_{kit-1} given $\eta_{kit-1}, \eta_{kit-2}, \ldots$ has constant variance and mean $\hat{\nu}_{kit-1} = \mathbb{E}\left[\nu_{kit-1} \mid \eta_{kit-1}, \eta_{kit-2}, \ldots\right]$ that evolves as

$$\hat{\nu}_{kit} = g_{k1}\left(\eta_{kit} - \rho_k \eta_{kit-1} - \vartheta_k \hat{\nu}_{kit-1}\right). \tag{41}$$

for some gain parameter g_{k1} that satisfies $g_{k1}\vartheta_k \in (-1, 1)$. The tracking error for ν_{kit} is an AR(1) process:

$$\nu_{kit} - \hat{\nu}_{kit} = g_{k1}\nu_{kit} + g_{k1}\vartheta_k\left(\nu_{kit-1} - \hat{\nu}_{kit-1}\right). \tag{42}$$

In the second step, we want to find the expectation $\mathbb{E}\left[\varepsilon_{kt} \mid s_{kit}, s_{kit-1}, \ldots\right]$. Let $S_{kit} = \varepsilon_{kt} + \eta_{kit}$. We know that:

$$\mathbb{E}\left[\varepsilon_{kt} \mid s_{kit}, s_{kit-1}, \ldots\right] = \mathbb{E}\left[\varepsilon_{kt} \mid S_{kit}, \varepsilon_{kt-1}, \varepsilon_{kt-2}, \ldots\right].$$

Using this fact and the joint normal distribution of all variables, we can express the conditional expectation of the structural shocks given the time-t information set. There exists another gain parameter $g_{k2} \in (0,1)$ such that:

$$\begin{aligned}\mathbb{E}_{it}\varepsilon_{kt} &= g_{k2}\left(S_{kit} - \mathbb{E}\left[S_{kit} \mid \varepsilon_{t-H}, \varepsilon_{t-H-1}, \ldots\right]\right) \\ &= g_{k2}\left(\varepsilon_{kt} + \nu_{kit} + \vartheta_k\left(\nu_{kit-1} - \hat{\nu}_{kit-1}\right)\right). \end{aligned} \tag{43}$$

With this we have completely characterized expectations. We now move from expectations to disagreement. Define

$$f_{kit} = g_{k1}\vartheta_k f_{kit-1} + u_{kit}.$$

Note that $u_{kit} = \nu_{kit} - \int \nu_{k\iota t} d\iota$ and $g_{k1} f_{kit} = (\nu_{kit} - \hat{\nu}_{kit}) - \int (\nu_{k\iota t} - \hat{\nu}_{k\iota t})\, d\iota$. Furthermore, we can write $u_{kit-s} = f_{kit-s} - g_{k1}\vartheta_k f_{kit-s-1}$. Then disagreement about the structural shocks is nothing else than

$$\mathbb{E}_{it}\varepsilon_{kt} - \int_0^1 \mathbb{E}_{\iota t}\varepsilon_{kt} d\iota = g_{k2} f_{kit}. \tag{44}$$

Finally, expectations about the model variables y_{t+h} at a forecast horizon horizon h are directly related to the expectations about the structural shocks through their impulse responses:

$$\mathbb{E}_{it} y_{t+h} = C \sum_{s=0}^{\infty} A^{s+h} B \mathbb{E}_{it}\varepsilon_{t-s}. \tag{45}$$

Therefore, disagreement about y_{t+h} can be written as:

$$\begin{aligned} y_{hit} &= CA^h B\left(\mathbb{E}_{it}\varepsilon_{t-s} - \int_0^1 \mathbb{E}_{\iota t}\varepsilon_{t-s} d\iota\right) + \xi_{hit} \\ &= \sum_{k=1}^{p} CA^h B_{\cdot k} g_{k2} f_{kit} + \xi_{hit}. \end{aligned} \tag{46}$$

The sum over k can end at p because for $k > p$ we have $\sigma_{uk}^2 = 0$ and therefore $f_{kit} \equiv 0$. This establishes the proposition with parameters $a_k = g_{k1}\vartheta_k$ and $c_k = g_{k2}$. □

C Additional Tables and Figures for the Baseline Model

Table C.1: List of SPF Variables.

Symbol	Variable	Forecast horizon	Freq.	Included since
RGDP	Real GDP (s.a.)	0–4 q., 10 y.*	Q	1968:Q4
NGDP	Nominal GDP (s.a.)	0–4 q.	Q	1968:Q4
PGDP	GDP price index (s.a.)	0–4 q.	Q	1968:Q4
UNEMP	Unemployment rate (s.a.)	0–4 q.	Q	1968:Q4
INDPROD	Industrial production (s.a.)	0–4 q.	Q	1968:Q4
CPROF	Corporate profits after tax (s.a.)	0–4 q.	Q	1968:Q4
HOUSING	Housing starts (s.a.)	0–4 q.	Q	1968:Q4
RECESS	Probability of recession	0–4 q.	Q	1968:Q4
RCONSUM	Real personal consumption expenditures (s.a.)	0–4 q.	Q	1981:Q3
RNRESIN	Real nonresidential fixed investment (s.a)	0–4 q.	Q	1981:Q3
RRESINV	Real residential fixed investment (s.a.)	0–4 q.	Q	1981:Q3
RFEDGOV	Real federal govt. cons. and gross inv. (s.a.)	0–4 q.	Q	1981:Q3
RSLGOV	Real state and local govt. cons. and gross inv. (s.a.)	0–4 q.	Q	1981:Q3
RCBI	Real change in private inventories (s.a.)	0–4 q.	Q	1981:Q3
REXPORT	Real net exports (s.a.)	0–4 q.	Q	1981:Q3
CPI	Headline CPI inflation (s.a.)	0–4 q., 5&10 y.*	Q	1981:Q3
TBILL	Average yield on 3-month Treasury bills	0–4 q., 10 y.*	Q	1981:Q3
BOND	Average yield on Moody's Aaa corporate bonds	0–4 q.	Q	1981:Q3
TBOND	Average yield on 10-year Treasury bonds	0–4 q., 10 y.*	Q	1992:Q1
STOCK	Average return on S&P500	10 y.	A	1992:Q1
PROD	Average productivity growth	10 y.	A	1992:Q1
UBAR	Natural rate of unemployment	n/a	A	1996:Q3
EMP	Non-farm payroll employment (s.a.)	0–4 q.	Q	2003:Q4
CORECPI	Core CPI inflation (s.a.)	0–4 q.	Q	2007:Q1
PCE	Headline PCE inflation (s.a.)	0–4 q., 5&10 y.	Q	2007:Q1
COREPCE	Core PCE inflation (s.a.)	0–4 q.	Q	2007:Q1
BAABOND	Average yield on Moody's Baa corporate bonds	0–4 q.	Q	2010:Q1

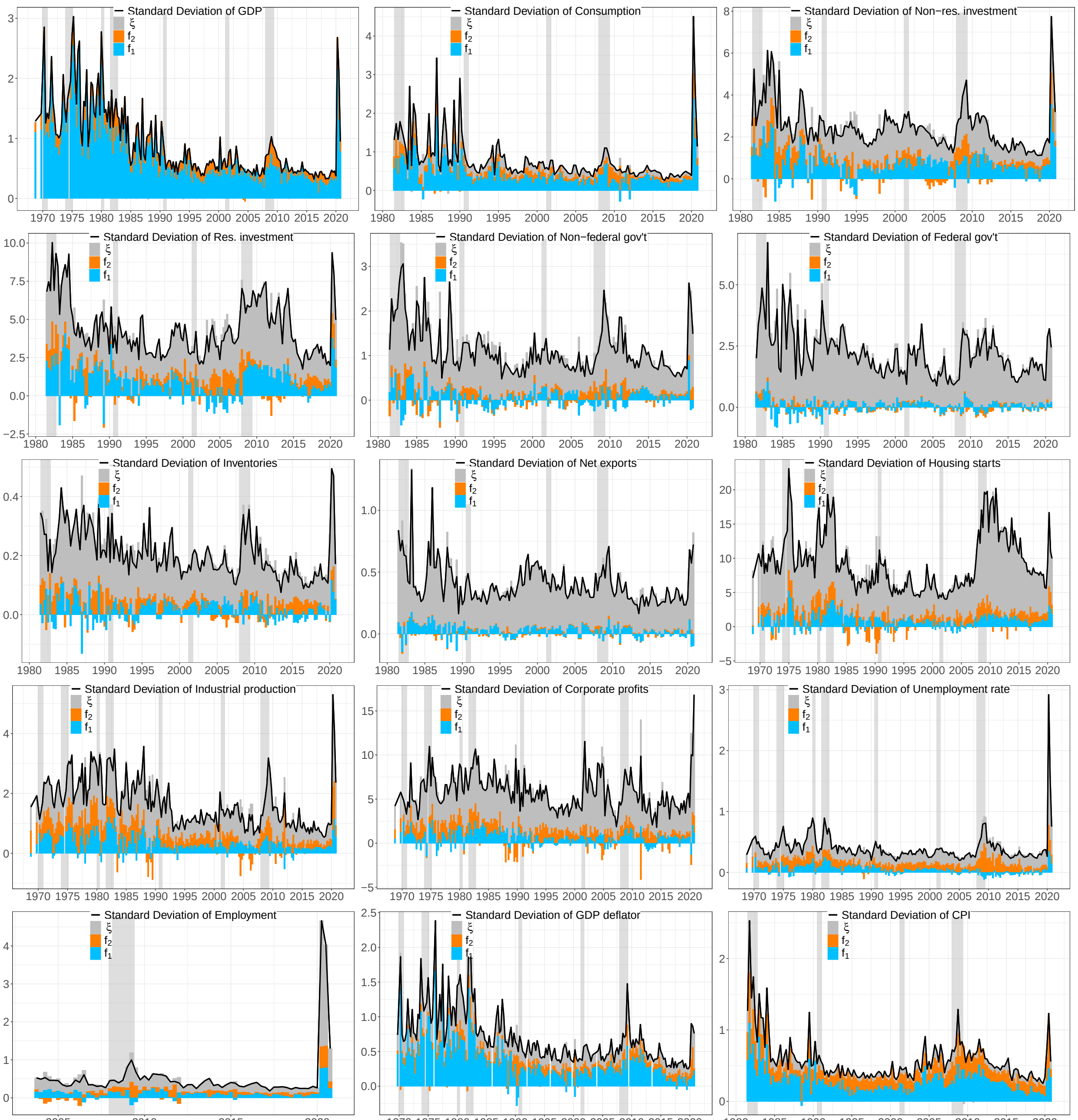
Note: List is sorted by date of first inclusion in the survey. The list excludes probability density forecasts that were started in 2007 and forecasts at annual horizons for which quarterly horizons are also available.
* For CPI, 10-year forecasts start in 1991:Q4 and 5-year forecasts in 2005:Q3. For RGDP, TBILL and TBOND, 10-year forecasts start in 1992:Q1 and are conducted annually.

Table C.2: Posterior of ρ and σ^2

Variable	ρ		σ^2	
	Mean	[5, 95]	Mean	[5, 95]
GDP Components				
GDP	-0.14 (0.00)	[-0.24, -0.04]	0.04 (0.00)	[0.04, 0.05]
Consumption	0.50 (0.00)	[0.48, 0.53]	0.33 (0.00)	[0.31, 0.34]
Non-res. investment	0.61 (0.00)	[0.59, 0.63]	3.27 (0.00)	[3.15, 3.38]
Res. investment	0.67 (0.00)	[0.65, 0.69]	10.37 (0.01)	[10.01, 10.74]
Non-federal gov't	0.63 (0.00)	[0.61, 0.65]	0.83 (0.00)	[0.81, 0.86]
Federal gov't	0.66 (0.00)	[0.64, 0.68]	2.96 (0.00)	[2.86, 3.06]
Inventories	0.48 (0.00)	[0.45, 0.50]	0.03 (0.00)	[0.03, 0.03]
Net exports	0.59 (0.00)	[0.57, 0.61]	0.12 (0.00)	[0.11, 0.12]
Other Real Activity				
Housing starts	0.66 (0.00)	[0.65, 0.68]	55.59 (0.02)	[53.99, 57.18]
Industrial production	0.54 (0.00)	[0.52, 0.56]	1.76 (0.00)	[1.71, 1.82]
Corporate profits	0.49 (0.00)	[0.47, 0.51]	25.47 (0.02)	[24.64, 26.33]
Labor Market				
Unemployment rate	0.50 (0.00)	[0.48, 0.51]	0.14 (0.00)	[0.13, 0.14]
Employment	0.50 (0.00)	[0.47, 0.54]	0.50 (0.00)	[0.47, 0.52]
Inflation				
GDP deflator	0.53 (0.00)	[0.50, 0.56]	0.27 (0.00)	[0.26, 0.29]
CPI	0.33 (0.01)	[0.21, 0.45]	0.14 (0.00)	[0.12, 0.16]
Core CPI	0.63 (0.01)	[0.55, 0.69]	0.03 (0.00)	[0.03, 0.03]
PCE	0.39 (0.01)	[0.26, 0.51]	0.07 (0.00)	[0.07, 0.08]
Core PCE	0.63 (0.01)	[0.53, 0.70]	0.02 (0.00)	[0.02, 0.02]
Interest rates				
3m T-bill rate	0.70 (0.00)	[0.69, 0.72]	0.22 (0.00)	[0.22, 0.23]
10y term spread	0.55 (0.00)	[0.53, 0.57]	0.13 (0.00)	[0.12, 0.13]
Aaa spread	0.47 (0.00)	[0.44, 0.49]	0.08 (0.00)	[0.08, 0.08]
Baa-Aaa spread	0.24 (0.00)	[0.19, 0.29]	0.05 (0.00)	[0.05, 0.06]
Recession prob.				
Recession prob.	0.61 (0.00)	[0.60, 0.63]	95.42 (0.05)	[92.75, 98.15]
Long-term forecasts				
Natural rate of unemp.	0.91 (0.00)	[0.89, 0.93]	0.03 (0.00)	[0.02, 0.03]
Stock Prices	0.83 (0.01)	[0.81, 0.86]	1.18 (0.01)	[1.04, 1.34]
Productivity	0.86 (0.00)	[0.84, 0.88]	0.05 (0.00)	[0.05, 0.06]
Real GDP	0.85 (0.00)	[0.83, 0.87]	0.03 (0.00)	[0.03, 0.03]
Treasury Bill	0.80 (0.01)	[0.77, 0.83]	0.22 (0.00)	[0.19, 0.25]
Term Spread	0.79 (0.00)	[0.76, 0.82]	0.25 (0.00)	[0.23, 0.29]
Inflation	0.81 (0.00)	[0.79, 0.82]	0.06 (0.00)	[0.06, 0.07]

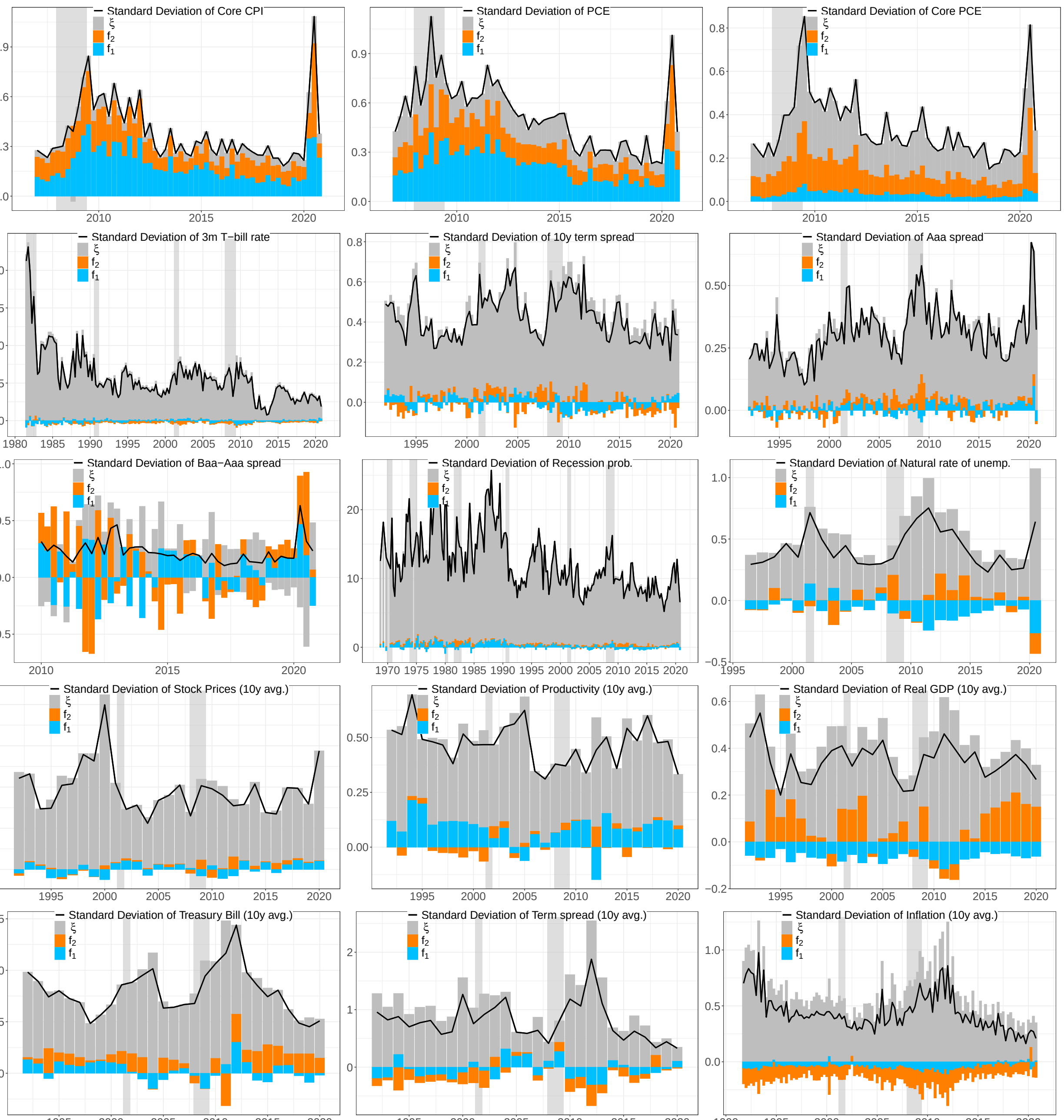
Note: Parameter estimates are computed using pooled draws from 10 MCMC chains each of length 40,000 after a 10,000 period burn-in. Standard deviation of the posterior mean across chains in parentheses. See the Appendix for details.

Figure C.1: Decompositions of Forecast Dispersion over time (I).



Note: Decompositions of the sample variance of disagreement across forecasters into sample covariances with both common components and the idiosyncratic component. Full sample. For each variable, factors and error terms are mean smoothed values at estimated parameter values. Variances and covariances are transformed by taking the signed squared root. Black line is the sample standard deviation.

Figure C.2: Decompositions of Forecast Dispersion over time (II).



Note: Decompositions of the sample variance of disagreement across forecasters into sample covariances with both common components and the idiosyncratic component. Full sample. For each variable, factors and error terms are mean smoothed values at estimated parameter values. Variances and covariances are transformed by taking the signed squared root. Black line is the sample standard deviation.

D Additional Figures and Tables for Alternative Specifications

D.1 Reduced set of variables

Table D.1: Reduced Set of Variables: Posterior of Λ and ϕ

Variable	$\Lambda_{\cdot 1}$		$\Lambda_{\cdot 2}$	
	Mean	[5, 95]	Mean	[5, 95]
GDP Components				
GDP	0.33	[0.30, 0.36]	0.49	[0.46, 0.52]
Other Real Activity				
Housing starts	-0.20	[-0.41, 0.07]	2.23	[2.04, 2.42]
Industrial production	-0.12	[-0.18, -0.00]	0.84	[0.81, 0.88]
Corporate profits	-0.36	[-0.53, -0.12]	1.69	[1.57, 1.80]
Labor Market				
Unemployment rate	0.02	[0.00, 0.03]	-0.17	[-0.18, -0.16]
Inflation				
GDP deflator	-0.58	[-0.61, -0.57]	-0.03	[-0.07, 0.01]
autoregressive coef. (ϕ)	0.31	[0.23, 0.64]	0.76	[0.75, 0.78]

Note: Parameter estimates are computed using pooled draws from 10 MCMC chains each of length 40,000 after a 10,000 period burn-in. See the Appendix for details.

Table D.2: REDUCED SET OF VARIABLES: POSTERIOR OF ρ AND σ^2

Variable	ρ		σ^2	
	Mean	[5, 95]	Mean	[5, 95]
GDP Components				
GDP	0.39	[0.34, 0.49]	0.48	[0.45, 0.51]
Other Real Activity				
Housing starts	0.65	[0.64, 0.67]	52.97	[51.38, 54.62]
Industrial production	0.44	[0.41, 0.47]	1.32	[1.25, 1.38]
Corporate profits	0.45	[0.43, 0.47]	23.70	[22.90, 24.54]
Labor Market				
Unemployment rate	0.47	[0.45, 0.49]	0.12	[0.11, 0.12]
Inflation				
GDP deflator	0.78	[-0.24, 0.92]	0.06	[0.05, 0.07]

Note: Parameter estimates are computed using pooled draws from 10 MCMC chains each of length 40,000 after a 10,000 period burn-in. See the Appendix for details.

D.2 Varying forecast horizons

Table D.3: Forecast Horizon $h = 0$: Posterior of Λ

Variable	$\Lambda_{\cdot 1}$		$\Lambda_{\cdot 2}$	
	Mean	[5, 95]	Mean	[5, 95]
GDP Components				
GDP	1.21	[0.41, 1.65]	1.16	[0.60, 1.70]
Consumption	1.16	[0.46, 1.52]	0.96	[0.45, 1.50]
Non-res. investment	1.11	[0.33, 1.58]	1.19	[0.64, 1.69]
Res. investment	1.78	[0.90, 2.26]	1.09	[0.25, 2.01]
Non-federal gov't	0.28	[0.09, 0.39]	0.23	[0.08, 0.38]
Federal gov't	0.41	[0.08, 0.64]	0.49	[0.25, 0.72]
Inventories	0.02	[0.01, 0.03]	0.01	[0.00, 0.02]
Net exports	0.01	[-0.00, 0.02]	0.02	[0.01, 0.03]
Other Real Activity				
Industrial production	0.55	[0.32, 0.71]	0.20	[-0.08, 0.53]
Housing starts	1.10	[0.76, 1.29]	0.32	[-0.15, 0.95]
Corporate profits	2.17	[1.34, 2.73]	0.80	[-0.24, 2.05]
Labor Market				
Unemployment rate	-0.03	[-0.04, -0.02]	-0.02	[-0.04, -0.00]
Employment	0.65	[0.27, 0.86]	0.51	[0.18, 0.83]
Inflation				
GDP deflator	0.07	[-0.26, 0.36]	-0.53	[-0.60, -0.36]
CPI	0.50	[0.12, 0.79]	-0.53	[-0.75, -0.05]
Core CPI	0.15	[0.08, 0.20]	-0.08	[-0.15, 0.04]
PCE	0.48	[0.10, 0.76]	-0.54	[-0.75, -0.06]
Core PCE	0.15	[0.08, 0.19]	-0.08	[-0.14, 0.05]
Interest rates				
3m T-bill rate	0.01	[0.00, 0.02]	0.01	[-0.00, 0.02]
10y term spread	0.01	[0.00, 0.02]	-0.01	[-0.02, 0.00]
Aaa spread	-0.00	[-0.01, 0.01]	0.00	[-0.01, 0.01]
Baa-Aaa spread	-0.02	[-0.04, -0.00]	0.03	[0.00, 0.04]
Recession prob.				
Recession prob.	-2.60	[-3.32, -1.15]	-1.83	[-3.12, -0.66]
Long-term forecasts				

Note: Parameter estimates are computed using pooled draws from 10 MCMC chains each of length 40,000 after a 10,000 period burn-in. See the Appendix for details.

Table D.4: Forecast Horizon $h = 1$: Posterior of Λ

Variable	$\Lambda_{\cdot 1}$		$\Lambda_{\cdot 2}$	
	Mean	[5, 95]	Mean	[5, 95]
GDP Components				
GDP	0.76	[0.13, 1.08]	0.62	[0.09, 1.07]
Consumption	0.64	[0.10, 0.90]	0.48	[0.03, 0.88]
Non-res. investment	1.10	[0.18, 1.57]	0.83	[0.04, 1.52]
Res. investment	1.85	[0.07, 2.61]	1.28	[-0.16, 2.53]
Non-federal gov't	0.20	[0.03, 0.31]	0.17	[0.02, 0.30]
Federal gov't	0.21	[-0.03, 0.39]	0.22	[-0.04, 0.40]
Inventories	0.03	[0.00, 0.05]	0.02	[-0.00, 0.04]
Net exports	-0.02	[-0.04, 0.02]	-0.01	[-0.04, 0.02]
Other Real Activity				
Industrial production	1.15	[-0.30, 1.66]	0.64	[-0.43, 1.58]
Housing starts	0.85	[-0.28, 1.20]	0.43	[-0.38, 1.16]
Corporate profits	1.82	[-0.47, 2.64]	1.01	[-0.71, 2.50]
Labor Market				
Unemployment rate	-0.10	[-0.14, 0.02]	-0.06	[-0.14, 0.03]
Employment	0.67	[0.08, 0.96]	0.49	[-0.02, 0.93]
Inflation				
GDP deflator	-0.08	[-0.49, 0.35]	-0.28	[-0.50, 0.37]
CPI	0.07	[-0.59, 0.55]	-0.24	[-0.59, 0.56]
Core CPI	0.04	[-0.23, 0.22]	-0.09	[-0.24, 0.23]
PCE	0.04	[-0.58, 0.52]	-0.25	[-0.58, 0.53]
Core PCE	0.05	[-0.21, 0.21]	-0.07	[-0.22, 0.21]
Interest rates				
3m T-bill rate	0.02	[-0.02, 0.03]	0.00	[-0.02, 0.03]
10y term spread	0.01	[-0.01, 0.02]	0.00	[-0.02, 0.02]
Aaa spread	-0.02	[-0.04, -0.00]	-0.02	[-0.03, 0.00]
Baa-Aaa spread	0.01	[-0.03, 0.04]	0.02	[-0.03, 0.05]
Recession prob.				
Recession prob.	-2.89	[-4.07, -0.02]	-1.95	[-3.94, 0.37]
Long-term forecasts				

Note: Parameter estimates are computed using pooled draws from 10 MCMC chains each of length 40,000 after a 10,000 period burn-in. See the Appendix for details.

Table D.5: Forecast Horizon $h = 2$: Posterior of Λ

Variable	$\Lambda_{\cdot 1}$		$\Lambda_{\cdot 2}$	
	Mean	[5, 95]	Mean	[5, 95]
GDP Components				
GDP	0.90	[0.83, 0.96]	0.46	[0.34, 0.57]
Consumption	0.67	[0.61, 0.73]	0.39	[0.30, 0.48]
Non-res. investment	1.36	[1.19, 1.52]	0.91	[0.71, 1.10]
Res. investment	2.17	[1.90, 2.43]	1.38	[1.06, 1.68]
Non-federal gov't	0.32	[0.27, 0.37]	0.11	[0.06, 0.16]
Federal gov't	0.40	[0.30, 0.49]	-0.01	[-0.10, 0.07]
Inventories	0.05	[0.04, 0.05]	0.04	[0.03, 0.05]
Net exports	0.02	[0.01, 0.03]	-0.03	[-0.03, -0.02]
Other Real Activity				
Industrial production	0.95	[0.69, 1.21]	1.48	[1.26, 1.71]
Housing starts	0.48	[0.36, 0.59]	0.75	[0.66, 0.83]
Corporate profits	1.19	[0.91, 1.46]	1.51	[1.24, 1.76]
Labor Market				
Unemployment rate	-0.08	[-0.10, -0.06]	-0.11	[-0.12, -0.10]
Employment	0.48	[0.40, 0.56]	0.37	[0.30, 0.45]
Inflation				
GDP deflator	-0.40	[-0.43, -0.36]	0.22	[0.17, 0.27]
CPI	-0.33	[-0.39, -0.27]	0.35	[0.30, 0.39]
Core CPI	-0.21	[-0.25, -0.18]	0.22	[0.19, 0.25]
PCE	-0.32	[-0.38, -0.27]	0.32	[0.27, 0.37]
Core PCE	-0.17	[-0.21, -0.14]	0.20	[0.17, 0.22]
Interest rates				
3m T-bill rate	-0.02	[-0.03, 0.00]	0.05	[0.04, 0.06]
10y term spread	0.01	[-0.00, 0.02]	0.02	[0.01, 0.03]
Aaa spread	-0.02	[-0.03, -0.01]	-0.03	[-0.03, -0.02]
Baa-Aaa spread	0.02	[0.00, 0.03]	-0.02	[-0.03, -0.01]
Recession prob.				
Recession prob.	-1.89	[-2.27, -1.48]	-2.28	[-2.64, -1.89]
Long-term forecasts				

Note: Parameter estimates are computed using pooled draws from 10 MCMC chains each of length 40,000 after a 10,000 period burn-in.See the Appendix for details.

Table D.6: Forecast Horizon $h = 3$: Posterior of Λ

Variable	$\Lambda_{\cdot 1}$ Mean	$\Lambda_{\cdot 1}$ [5, 95]	$\Lambda_{\cdot 2}$ Mean	$\Lambda_{\cdot 2}$ [5, 95]
GDP Components				
GDP	0.81	[0.77, 0.85]	0.46	[0.40, 0.52]
Consumption	0.65	[0.61, 0.70]	0.40	[0.34, 0.45]
Non-res. investment	1.18	[1.06, 1.30]	0.91	[0.79, 1.02]
Res. investment	1.85	[1.66, 2.05]	1.29	[1.10, 1.48]
Non-federal gov't	0.31	[0.26, 0.35]	0.14	[0.10, 0.17]
Federal gov't	0.24	[0.16, 0.33]	-0.05	[-0.11, 0.01]
Inventories	0.04	[0.03, 0.05]	0.04	[0.04, 0.05]
Net exports	0.04	[0.03, 0.05]	-0.02	[-0.02, -0.01]
Other Real Activity				
Industrial production	0.76	[0.54, 0.97]	1.51	[1.32, 1.70]
Housing starts	0.33	[0.25, 0.39]	0.64	[0.58, 0.69]
Corporate profits	0.75	[0.57, 0.93]	1.35	[1.18, 1.52]
Labor Market				
Unemployment rate	-0.06	[-0.08, -0.05]	-0.12	[-0.13, -0.11]
Employment	0.31	[0.25, 0.36]	0.27	[0.23, 0.31]
Inflation				
GDP deflator	-0.41	[-0.43, -0.38]	0.18	[0.15, 0.22]
CPI	-0.34	[-0.37, -0.31]	0.28	[0.25, 0.31]
Core CPI	-0.24	[-0.27, -0.22]	0.21	[0.19, 0.23]
PCE	-0.29	[-0.33, -0.26]	0.25	[0.23, 0.28]
Core PCE	-0.21	[-0.24, -0.19]	0.19	[0.17, 0.21]
Interest rates				
3m T-bill rate	-0.02	[-0.04, -0.01]	0.07	[0.05, 0.08]
10y term spread	0.02	[0.00, 0.03]	0.01	[-0.00, 0.02]
Aaa spread	-0.02	[-0.04, -0.01]	-0.03	[-0.04, -0.02]
Baa-Aaa spread	-0.00	[-0.02, 0.01]	-0.02	[-0.03, -0.01]
Recession prob.				
Recession prob.	-1.36	[-1.63, -1.09]	-1.83	[-2.09, -1.58]
Long-term forecasts				

Note: Parameter estimates are computed using pooled draws from 10 MCMC chains each of length 40,000 after a 10,000 period burn-in. See the Appendix for details.

D.3 Varying the number of factors

Table D.7: MODEL WITH $p = 1$: POSTERIOR OF Λ AND ϕ

Variable	$\Lambda_{\cdot 1}$	
	Mean	[5, 95]
GDP Components		
GDP	0.89	[0.88, 0.91]
Consumption	0.76	[0.74, 0.79]
Non-res. investment	1.42	[1.35, 1.49]
Res. investment	2.03	[1.89, 2.16]
Non-federal gov't	0.31	[0.27, 0.35]
Federal gov't	0.10	[0.02, 0.17]
Inventories	0.06	[0.05, 0.06]
Net exports	0.03	[0.02, 0.04]
Other Real Activity		
Housing starts	1.37	[1.21, 1.53]
Industrial production	0.54	[0.51, 0.57]
Corporate profits	1.10	[1.00, 1.21]
Labor Market		
Unemployment rate	-0.11	[-0.12, -0.11]
Employment	0.33	[0.30, 0.38]
Inflation		
GDP deflator	-0.23	[-0.24, -0.22]
CPI	0.03	[0.01, 0.06]
Core CPI	0.02	[0.01, 0.04]
PCE	0.05	[0.02, 0.07]
Core PCE	0.02	[0.00, 0.04]
Interest rates		
3m T-bill rate	0.04	[0.03, 0.06]
10y term spread	0.00	[-0.01, 0.02]
Aaa spread	-0.04	[-0.06, -0.03]
Baa-Aaa spread	-0.03	[-0.05, -0.02]
Recession prob.		
Recession prob.	-1.84	[-2.05, -1.63]
Long-term forecasts		
Natural rate of unemp.	-0.09	[-0.14, -0.05]
Stock Prices	0.18	[-0.00, 0.36]
Productivity	0.09	[0.06, 0.13]
Real GDP	0.14	[0.11, 0.18]
Treasury Bill	0.11	[0.04, 0.19]
Term Spread	-0.10	[-0.18, -0.02]
Inflation	0.04	[0.02, 0.05]
autoregressive coef. (ϕ)	0.58	[0.57, 0.60]

Notes: Parameter estimates are computed using pooled draws from 10 MCMC chains each of length 40,000 after a 10,000 period burn-in. See the Appendix for details.

Table D.8: Model With $p = 1$: Posterior of ρ and σ^2

Variable	ρ		σ^2	
	Mean	[5, 95]	Mean	[5, 95]
GDP Components				
GDP	-0.14	[-0.24, -0.02]	0.04	[0.03, 0.05]
Consumption	0.51	[0.48, 0.53]	0.33	[0.31, 0.34]
Non-res. investment	0.61	[0.59, 0.62]	3.28	[3.17, 3.40]
Res. investment	0.67	[0.65, 0.69]	10.41	[10.05, 10.78]
Non-federal gov't	0.63	[0.61, 0.65]	0.84	[0.81, 0.86]
Federal gov't	0.66	[0.64, 0.68]	2.98	[2.87, 3.08]
Inventories	0.47	[0.45, 0.50]	0.03	[0.03, 0.03]
Net exports	0.59	[0.57, 0.61]	0.12	[0.11, 0.12]
Other Real Activity				
Housing starts	0.66	[0.65, 0.68]	56.32	[54.79, 57.93]
Industrial production	0.54	[0.52, 0.55]	1.89	[1.83, 1.94]
Corporate profits	0.49	[0.47, 0.51]	25.99	[25.15, 26.85]
Labor Market				
Unemployment rate	0.51	[0.49, 0.53]	0.14	[0.14, 0.15]
Employment	0.50	[0.46, 0.53]	0.50	[0.48, 0.53]
Inflation				
GDP deflator	0.61	[0.59, 0.62]	0.40	[0.39, 0.41]
CPI	0.60	[0.58, 0.62]	0.30	[0.29, 0.31]
Core CPI	0.66	[0.64, 0.69]	0.10	[0.10, 0.11]
PCE	0.67	[0.64, 0.69]	0.17	[0.17, 0.18]
Core PCE	0.69	[0.66, 0.72]	0.07	[0.07, 0.08]
Interest rates				
3m T-bill rate	0.71	[0.70, 0.73]	0.23	[0.22, 0.23]
10y term spread	0.55	[0.53, 0.57]	0.13	[0.12, 0.13]
Aaa spread	0.47	[0.44, 0.49]	0.08	[0.08, 0.08]
Baa-Aaa spread	0.23	[0.18, 0.29]	0.05	[0.05, 0.06]
Recession prob.				
Recession prob.	0.62	[0.60, 0.63]	95.92	[93.23, 98.64]
Long-term forecasts				
Natural rate of unemp.	0.91	[0.89, 0.93]	0.03	[0.02, 0.03]
Stock Prices	0.83	[0.80, 0.86]	1.18	[1.04, 1.34]
Productivity	0.86	[0.84, 0.88]	0.05	[0.05, 0.06]
Real GDP	0.81	[0.82, 0.87]	0.03	[0.03, 0.03]
Treasury Bill	0.80	[0.77, 0.83]	0.23	[0.21, 0.26]
Term Spread	0.77	[0.76, 0.82]	0.27	[0.23, 0.30]
Inflation	0.83	[0.82, 0.84]	0.07	[0.07, 0.08]

Notes: Parameter estimates are computed using pooled draws from 10 MCMC chains each of length 40,000 after a 10,000 period burn-in. See the Appendix for details.

Table D.9: MODEL WITH $p = 3$: POSTERIOR OF Λ AND ϕ

Variable	$\Lambda_{\cdot 1}$		$\Lambda_{\cdot 2}$		$\Lambda_{\cdot 3}$	
	Mean	[5, 95]	Mean	[5, 95]	Mean	[5, 95]
GDP Components						
GDP	0.70	[0.67, 0.74]	0.55	[0.51, 0.59]	-0.00	[-0.09, 0.09]
Consumption	0.66	[0.63, 0.70]	0.43	[0.39, 0.47]	0.02	[-0.05, 0.10]
Non-res. investment	0.57	[0.47, 0.67]	1.21	[1.14, 1.29]	-0.13	[-0.31, 0.09]
Res. investment	0.42	[0.24, 0.61]	1.93	[1.80, 2.06]	-0.36	[-0.66, -0.00]
Non-federal gov't	0.27	[0.23, 0.31]	0.18	[0.14, 0.21]	0.01	[-0.03, 0.06]
Federal gov't	0.38	[0.30, 0.46]	-0.09	[-0.14, -0.03]	-0.01	[-0.07, 0.05]
Inventories	0.00	[-0.01, 0.01]	0.06	[0.05, 0.06]	-0.01	[-0.02, 0.00]
Net exports	0.12	[0.11, 0.14]	-0.03	[-0.04, -0.01]	0.00	[-0.01, 0.02]
Other Real Activity						
Housing starts	-0.59	[-0.81, -0.37]	2.53	[2.32, 2.75]	-0.61	[-1.03, -0.13]
Industrial production	-0.07	[-0.13, -0.02]	0.81	[0.78, 0.85]	-0.08	[-0.20, 0.06]
Corporate profits	-0.15	[-0.30, 0.01]	1.64	[1.52, 1.77]	-0.17	[-0.43, 0.13]
Labor Market						
Unemployment rate	0.03	[0.02, 0.05]	-0.18	[-0.19, -0.17]	0.01	[-0.02, 0.04]
Employment	0.00	[-0.05, 0.05]	0.34	[0.31, 0.38]	-0.02	[-0.08, 0.05]
Inflation						
GDP deflator	-0.32	[-0.34, -0.30]	0.01	[-0.04, 0.06]	0.26	[0.24, 0.28]
CPI	-0.13	[-0.16, -0.10]	0.06	[-0.02, 0.13]	0.38	[0.35, 0.40]
Core CPI	-0.09	[-0.11, -0.06]	0.03	[-0.03, 0.07]	0.28	[0.27, 0.29]
PCE	-0.14	[-0.17, -0.11]	0.05	[-0.01, 0.11]	0.33	[0.31, 0.35]
Core PCE	-0.08	[-0.10, -0.06]	0.03	[-0.02, 0.07]	0.24	[0.23, 0.26]
Interest rates						
3m T-bill rate	-0.00	[-0.02, 0.02]	0.05	[0.03, 0.07]	0.05	[0.03, 0.07]
10y term spread	0.01	[-0.00, 0.03]	-0.00	[-0.02, 0.01]	-0.00	[-0.01, 0.01]
Aaa spread	-0.01	[-0.03, 0.00]	-0.04	[-0.05, -0.03]	-0.00	[-0.01, 0.01]
Baa-Aaa spread	-0.01	[-0.03, 0.01]	-0.03	[-0.04, -0.02]	-0.01	[-0.02, 0.01]
Recession prob.						
Recession prob.	-0.46	[-0.71, -0.20]	-2.09	[-2.34, -1.84]	0.20	[-0.24, 0.61]
Long-term forecasts						
Natural rate of unemp.	-0.07	[-0.12, -0.02]	-0.06	[-0.09, -0.02]	0.01	[-0.02, 0.04]
Stock Prices	0.08	[-0.15, 0.30]	0.14	[0.01, 0.29]	0.05	[-0.07, 0.18]
Productivity	0.03	[-0.02, 0.08]	0.07	[0.04, 0.10]	-0.02	[-0.05, 0.01]
Real GDP	0.11	[0.08, 0.15]	0.08	[0.06, 0.11]	-0.02	[-0.05, 0.00]
Treasury Bill	0.35	[0.22, 0.47]	-0.05	[-0.13, 0.02]	0.19	[0.14, 0.25]
Term Spread	-0.37	[-0.49, -0.24]	0.06	[-0.01, 0.14]	-0.14	[-0.19, -0.09]
Inflation	-0.04	[-0.05, -0.02]	0.02	[-0.00, 0.05]	0.13	[0.11, 0.14]
autoregressive coef. (ϕ)	0.34	[0.30, 0.38]	0.73	[0.71, 0.75]	0.82	[0.80, 0.84]

Notes: Parameter estimates are computed using pooled draws from 10 MCMC chains each of length 40,000 after a 10,000 period burn-in. See the Appendix for details.

Table D.10: Model With $p = 3$: Posterior of ρ and σ^2

Variable	ρ		σ^2	
	Mean	[5, 95]	Mean	[5, 95]
GDP Components				
GDP	0.30	[0.13, 0.48]	0.03	[0.02, 0.03]
Consumption	0.51	[0.48, 0.53]	0.32	[0.31, 0.33]
Non-res. investment	0.59	[0.57, 0.61]	3.04	[2.93, 3.16]
Res. investment	0.68	[0.66, 0.70]	9.16	[8.78, 9.56]
Non-federal gov't	0.63	[0.61, 0.65]	0.83	[0.80, 0.86]
Federal gov't	0.66	[0.64, 0.68]	2.89	[2.79, 2.99]
Inventories	0.47	[0.45, 0.49]	0.03	[0.03, 0.03]
Net exports	0.61	[0.59, 0.63]	0.11	[0.10, 0.11]
Other Real Activity				
Housing starts	0.65	[0.64, 0.67]	50.76	[49.13, 52.43]
Industrial production	0.46	[0.43, 0.49]	1.42	[1.36, 1.48]
Corporate profits	0.46	[0.43, 0.48]	24.08	[23.27, 24.91]
Labor Market				
Unemployment rate	0.49	[0.46, 0.51]	0.11	[0.11, 0.12]
Employment	0.49	[0.45, 0.53]	0.44	[0.42, 0.47]
Inflation				
GDP deflator	0.44	[0.41, 0.48]	0.27	[0.26, 0.29]
CPI	0.15	[0.09, 0.21]	0.12	[0.11, 0.13]
Core CPI	0.70	[0.66, 0.74]	0.03	[0.03, 0.03]
PCE	0.25	[0.17, 0.32]	0.07	[0.06, 0.07]
Core PCE	0.71	[0.67, 0.75]	0.02	[0.02, 0.02]
Interest rates				
3m T-bill rate	0.70	[0.69, 0.72]	0.22	[0.22, 0.23]
10y term spread	0.55	[0.53, 0.57]	0.13	[0.12, 0.13]
Aaa spread	0.46	[0.44, 0.49]	0.08	[0.08, 0.08]
Baa-Aaa spread	0.23	[0.18, 0.29]	0.05	[0.05, 0.06]
Recession prob.				
Recession prob.	0.61	[0.60, 0.63]	94.37	[91.69, 97.15]
Long-term forecasts				
Natural rate of unemp.	0.91	[0.89, 0.93]	0.03	[0.02, 0.03]
Stock Prices	0.83	[0.80, 0.86]	1.19	[1.04, 1.34]
Productivity	0.86	[0.84, 0.88]	0.05	[0.05, 0.06]
Real GDP	0.85	[0.83, 0.87]	0.03	[0.03, 0.03]
Treasury Bill	0.83	[0.80, 0.86]	0.17	[0.14, 0.20]
Term Spread	0.82	[0.79, 0.85]	0.20	[0.16, 0.24]
Inflation	0.80	[0.78, 0.81]	0.06	[0.06, 0.07]

Notes: Parameter estimates are computed using pooled draws from 10 MCMC chains each of length 40,000 after a 10,000 period burn-in. See the Appendix for details.

Table D.11: Model With $p = 4$: Posterior of Λ and ϕ

Variable	$\Lambda_{\cdot 1}$		$\Lambda_{\cdot 2}$		$\Lambda_{\cdot 3}$		$\Lambda_{\cdot 4}$	
	Mean	[5, 95]	Mean	[5, 95]	Mean	[5, 95]	Mean	[5, 95]
GDP Components								
GDP	0.68	[0.64, 0.71]	0.07	[-0.30, 0.31]	0.53	[0.41, 0.59]	-0.03	[-0.18, 0.11]
Consumption	0.63	[0.60, 0.67]	0.01	[-0.29, 0.21]	0.43	[0.32, 0.49]	-0.00	[-0.14, 0.12]
Non-res. investment	0.60	[0.51, 0.70]	0.48	[-0.23, 0.93]	1.01	[0.68, 1.23]	-0.16	[-0.43, 0.10]
Res. investment	0.39	[0.22, 0.56]	0.65	[-0.49, 1.41]	1.69	[1.18, 2.00]	-0.47	[-0.96, -0.01]
Non-federal gov't	0.25	[0.21, 0.29]	-0.00	[-0.13, 0.09]	0.18	[0.13, 0.22]	0.00	[-0.06, 0.06]
Federal gov't	0.35	[0.28, 0.43]	-0.14	[-0.20, -0.08]	-0.03	[-0.12, 0.06]	-0.01	[-0.07, 0.05]
Inventories	0.01	[-0.00, 0.01]	0.03	[0.00, 0.05]	0.04	[0.02, 0.06]	-0.01	[-0.02, 0.00]
Net exports	0.11	[0.09, 0.12]	-0.05	[-0.06, -0.03]	-0.00	[-0.03, 0.02]	0.00	[-0.01, 0.02]
Other Real Activity								
Housing starts	-0.61	[-0.82, -0.39]	1.02	[-0.42, 1.98]	2.17	[1.42, 2.67]	-0.76	[-1.40, -0.16]
Industrial production	-0.01	[-0.06, 0.04]	0.47	[0.03, 0.74]	0.61	[0.34, 0.82]	-0.10	[-0.26, 0.05]
Corporate profits	-0.12	[-0.27, 0.04]	0.76	[-0.20, 1.39]	1.34	[0.85, 1.69]	-0.23	[-0.59, 0.11]
Labor Market								
Unemployment rate	0.01	[0.00, 0.03]	-0.12	[-0.18, -0.03]	-0.13	[-0.18, -0.06]	0.01	[-0.02, 0.04]
Employment	0.02	[-0.03, 0.06]	0.18	[-0.02, 0.30]	0.27	[0.15, 0.35]	-0.03	[-0.10, 0.04]
Inflation								
GDP deflator	-0.31	[-0.33, -0.29]	0.06	[0.02, 0.11]	0.00	[-0.10, 0.09]	0.26	[0.23, 0.28]
CPI	-0.15	[-0.18, -0.13]	-0.00	[-0.09, 0.07]	0.09	[-0.03, 0.20]	0.36	[0.32, 0.39]
Core CPI	-0.10	[-0.12, -0.08]	-0.00	[-0.05, 0.04]	0.05	[-0.04, 0.13]	0.27	[0.24, 0.29]
PCE	-0.15	[-0.18, -0.12]	0.02	[-0.05, 0.08]	0.07	[-0.04, 0.16]	0.32	[0.28, 0.34]
Core PCE	-0.09	[-0.11, -0.07]	0.01	[-0.03, 0.05]	0.04	[-0.03, 0.11]	0.23	[0.21, 0.25]
Interest rates								
3m T-bill rate	0.13	[0.10, 0.16]	0.34	[0.25, 0.39]	-0.14	[-0.31, 0.07]	0.13	[0.05, 0.20]
10y term spread	-0.10	[-0.13, -0.08]	-0.23	[-0.29, -0.14]	0.16	[0.01, 0.26]	-0.04	[-0.10, 0.02]
Aaa spread	-0.01	[-0.03, 0.00]	-0.02	[-0.04, -0.00]	-0.03	[-0.05, -0.01]	-0.00	[-0.02, 0.01]
Baa-Aaa spread	-0.01	[-0.03, 0.01]	-0.00	[-0.02, 0.02]	-0.03	[-0.04, -0.01]	-0.00	[-0.02, 0.01]
Recession prob.								
Recession prob.	-0.48	[-0.72, -0.23]	-0.78	[-1.59, 0.44]	-1.79	[-2.21, -1.20]	0.28	[-0.24, 0.85]
Long-term forecasts								
Natural rate of unemp.	-0.06	[-0.12, -0.01]	0.01	[-0.04, 0.06]	-0.07	[-0.11, -0.03]	0.02	[-0.02, 0.05]
Stock Prices	0.06	[-0.15, 0.27]	0.04	[-0.11, 0.19]	0.13	[-0.01, 0.27]	0.05	[-0.09, 0.17]
Productivity	0.04	[-0.01, 0.08]	0.03	[-0.02, 0.07]	0.06	[0.02, 0.09]	-0.02	[-0.05, 0.01]
Real GDP	0.11	[0.08, 0.14]	0.01	[-0.06, 0.05]	0.08	[0.05, 0.11]	-0.02	[-0.06, 0.01]
Treasury Bill	0.35	[0.25, 0.45]	-0.09	[-0.15, -0.02]	-0.02	[-0.11, 0.08]	0.19	[0.14, 0.25]
Term Spread	-0.38	[-0.48, -0.27]	0.10	[0.02, 0.16]	0.04	[-0.05, 0.12]	-0.14	[-0.20, -0.09]
Inflation	-0.04	[-0.06, -0.02]	0.00	[-0.02, 0.03]	0.03	[-0.01, 0.07]	0.12	[0.10, 0.14]
autoregressive coef. (ϕ)	0.28	[0.24, 0.32]	0.67	[0.63, 0.71]	0.74	[0.72, 0.77]	0.83	[0.80, 0.85]

Notes: Parameter estimates are computed using pooled draws from 10 MCMC chains each of length 40,000 after a 10,000 period burn-in. See the Appendix for details.

Table D.12: Model With $p = 4$: Posterior of ρ and σ^2

Variable	ρ		σ^2	
	Mean	[5, 95]	Mean	[5, 95]
GDP Components				
GDP	0.26	[0.11, 0.42]	0.03	[0.02, 0.03]
Consumption	0.51	[0.48, 0.53]	0.32	[0.30, 0.33]
Non-res. investment	0.59	[0.57, 0.61]	3.04	[2.93, 3.16]
Res. investment	0.68	[0.66, 0.69]	9.08	[8.72, 9.46]
Non-federal gov't	0.63	[0.61, 0.65]	0.83	[0.80, 0.86]
Federal gov't	0.66	[0.64, 0.68]	2.89	[2.79, 2.99]
Inventories	0.47	[0.45, 0.49]	0.03	[0.03, 0.03]
Net exports	0.61	[0.59, 0.63]	0.11	[0.11, 0.11]
Other Real Activity				
Housing starts	0.65	[0.64, 0.67]	50.43	[48.80, 52.10]
Industrial production	0.45	[0.43, 0.48]	1.41	[1.35, 1.47]
Corporate profits	0.46	[0.43, 0.48]	24.03	[23.21, 24.85]
Labor Market				
Unemployment rate	0.47	[0.45, 0.49]	0.11	[0.11, 0.12]
Employment	0.49	[0.45, 0.53]	0.44	[0.42, 0.47]
Inflation				
GDP deflator	0.45	[0.41, 0.48]	0.27	[0.26, 0.28]
CPI	0.14	[0.08, 0.20]	0.12	[0.11, 0.13]
Core CPI	0.70	[0.66, 0.74]	0.03	[0.03, 0.03]
PCE	0.25	[0.17, 0.32]	0.07	[0.06, 0.07]
Core PCE	0.71	[0.67, 0.74]	0.02	[0.02, 0.02]
Interest rates				
3m T-bill rate	0.88	[0.83, 0.92]	0.04	[0.02, 0.06]
10y term spread	0.31	[0.19, 0.39]	0.06	[0.05, 0.07]
Aaa spread	0.46	[0.43, 0.49]	0.08	[0.08, 0.08]
Baa-Aaa spread	0.23	[0.18, 0.29]	0.05	[0.05, 0.06]
Recession prob.				
Recession prob.	0.61	[0.60, 0.63]	94.33	[91.69, 97.02]
Long-term forecasts				
Natural rate of unemp.	0.91	[0.89, 0.93]	0.03	[0.02, 0.03]
Stock Prices	0.83	[0.81, 0.86]	1.17	[1.03, 1.32]
Productivity	0.86	[0.84, 0.88]	0.05	[0.05, 0.06]
Real GDP	0.85	[0.82, 0.87]	0.03	[0.03, 0.03]
Treasury Bill	0.84	[0.81, 0.87]	0.16	[0.13, 0.19]
Term Spread	0.82	[0.79, 0.85]	0.19	[0.16, 0.23]
Inflation	0.80	[0.78, 0.81]	0.06	[0.06, 0.07]

Notes: Parameter estimates are computed using pooled draws from 10 MCMC chains each of length 40,000 after a 10,000 period burn-in. See the Appendix for details.

D.4 Subsamples

Table D.13: Posterior of Λ and ϕ for Subsamples (A)

	$\Lambda_{1.}$			$\Lambda_{2.}$		
	1968Q4-1981Q2	1981Q3-2006Q4	2007Q1-2017Q3	1968Q4-1981Q2	1981Q3-2006Q4	2007Q1-2017Q3
GDP Components						
GDP	0.75 [0.66, 0.83]	−0.04 [−0.10, 0.02]	0.53 [0.50, 0.55]	0.78 [0.70, 0.85]	0.47 [0.45, 0.49]	0.17 [0.09, 0.24]
Consumption		−0.01 [−0.06, 0.05]	0.55 [0.51, 0.58]		0.42 [0.40, 0.45]	0.17 [0.10, 0.25]
Non-res. investment		−0.12 [−0.23, −0.02]	1.06 [0.96, 1.15]		0.76 [0.69, 0.83]	0.45 [0.29, 0.62]
Res. investment		0.28 [0.10, 0.45]	1.50 [1.33, 1.67]		1.27 [1.15, 1.39]	0.48 [0.22, 0.74]
Non-federal gov't		0.01 [−0.03, 0.05]	0.21 [0.17, 0.24]		0.17 [0.14, 0.21]	0.05 [0.01, 0.10]
Federal gov't		0.00 [−0.07, 0.07]	0.10 [0.04, 0.17]		−0.01 [−0.07, 0.06]	−0.01 [−0.08, 0.05]
Inventories		−0.02 [−0.03, −0.01]	0.03 [0.03, 0.04]		0.04 [0.03, 0.05]	0.02 [0.01, 0.03]
Net exports		0.00 [−0.01, 0.02]	0.02 [0.01, 0.04]		0.01 [−0.00, 0.02]	−0.03 [−0.04, −0.01]
Other Real Activity						
Housing starts	0.02 [−0.38, 0.40]	0.68 [0.43, 0.92]	2.18 [1.75, 2.61]	2.51 [2.13, 2.90]	1.59 [1.40, 1.79]	1.12 [0.63, 1.62]
Industrial production	−0.20 [−0.31, −0.08]	−0.11 [−0.17, −0.04]	0.43 [0.36, 0.49]	1.17 [1.09, 1.26]	0.55 [0.51, 0.59]	0.27 [0.20, 0.35]
Corporate profits	−0.40 [−0.69, −0.10]	−0.02 [−0.27, 0.22]	0.57 [0.34, 0.80]	2.15 [1.93, 2.37]	1.56 [1.40, 1.71]	0.24 [0.01, 0.48]
Labor Market						
Unemployment rate	0.02 [−0.00, 0.04]	0.04 [0.02, 0.05]	−0.13 [−0.15, −0.10]	−0.18 [−0.20, −0.17]	−0.11 [−0.12, −0.10]	−0.12 [−0.15, −0.10]
Employment		−0.05 [−0.09, −0.01]	0.19 [0.16, 0.23]		0.12 [0.08, 0.17]	0.13 [0.09, 0.17]
Inflation						
GDP deflator	−0.85 [−0.89, −0.81]	−0.03 [−0.04, −0.01]	−0.05 [−0.08, −0.01]	−0.14 [−0.21, −0.07]	−0.02 [−0.04, −0.00]	0.23 [0.22, 0.25]
CPI		−0.04 [−0.06, −0.02]	−0.08 [−0.14, −0.03]		−0.01 [−0.03, 0.01]	0.36 [0.34, 0.37]
Core CPI			−0.07 [−0.10, −0.03]			0.24 [0.23, 0.26]
PCE			−0.09 [−0.14, −0.04]			0.33 [0.31, 0.35]
Core PCE			−0.06 [−0.09, −0.03]			0.22 [0.20, 0.23]
Interest rates						
3m T-bill rate		−0.50 [−0.52, −0.49]	0.01 [−0.01, 0.02]		0.01 [−0.04, 0.07]	0.05 [0.04, 0.07]
10y term spread		0.32 [0.30, 0.34]	0.02 [0.00, 0.03]		0.04 [0.01, 0.08]	0.01 [−0.00, 0.02]
Aaa spread		0.02 [0.00, 0.04]	−0.03 [−0.04, −0.01]		−0.02 [−0.04, −0.01]	−0.02 [−0.03, −0.00]
Baa-Aaa spread			−0.02 [−0.03, −0.01]			−0.02 [−0.03, −0.01]
Recession prob.						
Recession prob.	−0.70 [−1.27, −0.12]	−0.01 [−0.41, 0.38]	−0.90 [−1.20, −0.60]	−3.31 [−3.84, −2.78]	−2.28 [−2.60, −1.97]	−0.53 [−0.86, −0.22]
Long-term forecasts						
Natural rate of unemp.		0.01 [−0.05, 0.06]	−0.07 [−0.10, −0.03]		−0.03 [−0.09, 0.03]	0.00 [−0.03, 0.03]
Stock Prices		−0.06 [−0.25, 0.14]	0.05 [−0.12, 0.23]		0.08 [−0.09, 0.24]	0.16 [0.03, 0.29]
Productivity		−0.00 [−0.04, 0.04]	0.05 [0.02, 0.09]		0.06 [0.02, 0.09]	0.02 [−0.01, 0.05]
Real GDP		0.01 [−0.02, 0.05]	0.07 [0.04, 0.09]		0.11 [0.08, 0.13]	0.01 [−0.01, 0.03]
Treasury Bill		−0.19 [−0.25, −0.12]	0.06 [−0.02, 0.14]		0.01 [−0.05, 0.07]	0.16 [0.10, 0.23]
Term Spread		0.13 [0.05, 0.22]	−0.03 [−0.11, 0.04]		−0.04 [−0.11, 0.03]	−0.08 [−0.14, −0.03]
Inflation		−0.01 [−0.02, 0.01]	−0.03 [−0.05, −0.01]		0.03 [0.01, 0.05]	0.11 [0.10, 0.12]
autoregressive coef. (ϕ)	0.17 [0.10, 0.24]	0.56 [0.52, 0.60]	0.58 [0.55, 0.61]	0.69 [0.66, 0.73]	0.78 [0.76, 0.80]	0.73 [0.70, 0.75]

Table D.14: Posterior for ρ and σ^2 for Subsamples (A)

	ρ			σ^2		
	1968Q4-1981Q2	1981Q3-2006Q4	2007Q1-2017Q3	1968Q4-1981Q2	1981Q3-2006Q4	2007Q1-2017Q3
GDP Components						
GDP	0.28 [0.19, 0.37]	0.18 [0.10, 0.27]	0.02 [−0.14, 0.19]	0.98 [0.88, 1.07]	0.10 [0.08, 0.11]	0.02 [0.02, 0.03]
Consumption		0.50 [0.46, 0.53]	0.46 [0.42, 0.50]		0.37 [0.35, 0.39]	0.27 [0.25, 0.28]
Non-res. investment		0.63 [0.61, 0.66]	0.54 [0.51, 0.57]		3.42 [3.27, 3.58]	3.00 [2.84, 3.17]
Res. investment		0.63 [0.61, 0.66]	0.68 [0.65, 0.71]		9.35 [8.92, 9.80]	11.48 [10.88, 12.11]
Non-federal gov't		0.59 [0.57, 0.62]	0.68 [0.65, 0.71]		0.91 [0.87, 0.96]	0.73 [0.69, 0.77]
Federal gov't		0.64 [0.61, 0.66]	0.69 [0.66, 0.72]		3.56 [3.40, 3.73]	2.20 [2.08, 2.32]
Inventories		0.47 [0.44, 0.50]	0.47 [0.43, 0.51]		0.03 [0.03, 0.04]	0.03 [0.03, 0.03]
Net exports		0.57 [0.55, 0.60]	0.59 [0.56, 0.63]		0.13 [0.13, 0.14]	0.09 [0.09, 0.10]
Other Real Activity						
Housing starts	0.63 [0.60, 0.66]	0.60 [0.58, 0.63]	0.68 [0.66, 0.71]	66.08 [62.31, 69.94]	30.23 [28.90, 31.60]	77.05 [73.06, 81.29]
Industrial production	0.35 [0.28, 0.42]	0.62 [0.60, 0.65]	0.40 [0.36, 0.44]	2.01 [1.82, 2.20]	1.08 [1.03, 1.13]	1.57 [1.49, 1.66]
Corporate profits	0.39 [0.34, 0.43]	0.51 [0.48, 0.54]	0.50 [0.44, 0.54]	24.62 [23.15, 26.20]	23.79 [22.61, 25.04]	23.48 [21.92, 25.10]
Labor Market						
Unemployment rate	0.60 [0.57, 0.63]	0.62 [0.60, 0.65]	0.37 [0.34, 0.41]	0.10 [0.09, 0.10]	0.05 [0.05, 0.06]	0.25 [0.24, 0.26]
Employment		0.61 [0.55, 0.68]	0.50 [0.46, 0.54]		0.08 [0.07, 0.09]	0.60 [0.57, 0.63]
Inflation						
GDP deflator	0.84 [0.80, 0.88]	0.64 [0.62, 0.66]	0.47 [0.43, 0.50]	0.17 [0.13, 0.23]	0.27 [0.26, 0.28]	0.14 [0.13, 0.15]
CPI		0.55 [0.52, 0.58]	0.21 [0.15, 0.27]		0.34 [0.33, 0.36]	0.08 [0.07, 0.08]
Core CPI			0.67 [0.63, 0.71]			0.04 [0.04, 0.04]
PCE			0.25 [0.18, 0.32]			0.06 [0.05, 0.06]
Core PCE			0.68 [0.64, 0.72]			0.03 [0.02, 0.03]
Interest rates						
3m T-bill rate		0.85 [0.81, 0.88]	0.65 [0.63, 0.68]		0.04 [0.03, 0.05]	0.09 [0.09, 0.10]
10y term spread		0.40 [0.33, 0.47]	0.60 [0.57, 0.63]		0.08 [0.07, 0.08]	0.12 [0.11, 0.12]
Aaa spread		0.41 [0.37, 0.45]	0.50 [0.46, 0.54]		0.06 [0.06, 0.07]	0.10 [0.09, 0.10]
Baa-Aaa spread			0.24 [0.19, 0.29]			0.05 [0.05, 0.06]
Recession prob.						
Recession prob.	0.55 [0.52, 0.58]	0.63 [0.61, 0.66]	0.64 [0.62, 0.67]	140.01 [132.39, 148.17]	88.03 [84.20, 91.94]	54.03 [51.33, 56.98]
Long-term forecasts						
Natural rate of unemp.		0.88 [0.83, 0.91]	0.92 [0.89, 0.94]		0.03 [0.03, 0.04]	0.03 [0.02, 0.04]
Stock Prices		0.81 [0.76, 0.85]	0.85 [0.81, 0.88]		1.48 [1.24, 1.77]	0.91 [0.76, 1.09]
Productivity		0.86 [0.83, 0.88]	0.88 [0.85, 0.90]		0.06 [0.05, 0.07]	0.04 [0.04, 0.05]
Real GDP		0.81 [0.77, 0.84]	0.89 [0.87, 0.91]		0.04 [0.03, 0.05]	0.02 [0.02, 0.02]
Treasury Bill		0.84 [0.81, 0.87]	0.75 [0.69, 0.81]		0.16 [0.14, 0.19]	0.31 [0.26, 0.38]
Term Spread		0.76 [0.71, 0.81]	0.81 [0.77, 0.85]		0.29 [0.24, 0.34]	0.24 [0.20, 0.28]
Inflation		0.83 [0.82, 0.85]	0.77 [0.74, 0.79]		0.08 [0.07, 0.08]	0.07 [0.06, 0.07]

Table D.15: Posterior of Λ and ϕ for Subsamples (B)

	$\Lambda_{1\cdot}$		$\Lambda_{2\cdot}$	
	1968Q4-1990Q2	1990Q3-2020Q4	1968Q4-1990Q2	1990Q3-2020Q4
GDP Components				
GDP	0.97 $[0.88, 1.05]$	0.43 $[0.37, 0.46]$	0.69 $[0.62, 0.76]$	0.19 $[0.12, 0.29]$
Consumption	0.72 $[0.56, 0.86]$	0.40 $[0.34, 0.43]$	0.60 $[0.51, 0.68]$	0.19 $[0.12, 0.28]$
Non-res. investment	0.81 $[0.52, 1.10]$	0.80 $[0.68, 0.88]$	1.16 $[0.99, 1.34]$	0.42 $[0.28, 0.61]$
Res. investment	0.57 $[-0.04, 1.21]$	1.03 $[0.86, 1.16]$	2.30 $[1.98, 2.62]$	0.54 $[0.34, 0.79]$
Non-federal gov't	0.77 $[0.60, 0.93]$	0.15 $[0.12, 0.18]$	0.14 $[0.04, 0.25]$	0.07 $[0.03, 0.11]$
Federal gov't	1.11 $[0.74, 1.47]$	0.13 $[0.09, 0.18]$	-0.49 $[-0.68, -0.30]$	0.02 $[-0.03, 0.07]$
Inventories	0.03 $[0.00, 0.06]$	0.03 $[0.03, 0.04]$	0.06 $[0.04, 0.07]$	0.02 $[0.01, 0.03]$
Net exports	0.07 $[0.01, 0.14]$	0.03 $[0.02, 0.04]$	0.03 $[0.00, 0.07]$	-0.01 $[-0.02, -0.00]$
Other Real Activity				
Housing starts	-0.16 $[-0.53, 0.21]$	1.37 $[1.04, 1.64]$	2.70 $[2.41, 3.01]$	0.98 $[0.66, 1.36]$
Industrial production	-0.10 $[-0.20, 0.00]$	0.35 $[0.28, 0.40]$	1.09 $[1.03, 1.16]$	0.25 $[0.18, 0.33]$
Corporate profits	-0.23 $[-0.53, 0.06]$	0.70 $[0.53, 0.86]$	2.24 $[2.06, 2.42]$	0.32 $[0.12, 0.54]$
Labor Market				
Unemployment rate	0.00 $[-0.01, 0.02]$	-0.09 $[-0.11, -0.06]$	-0.18 $[-0.19, -0.16]$	-0.10 $[-0.13, -0.08]$
Employment		0.15 $[0.12, 0.18]$		0.11 $[0.08, 0.16]$
Inflation				
GDP deflator	-0.59 $[-0.64, -0.54]$	-0.07 $[-0.11, -0.03]$	-0.05 $[-0.09, -0.01]$	0.19 $[0.17, 0.20]$
CPI	-0.11 $[-0.23, 0.00]$	-0.10 $[-0.17, -0.06]$	-0.02 $[-0.08, 0.04]$	0.28 $[0.25, 0.30]$
Core CPI		-0.09 $[-0.13, -0.05]$		0.20 $[0.18, 0.22]$
PCE		-0.12 $[-0.18, -0.07]$		0.27 $[0.24, 0.29]$
Core PCE		-0.08 $[-0.12, -0.04]$		0.18 $[0.16, 0.20]$
Interest rates				
3m T-bill rate	0.02 $[-0.06, 0.10]$	0.02 $[-0.01, 0.03]$	-0.06 $[-0.11, 0.00]$	0.08 $[0.07, 0.10]$
10y term spread		0.01 $[-0.00, 0.02]$		-0.01 $[-0.02, 0.00]$
Aaa spread		-0.02 $[-0.03, -0.01]$		-0.02 $[-0.02, -0.01]$
Baa-Aaa spread		-0.01 $[-0.02, -0.01]$		-0.01 $[-0.02, -0.01]$
Recession prob.				
Recession prob.	-0.90 $[-1.43, -0.35]$	-0.84 $[-1.10, -0.57]$	-3.17 $[-3.60, -2.74]$	-0.72 $[-1.01, -0.45]$
Long-term forecasts				
Natural rate of unemp.		-0.05 $[-0.08, -0.03]$		-0.01 $[-0.04, 0.01]$
Stock Prices		0.05 $[-0.05, 0.16]$		0.09 $[-0.01, 0.19]$
Productivity		0.05 $[0.03, 0.07]$		0.01 $[-0.01, 0.03]$
Real GDP		0.08 $[0.06, 0.10]$		0.02 $[-0.00, 0.04]$
Treasury Bill		0.01 $[-0.04, 0.05]$		0.12 $[0.08, 0.16]$
Term Spread		-0.03 $[-0.07, 0.02]$		-0.07 $[-0.11, -0.03]$
Inflation		-0.03 $[-0.06, -0.01]$		0.10 $[0.08, 0.11]$
autoregressive coef. (ϕ)	0.36 $[0.30, 0.41]$	0.64 $[0.62, 0.67]$	0.75 $[0.73, 0.78]$	0.75 $[0.72, 0.77]$

Table D.16: Posterior for ρ and σ^2 for Subsamples (B)

	ρ		σ^2	
	1968Q4-1990Q2	1990Q3-2020Q4	1968Q4-1990Q2	1990Q3-2020Q4
GDP Components				
GDP	-0.15 $[-0.27, -0.02]$	-0.11 $[-0.23, 0.00]$	0.28 [0.21, 0.37]	0.02 [0.01, 0.02]
Consumption	0.34 [0.25, 0.43]	0.52 [0.49, 0.55]	1.06 [0.95, 1.17]	0.21 [0.20, 0.22]
Non-res. investment	0.52 [0.46, 0.57]	0.62 [0.60, 0.64]	6.34 [5.76, 6.96]	2.71 [2.61, 2.81]
Res. investment	0.53 [0.47, 0.59]	0.69 [0.67, 0.71]	19.44 [17.56, 21.38]	8.70 [8.38, 9.03]
Non-federal gov't	0.56 [0.51, 0.61]	0.65 [0.63, 0.67]	1.79 [1.62, 1.98]	0.63 [0.61, 0.66]
Federal gov't	0.56 [0.49, 0.62]	0.67 [0.65, 0.69]	6.33 [5.66, 7.05]	2.21 [2.13, 2.29]
Inventories	0.30 [0.22, 0.38]	0.52 [0.49, 0.54]	0.07 [0.06, 0.07]	0.03 [0.02, 0.03]
Net exports	0.36 [0.29, 0.43]	0.65 [0.63, 0.67]	0.29 [0.27, 0.32]	0.09 [0.08, 0.09]
Other Real Activity				
Housing starts	0.62 [0.60, 0.65]	0.67 [0.65, 0.68]	60.53 [57.70, 63.45]	49.29 [47.52, 51.11]
Industrial production	0.40 [0.34, 0.45]	0.48 [0.45, 0.50]	1.88 [1.74, 2.02]	1.20 [1.15, 1.24]
Corporate profits	0.39 [0.36, 0.43]	0.53 [0.50, 0.56]	26.55 [25.25, 27.91]	21.33 [20.39, 22.29]
Labor Market				
Unemployment rate	0.58 [0.55, 0.61]	0.43 [0.41, 0.45]	0.09 [0.09, 0.10]	0.15 [0.14, 0.15]
Employment		0.50 [0.46, 0.53]		0.50 [0.48, 0.53]
Inflation				
GDP deflator	0.64 [0.61, 0.68]	0.51 [0.48, 0.54]	0.53 [0.47, 0.58]	0.12 [0.12, 0.13]
CPI	0.41 [0.33, 0.49]	0.23 [0.17, 0.28]	0.99 [0.91, 1.07]	0.07 [0.06, 0.07]
Core CPI		0.66 [0.62, 0.69]		0.04 [0.04, 0.04]
PCE		0.25 [0.17, 0.32]		0.06 [0.05, 0.06]
Core PCE		0.67 [0.64, 0.71]		0.03 [0.03, 0.03]
Interest rates				
3m T-bill rate	0.69 [0.64, 0.73]	0.61 [0.59, 0.63]	0.71 [0.65, 0.77]	0.12 [0.12, 0.13]
10y term spread		0.55 [0.53, 0.57]		0.13 [0.12, 0.13]
Aaa spread		0.47 [0.44, 0.49]		0.08 [0.08, 0.08]
Baa-Aaa spread		0.24 [0.19, 0.29]		0.05 [0.05, 0.06]
Recession prob.				
Recession prob.	0.57 [0.55, 0.60]	0.63 [0.61, 0.65]	139.29 [132.87, 145.83]	62.11 [59.86, 64.44]
Long-term forecasts				
Natural rate of unemp.		0.91 [0.89, 0.93]		0.03 [0.02, 0.03]
Stock Prices		0.83 [0.81, 0.86]		1.18 [1.05, 1.34]
Productivity		0.86 [0.84, 0.88]		0.06 [0.05, 0.06]
Real GDP		0.85 [0.82, 0.87]		0.03 [0.03, 0.03]
Treasury Bill		0.80 [0.77, 0.83]		0.22 [0.20, 0.25]
Term Spread		0.79 [0.76, 0.82]		0.26 [0.23, 0.29]
Inflation		0.80 [0.78, 0.81]		0.07 [0.06, 0.07]

Table D.17: Posterior of Λ and ϕ for Subsamples (C)

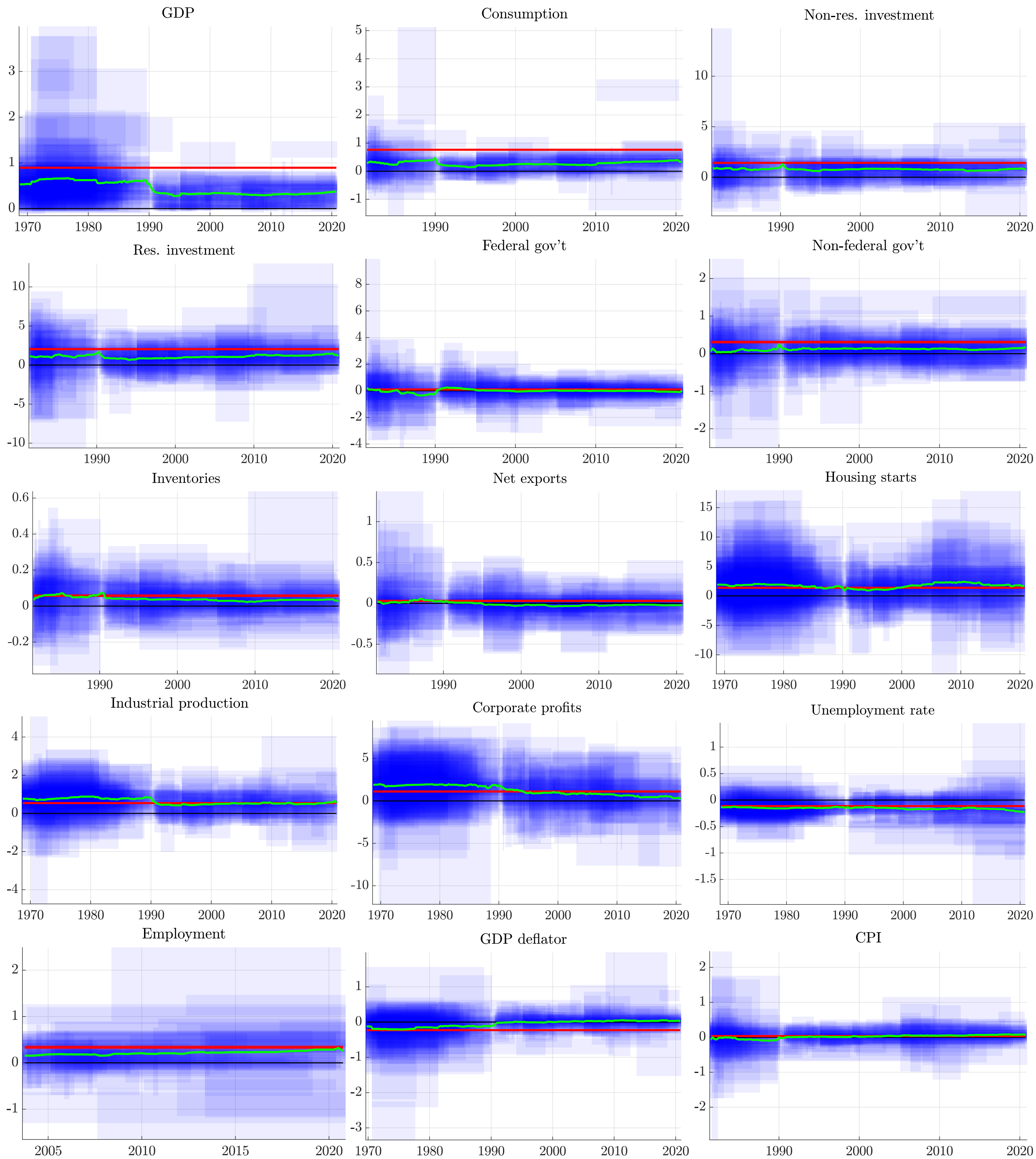
		$\Lambda_{1\cdot}$			$\Lambda_{2\cdot}$	
	1968Q4-1984Q3	1984Q4-2007Q4	2008Q1-2020Q4	1968Q4-1984Q3	1984Q4-2007Q4	2008Q1-2020Q4
GDP Components						
GDP	0.68 [0.61, 0.76]	0.15 [−0.15, 0.41]	0.53 [0.49, 0.56]	0.80 [0.74, 0.86]	0.34 [0.13, 0.43]	0.19 [0.10, 0.28]
Consumption	0.28 [0.12, 0.43]	0.16 [−0.09, 0.37]	0.56 [0.51, 0.60]	0.66 [0.56, 0.76]	0.29 [0.07, 0.38]	0.20 [0.11, 0.30]
Non-res. investment	0.74 [0.32, 1.16]	0.10 [−0.35, 0.56]	1.07 [0.95, 1.17]	1.35 [1.07, 1.62]	0.56 [0.32, 0.68]	0.49 [0.29, 0.68]
Res. investment	−0.69 [−1.44, 0.08]	0.42 [−0.14, 0.85]	1.54 [1.36, 1.71]	2.86 [2.34, 3.40]	0.60 [0.11, 0.88]	0.43 [0.14, 0.74]
Non-federal gov't	0.37 [0.17, 0.56]	0.03 [−0.08, 0.13]	0.21 [0.17, 0.25]	0.35 [0.20, 0.50]	0.12 [0.06, 0.16]	0.06 [0.00, 0.11]
Federal gov't	1.27 [0.86, 1.66]	−0.05 [−0.15, 0.06]	0.10 [0.03, 0.17]	−0.16 [−0.46, 0.13]	0.11 [0.02, 0.17]	−0.02 [−0.09, 0.06]
Inventories	0.06 [0.03, 0.10]	−0.00 [−0.03, 0.02]	0.03 [0.02, 0.04]	0.09 [0.06, 0.11]	0.03 [0.02, 0.04]	0.02 [0.01, 0.03]
Net exports	0.09 [−0.00, 0.19]	0.01 [0.00, 0.03]	0.03 [0.01, 0.04]	0.03 [−0.02, 0.08]	0.01 [−0.01, 0.02]	−0.02 [−0.03, −0.01]
Other Real Activity						
Housing starts	−0.27 [−0.64, 0.09]	0.62 [0.04, 1.04]	2.23 [1.76, 2.70]	2.92 [2.60, 3.24]	0.57 [−0.12, 1.03]	1.09 [0.55, 1.67]
Industrial production	−0.18 [−0.28, −0.09]	0.08 [−0.21, 0.36]	0.42 [0.35, 0.49]	1.08 [1.02, 1.15]	0.35 [0.19, 0.43]	0.29 [0.20, 0.38]
Corporate profits	−0.60 [−0.86, −0.33]	0.51 [−0.31, 1.19]	0.53 [0.29, 0.78]	2.28 [2.08, 2.47]	0.91 [0.21, 1.28]	0.19 [−0.07, 0.46]
Labor Market						
Unemployment rate	0.01 [−0.00, 0.03]	0.00 [−0.08, 0.07]	−0.13 [−0.16, −0.09]	−0.18 [−0.19, −0.17]	−0.09 [−0.11, −0.06]	−0.13 [−0.16, −0.11]
Employment		0.01 [−0.07, 0.10]	0.19 [0.15, 0.23]		0.10 [0.05, 0.13]	0.14 [0.10, 0.19]
Inflation						
GDP deflator	−0.84 [−0.88, −0.80]	−0.06 [−0.08, −0.04]	−0.05 [−0.10, −0.01]	−0.11 [−0.17, −0.06]	0.00 [−0.05, 0.06]	0.24 [0.22, 0.26]
CPI	−0.27 [−0.43, −0.11]	−0.05 [−0.08, −0.01]	−0.10 [−0.16, −0.04]	−0.02 [−0.12, 0.07]	0.03 [−0.02, 0.07]	0.35 [0.33, 0.37]
Core CPI		−0.03 [−0.08, 0.03]	−0.08 [−0.13, −0.04]		0.05 [0.01, 0.09]	0.26 [0.24, 0.27]
PCE		−0.03 [−0.16, 0.12]	−0.10 [−0.16, −0.05]		0.12 [0.03, 0.21]	0.32 [0.30, 0.34]
Core PCE		−0.03 [−0.09, 0.04]	−0.07 [−0.11, −0.03]		0.06 [0.02, 0.11]	0.23 [0.21, 0.24]
Interest rates						
3m T-bill rate	0.04 [−0.07, 0.15]	−0.29 [−0.40, −0.05]	0.00 [−0.01, 0.02]	−0.08 [−0.18, 0.01]	0.19 [−0.06, 0.38]	0.05 [0.04, 0.07]
10y term spread		0.19 [0.07, 0.25]	0.01 [0.00, 0.03]		−0.08 [−0.23, 0.08]	0.01 [−0.00, 0.03]
Aaa spread		0.01 [−0.01, 0.03]	−0.02 [−0.04, −0.01]		−0.02 [−0.04, −0.01]	−0.02 [−0.03, −0.00]
Baa-Aaa spread			−0.02 [−0.03, −0.01]			−0.02 [−0.03, −0.01]
Recession prob.						
Recession prob.	−0.60 [−1.13, −0.09]	−0.90 [−2.02, 0.48]	−0.84 [−1.15, −0.53]	−3.44 [−3.89, −2.99]	−1.49 [−2.17, −0.32]	−0.52 [−0.86, −0.18]
Long-term forecasts						
Natural rate of unemp.		−0.02 [−0.07, 0.02]	−0.08 [−0.12, −0.04]		−0.02 [−0.06, 0.02]	−0.01 [−0.04, 0.03]
Stock Prices		0.03 [−0.13, 0.18]	0.07 [−0.13, 0.27]		0.07 [−0.06, 0.21]	0.21 [0.06, 0.36]
Productivity		0.02 [−0.02, 0.06]	0.05 [0.01, 0.09]		0.03 [−0.00, 0.07]	0.02 [−0.01, 0.06]
Real GDP		0.04 [−0.02, 0.09]	0.08 [0.05, 0.11]		0.07 [0.01, 0.10]	0.02 [−0.01, 0.04]
Treasury Bill		−0.11 [−0.18, −0.01]	0.07 [−0.03, 0.16]		0.07 [−0.03, 0.16]	0.21 [0.13, 0.27]
Term Spread		0.04 [−0.04, 0.11]	−0.03 [−0.12, 0.05]		−0.06 [−0.12, 0.00]	−0.11 [−0.17, −0.04]
Inflation		−0.01 [−0.03, 0.02]	−0.03 [−0.06, −0.01]		0.02 [0.01, 0.04]	0.11 [0.09, 0.12]
autoregressive coef. (ϕ)	0.20 [0.14, 0.26]	0.68 [0.65, 0.70]	0.58 [0.54, 0.61]	0.74 [0.71, 0.76]	0.71 [0.69, 0.74]	0.71 [0.68, 0.74]

Table D.18: Posterior for ρ and σ^2 for Subsamples (C)

	ρ			σ^2		
	1968Q4-1984Q3	1984Q4-2007Q4	2008Q1-2020Q4	1968Q4-1984Q3	1984Q4-2007Q4	2008Q1-2020Q4
GDP Components						
GDP	0.15 [0.05, 0.24]	-0.01 [-0.13, 0.11]	0.03 [-0.14, 0.21]	0.84 [0.74, 0.94]	0.03 [0.03, 0.04]	0.02 [0.02, 0.03]
Consumption	0.34 [0.20, 0.47]	0.53 [0.49, 0.56]	0.46 [0.42, 0.51]	1.17 [1.02, 1.35]	0.26 [0.25, 0.27]	0.28 [0.27, 0.30]
Non-res. investment	0.49 [0.40, 0.57]	0.65 [0.63, 0.68]	0.54 [0.50, 0.57]	9.56 [8.26, 10.97]	2.49 [2.38, 2.61]	3.17 [3.00, 3.36]
Res. investment	0.47 [0.36, 0.57]	0.68 [0.66, 0.71]	0.67 [0.64, 0.70]	30.72 [26.49, 35.49]	6.39 [6.09, 6.68]	11.35 [10.72, 12.02]
Non-federal gov't	0.57 [0.50, 0.63]	0.57 [0.54, 0.59]	0.69 [0.66, 0.71]	2.35 [2.06, 2.68]	0.70 [0.67, 0.73]	0.76 [0.72, 0.80]
Federal gov't	0.55 [0.46, 0.64]	0.63 [0.61, 0.66]	0.69 [0.66, 0.72]	8.22 [7.12, 9.42]	2.62 [2.50, 2.75]	2.35 [2.22, 2.48]
Inventories	0.37 [0.25, 0.49]	0.48 [0.44, 0.51]	0.46 [0.42, 0.50]	0.06 [0.05, 0.06]	0.03 [0.03, 0.03]	0.03 [0.03, 0.03]
Net exports	0.23 [0.10, 0.35]	0.65 [0.62, 0.67]	0.59 [0.55, 0.62]	0.37 [0.33, 0.43]	0.10 [0.10, 0.11]	0.10 [0.09, 0.10]
Other Real Activity						
Housing starts	0.62 [0.60, 0.65]	0.60 [0.57, 0.62]	0.69 [0.66, 0.71]	67.61 [64.03, 71.33]	25.12 [24.05, 26.26]	77.81 [73.68, 82.25]
Industrial production	0.42 [0.37, 0.46]	0.62 [0.59, 0.64]	0.40 [0.36, 0.44]	2.10 [1.96, 2.24]	0.95 [0.91, 0.99]	1.68 [1.59, 1.78]
Corporate profits	0.40 [0.36, 0.44]	0.52 [0.49, 0.55]	0.51 [0.46, 0.56]	25.88 [24.46, 27.41]	21.23 [20.08, 22.40]	24.55 [22.89, 26.35]
Labor Market						
Unemployment rate	0.59 [0.56, 0.62]	0.63 [0.61, 0.66]	0.37 [0.33, 0.40]	0.10 [0.09, 0.11]	0.04 [0.04, 0.04]	0.27 [0.26, 0.28]
Employment		0.60 [0.55, 0.65]	0.50 [0.46, 0.54]		0.08 [0.07, 0.08]	0.65 [0.61, 0.69]
Inflation						
GDP deflator	0.83 [0.79, 0.87]	0.64 [0.61, 0.66]	0.43 [0.39, 0.47]	0.20 [0.14, 0.26]	0.16 [0.15, 0.17]	0.15 [0.14, 0.15]
CPI	0.37 [0.23, 0.50]	0.57 [0.54, 0.60]	0.21 [0.15, 0.27]	1.67 [1.47, 1.88]	0.16 [0.15, 0.16]	0.08 [0.07, 0.08]
Core CPI		0.83 [0.77, 0.89]	0.67 [0.63, 0.70]		0.02 [0.01, 0.02]	0.04 [0.03, 0.04]
PCE		0.63 [0.51, 0.73]	0.26 [0.19, 0.33]		0.14 [0.12, 0.18]	0.06 [0.05, 0.06]
Core PCE		0.67 [0.55, 0.77]	0.69 [0.65, 0.73]		0.03 [0.02, 0.03]	0.03 [0.02, 0.03]
Interest rates						
3m T-bill rate	0.66 [0.59, 0.73]	-0.05 [-0.20, 0.12]	0.66 [0.63, 0.69]	1.10 [0.98, 1.25]	0.02 [0.02, 0.03]	0.09 [0.08, 0.09]
10y term spread		0.60 [0.56, 0.63]	0.61 [0.58, 0.64]		0.08 [0.08, 0.09]	0.12 [0.11, 0.12]
Aaa spread		0.39 [0.35, 0.43]	0.51 [0.47, 0.55]		0.06 [0.06, 0.07]	0.10 [0.09, 0.10]
Baa-Aaa spread			0.24 [0.19, 0.29]			0.05 [0.05, 0.06]
Recession prob.						
Recession prob.	0.54 [0.51, 0.57]	0.65 [0.62, 0.67]	0.64 [0.61, 0.66]	136.10 [129.50, 143.20]	81.90 [78.25, 85.62]	51.81 [49.03, 54.69]
Long-term forecasts						
Natural rate of unemp.		0.88 [0.84, 0.92]	0.91 [0.88, 0.94]		0.03 [0.02, 0.04]	0.03 [0.03, 0.04]
Stock Prices		0.82 [0.78, 0.86]	0.87 [0.83, 0.91]		1.39 [1.18, 1.64]	0.86 [0.71, 1.04]
Productivity		0.85 [0.82, 0.87]	0.87 [0.84, 0.90]		0.06 [0.06, 0.07]	0.05 [0.04, 0.05]
Real GDP		0.80 [0.77, 0.84]	0.89 [0.86, 0.91]		0.04 [0.03, 0.04]	0.02 [0.02, 0.02]
Treasury Bill		0.84 [0.80, 0.87]	0.74 [0.67, 0.80]		0.16 [0.14, 0.19]	0.33 [0.26, 0.41]
Term Spread		0.77 [0.72, 0.81]	0.81 [0.77, 0.85]		0.27 [0.23, 0.32]	0.26 [0.22, 0.31]
Inflation		0.84 [0.82, 0.85]	0.75 [0.73, 0.78]		0.07 [0.07, 0.08]	0.07 [0.07, 0.07]

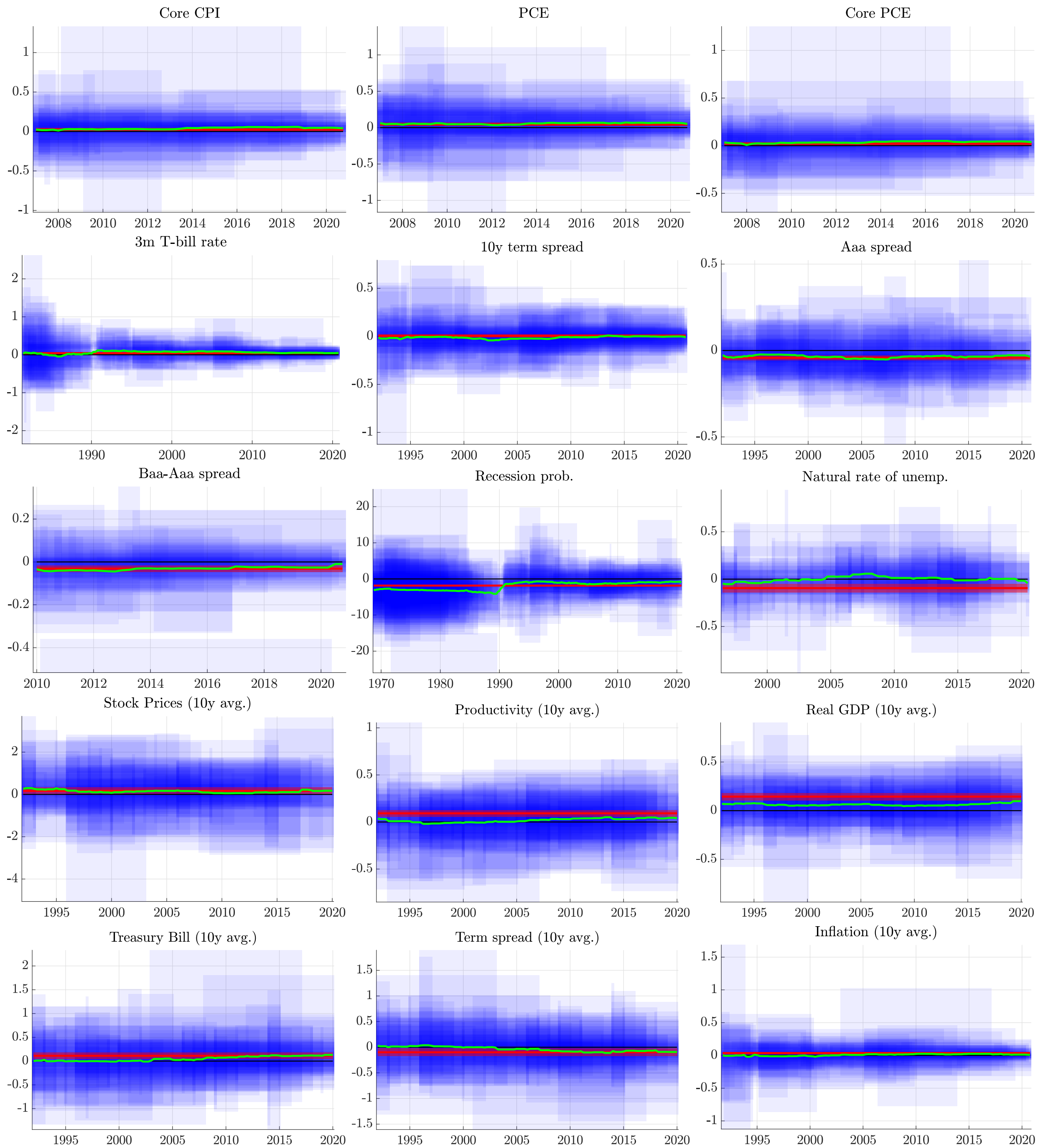
D.5 Individual estimates for $p = 1$

Figure D.1: Individual and Panel Estimates of Factor Loadings (I).



Note: Each panel plots the posterior distribution of the estimated factor loadings for the model with $p = 1$, estimated either on the entire panel or on each forecaster individually. Red lines represent the panel posterior mean. Blue rectangles represent individual estimates of a single forecaster, where the horizontal position indicates the time when that forecaster is present in the sample, and the vertical position indicates the range of values containing 67 percent of the posterior distribution. Green lines denote the average across forecasters of the individual posterior mean.

Figure D.2: Individual and Panel Estimates of Factor Loadings (II).



Note: Each panel plots the posterior distribution of the estimated factor loadings for the model with $p = 1$, estimated either on the entire panel or on each forecaster individually. Red lines represent the panel posterior mean. Blue rectangles represent individual estimates of a single forecaster, where the horizontal position indicates the time when that forecaster is present in the sample, and the vertical position indicates the range of values containing 67 percent of the posterior distribution. Green lines denote the average across forecasters of the individual posterior mean.

D.6 Model for Consensus Forecasts

Table D.19: POSTERIOR OF Λ AND ϕ IN THE CONSENSUS FORECAST MODEL

Variable	$\Lambda_{\cdot 1}$		$\Lambda_{\cdot 2}$	
	Mean	[5, 95]	Mean	[5, 95]
GDP Components				
GDP	0.16	[0.02, 0.30]	0.82	[0.75, 0.90]
Consumption	0.21	[0.04, 0.35]	0.82	[0.73, 0.92]
Non-res. investment	0.04	[-0.20, 0.30]	1.34	[1.17, 1.53]
Res. investment	0.09	[-0.20, 0.41]	1.49	[1.23, 1.77]
Non-federal gov't	0.02	[-0.01, 0.06]	0.09	[0.05, 0.13]
Federal gov't	0.01	[-0.11, 0.13]	-0.12	[-0.27, 0.04]
Inventories	0.01	[-0.00, 0.02]	0.02	[0.01, 0.03]
Net exports	-0.01	[-0.04, 0.03]	-0.00	[-0.05, 0.05]
Other Real Activity				
Housing starts	-0.33	[-0.91, 0.41]	1.97	[1.42, 2.52]
Industrial production	0.22	[0.02, 0.44]	1.26	[1.14, 1.39]
Corporate profits	0.46	[-0.00, 0.91]	1.88	[1.52, 2.24]
Labor Market				
Unemployment rate	0.02	[-0.06, 0.09]	-0.23	[-0.29, -0.18]
Employment	0.18	[0.01, 0.36]	1.04	[0.93, 1.15]
Inflation				
GDP deflator	0.59	[0.15, 0.93]	0.11	[-0.11, 0.32]
CPI	1.48	[1.41, 1.78]	-0.14	[-0.47, 0.21]
Core CPI	0.24	[0.11, 0.37]	0.16	[0.05, 0.27]
PCE	0.98	[0.90, 1.20]	-0.04	[-0.28, 0.20]
Core PCE	0.22	[0.12, 0.32]	0.17	[0.08, 0.26]
Interest rates				
3m T-bill rate	0.04	[-0.02, 0.10]	0.17	[0.11, 0.24]
10y term spread	-0.02	[-0.05, 0.02]	-0.02	[-0.05, 0.02]
Aaa spread	0.00	[-0.01, 0.02]	-0.05	[-0.07, -0.03]
Baa-Aaa spread	0.03	[-0.00, 0.05]	-0.01	[-0.03, 0.00]
Recession prob.				
Recession prob.	0.05	[-0.37, 0.46]	-0.38	[-0.81, 0.04]
Long-term forecasts				
Natural rate of unemp.	0.01	[-0.08, 0.11]	-0.03	[-0.16, 0.10]
Stock Prices	0.06	[-0.14, 0.28]	0.21	[-0.12, 0.55]
Productivity	0.02	[-0.04, 0.09]	0.08	[-0.02, 0.18]
Real GDP	0.04	[-0.02, 0.11]	0.12	[0.03, 0.22]
Treasury Bill	0.10	[-0.19, 0.38]	0.02	[-0.42, 0.45]
Term Spread	0.06	[-0.06, 0.18]	0.08	[-0.09, 0.27]
Inflation	1.50	[1.40, 1.83]	-0.26	[-0.60, 0.10]
autoregressive coef. (ϕ)	0.07	[0.01, 0.16]	0.56	[0.47, 0.65]

Notes: Parameter estimates are computed using pooled draws from 10 MCMC chains each of length 40,000 after a 10,000 period burn-in. See the Appendix for details.

Table D.20: Posterior of ρ and σ^2 in the Consensus Forecast Model

Variable	ρ		σ^2	
	Mean	[5, 95]	Mean	[5, 95]
GDP Components				
GDP	0.63	[0.39, 0.82]	0.07	[0.05, 0.08]
Consumption	0.38	[0.15, 0.59]	0.11	[0.08, 0.14]
Non-res. investment	0.89	[0.83, 0.95]	1.21	[0.99, 1.45]
Res. investment	0.93	[0.88, 0.97]	3.38	[2.80, 4.03]
Non-federal gov't	0.94	[0.90, 0.98]	0.10	[0.09, 0.12]
Federal gov't	0.85	[0.79, 0.92]	1.28	[1.08, 1.52]
Inventories	0.85	[0.79, 0.90]	0.01	[0.01, 0.01]
Net exports	0.99	[0.97, 1.00]	0.16	[0.13, 0.19]
Other Real Activity				
Housing starts	0.98	[0.95, 1.00]	23.06	[19.53, 27.10]
Industrial production	0.77	[0.67, 0.86]	0.30	[0.25, 0.37]
Corporate profits	0.77	[0.69, 0.84]	6.66	[5.64, 7.84]
Labor Market				
Unemployment rate	0.95	[0.92, 0.99]	0.23	[0.19, 0.27]
Employment	0.76	[0.55, 0.94]	0.15	[0.11, 0.20]
Inflation				
GDP deflator	0.08	[-0.04, 0.20]	4.29	[3.62, 5.09]
CPI	0.29	[-0.03, 0.59]	0.17	[0.13, 0.22]
Core CPI	-0.11	[-0.28, 0.12]	0.30	[0.23, 0.40]
PCE	0.24	[-0.12, 0.59]	0.19	[0.14, 0.26]
Core PCE	0.02	[-0.23, 0.30]	0.18	[0.14, 0.24]
Interest rates				
3m T-bill rate	0.74	[0.64, 0.83]	0.22	[0.18, 0.26]
10y term spread	0.96	[0.92, 0.99]	0.09	[0.07, 0.11]
Aaa spread	0.91	[0.85, 0.96]	0.01	[0.01, 0.02]
Baa-Aaa spread	0.63	[0.50, 0.76]	0.01	[0.01, 0.01]
Recession prob.				
Recession prob.	0.77	[0.69, 0.84]	12.17	[10.36, 14.29]
Long-term forecasts				
Natural rate of unemp.	0.92	[0.84, 0.98]	0.04	[0.02, 0.06]
Stock Prices	0.95	[0.89, 0.99]	0.19	[0.12, 0.27]
Productivity	0.91	[0.83, 0.97]	0.02	[0.01, 0.03]
Real GDP	0.91	[0.84, 0.98]	0.02	[0.01, 0.03]
Treasury Bill	0.89	[0.79, 0.96]	0.33	[0.21, 0.49]
Term Spread	0.88	[0.78, 0.96]	0.06	[0.04, 0.09]
Inflation	0.45	[0.17, 0.70]	0.20	[0.16, 0.26]

Notes: Parameter estimates are computed using pooled draws from 10 MCMC chains each of length 40,000 after a 10,000 period burn-in. See the Appendix for details.