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A Structural Measure of the Shadow Federal Funds Rate*

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Abstract

We propose a shadow policy interest rate based on an estimated structural model that accounts for the zero lower bound. The lower bound constraint, if expected to bind, is contractionary and increases the shadow rate compared to an unconstrained systematic policy response. By contrast, forward guidance and other unconventional policies that extend the expected duration of zero-interest-rate policy are expansionary and decrease the shadow rate. By quantifying these distinct effects, our structural shadow federal funds rate better captures the stance of monetary policy given economic conditions than a shadow rate based only on the term structure of interest rates.

Keywords: zero lower bound; forward guidance; shadow rate; monetary policy

JEL classifications: E52; E58

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1 Introduction

In response to the Great Recession and the COVID-19 crisis, the Federal Reserve, like many other central banks, cut its policy interest rate close to zero. When this happens, the lower bound constraint on nominal interest rates makes it difficult to determine the stance of monetary policy given prevailing economic conditions from the observed policy rate alone. In an influential paper, Wu and Xia (2016) use a term structure model to construct a 'shadow' policy rate intended to quantify the interest-rate-equivalent stance of policy at the zero lower bound (ZLB). The basic idea is that the shadow rate reflects the effects of unconventional policies in terms of a hypothetical unconstrained short-term interest rate.

Having a shadow rate that captures monetary policy at the ZLB is useful for two reasons. First, policymakers can gauge the scale of unconventional policy actions with a comparable measure to policy conducted during conventional times. Second, researchers can extend empirical analysis using linear models into periods in which the observed policy rate is at the ZLB, as done with the Wu-Xia shadow rate in studies such as Avdjiev et al. (2020) and Anderson et al. (2017).

We argue, however, that shadow rates derived from term structure models, including those developed in Ichiue and Ueno (2013) and Krippner (2013), fail to accurately reflect the stance of monetary policy because they do not disentangle movements in the term structure due to unconventional policies from those due to other shocks.[1] Term structure models can be used to extract market expectations about the duration of the ZLB, as in Ichiue and Ueno (2015). Yet, to capture the policy stance given economic conditions, it is critical to uncover the underlying determinants of the duration. Specifically, is the policy rate expected to be zero because deteriorating economic conditions suggest the ZLB constraint is likely to bind for a long time? Or does the expected duration reflect unconventional 'lower-for-longer' zero-interest-rate policy beyond what the ZLB constraint would imply on its own?

Shadow rate term structure models (SRTSMs) do not account for this distinction in terms of why the policy rate is expected to be zero. To do so, it is necessary to have a more structural model, which is what we propose in this paper. In the SRTSM approach, the actual policy rate

[1] Bauer and Rudebusch (2016) caution against using shadow rates from term structure models for the alternative reason that they are sensitive to model specification. Meanwhile, Johannsen and Mertens (2021) argue that it is important to augment term structure models with macroeconomic variables. However, we note that, like with the more basic term structure models, their shadow rate is restricted to be non-positive and they do not separately identify the effects of unconventional policies and other shocks.

follows $i_t = \max(i_t^*, 0)$, where i_t^* is the shadow policy rate. Therefore, it is impossible to have the shadow rate greater than zero when $i_t = 0$. In our approach, a binding constraint is equivalent to a contractionary policy shock in an unconstrained system that increases the shadow rate relative to the level implied by the systematic policy response to economic conditions. Our structural measure of the shadow rate thus allows the possibility that $i_t^* > i_t = 0$, which would occur, for example, if the unconstrained systematic policy response is to set the policy rate exactly at zero, but the ZLB constraint is expected to bind in the future. Meanwhile, forward guidance and other unconventional policies that extend the expected duration of zero-interest-rate policy act to offset the effects of the constraint, decreasing the shadow rate.

The overall level of our shadow rate at any point in time reflects the net effects of the ZLB constraint and unconventional policies. Depending on the implied systematic policy response and how much unconventional policies offset a binding ZLB constraint, our shadow rate can be positive or negative when the actual policy rate is zero. In the SRTSM approach, the shadow rate is estimated from yield-curve data and a 3-month yield is typically used as the short-term rate. Consequently, it is technically feasible for an SRTSM measure to be greater than the federal funds rate, as in Figure 4 of Wu and Xia (2016) at the start of the ZLB in 2009 when the 3-month yield is still positive. But, unlike with our approach, the reason is not directly related to the extent to which the ZLB constraint binds for the policy rate.

Section 2 describes how our structural shadow rate is constructed, illustrating its direct link to the stance of monetary policy with a simple analytical example. Section 3 presents the estimated structural shadow federal funds rate for the Smets and Wouters (2007) model allowing for the ZLB. Section 4 shows how our shadow rate closely aligns with notable unconventional policy actions in the aftermath of the Great Recession, compares it with an SRTSM measure in a VAR with data from the ZLB, and extends the sample to look at monetary policy during the recent pandemic. Section 5 concludes.

2 The Structural Shadow Rate

2.1 Decomposition of ZLB Durations

The structural shadow rate can be constructed given a structural model that accounts for the ZLB. In the next section, we consider the Smets and Wouters (2007) model allowing for the lower bound on nominal interest rates, but here we first describe how the structural shadow rate is constructed in general. Because the structural shadow rate is based on decomposing the expected duration of how long the ZLB will hold into its underlying sources, we begin with the details of this decomposition, as proposed in Jones et al. (2021).

Assume the policy rate, i_t, either follows an unconstrained Taylor-type policy rule or is fixed when at the ZLB. For expositional simplicity, we set the fixed level to zero, although it could be any feasible value, including an alternative effective lower bound. Thus, the policy rate can be described by

$$i_t = \begin{cases} \text{policy rule}, & \mathbb{I}_t = 0 \\ 0, & \mathbb{I}_t = 1, \end{cases} \tag{1}$$

where the indicator $\mathbb{I}_t$ keeps track of the policy-setting regime. When $\mathbb{I}_t = 0$, the policy rate follows the unconstrained policy rule and linearized structural equations are given by

$$\mathbf{A}x_t = \mathbf{C} + \mathbf{B}x_{t-1} + \mathbf{D}\mathbb{E}_t x_{t+1} + \mathbf{F}\varepsilon_t, \tag{2}$$

where x_t is an $n \times 1$ vector of model variables, ε_t is an $l \times 1$ vector of structural shocks, and $\mathbf{A}$, $\mathbf{C}$, $\mathbf{B}$, $\mathbf{D}$, and $\mathbf{F}$ are conformable matrices. The reduced-form solution to the $\mathbb{I}_t = 0$ regime is

$$x_t = \mathbf{J} + \mathbf{Q}x_{t-1} + \mathbf{G}\varepsilon_t, \tag{3}$$

where $\mathbf{J}$, $\mathbf{Q}$, and $\mathbf{G}$ are the standard linear rational expectations solution matrices. When $\mathbb{I}_t = 1$, the structural equations are described by

$$\bar{\mathbf{A}}x_t = \bar{\mathbf{C}} + \bar{\mathbf{B}}x_{t-1} + \bar{\mathbf{D}}\mathbb{E}_t x_{t+1} + \bar{\mathbf{F}}\varepsilon_t, \tag{4}$$

where the only equation that changes from (2) is the one corresponding to the policy rate, which now is fixed at zero.[2] As discussed in Jones (2017) and Kulish et al. (2017), the solution when $\mathbb{I}_t = 1$ is, following Kulish and Pagan (2017), a time-varying VAR of the form

$$x_t = \mathbf{J}_t + \mathbf{Q}_t x_{t-1} + \mathbf{G}_t \varepsilon_t, \tag{5}$$

where $\mathbf{J}_t$, $\mathbf{Q}_t$, and $\mathbf{G}_t$ are time-varying matrices that depend on the expected duration of the fixed-interest-rate regime, denoted $\mathbf{d}_t$. To keep track of the reduced-form that prevails at each point time, we allow $\bar{T}$ to be an arbitrarily large upper-bound duration and find the sequences $\{\mathbf{J}_d\}_{d=1}^{\bar{T}}$, $\{\mathbf{Q}_d\}_{d=1}^{\bar{T}}$, and $\{\mathbf{G}_d\}_{d=1}^{\bar{T}}$ such that $\mathbf{J}_d$, $\mathbf{Q}_d$, and $\mathbf{G}_d$ are reduced-form matrices corresponding to a particular expected duration d. The prevailing reduced-form in a given period can be

[2] It is possible to allow other structural equations to change in the fixed-interest-rate regime, such as would be the case if the propagation of certain shocks changes under the ZLB. However, we focus on the case where the relevant structural change is in terms of setting the policy rate to align with our application to the Smets and Wouters (2007) model.

relabeled as $\mathbf{J}_t = \mathbf{J}_{\mathbf{d}_t}$, $\mathbf{Q}_t = \mathbf{Q}_{\mathbf{d}_t}$, and $\mathbf{G}_t = \mathbf{G}_{\mathbf{d}_t}$ and, noting that $\mathbf{d}_t = 0$ in periods where $\mathbb{I}_t = 0$ and $\mathbf{J}_0 = \mathbf{J}$, $\mathbf{Q}_0 = \mathbf{Q}$, and $\mathbf{G}_0 = \mathbf{G}$, the reduced-form over the full sample is also given by (5), providing the basis of estimation. At the estimated parameter values with the estimated structural shocks, (5) returns back the data.

Following Jones et al. (2021), the duration of a fixed-interest-rate regime $\mathbf{d}_t$ corresponds to the actual duration expected by agents, which is not necessarily the same duration prescribed by the policy rule given the ZLB constraint. With the occasionally-binding-constraint solution of Jones (2017), we can find the duration in each period t prescribed by the policy rule – i.e. the expected duration implied by the estimated state x_{t-1}, the estimated non-policy structural shocks, and a given lower bound.[3] This duration corresponds to monetary policy following exactly $\max(\text{policy rule}, 0)$ in projecting when the policy rate lifts off from the ZLB. We denote this expected duration by $\mathbf{d}_t^{\text{lb}}$ and refer to it as the lower-bound duration. In the absence of future shocks, this duration is expected to fall by one period at a time as the effects of current shocks unwind.

Again following Jones et al. (2021), for each period of the fixed-interest-rate regime, we define the forward-guidance duration, $\mathbf{d}_t^{\text{fg}}$, as the difference between the actual duration $\mathbf{d}_t$ and the lower-bound duration $\mathbf{d}_t^{\text{lb}}$, so that the actual duration is decomposed as

$$\mathbf{d}_t = \mathbf{d}_t^{\text{lb}} + \mathbf{d}_t^{\text{fg}}. \tag{6}$$

The forward-guidance duration captures announcements and other factors that change the actual duration beyond the lower-bound duration.[4] This decomposition is important because changes in $\mathbf{d}_t$ that originate from $\mathbf{d}_t^{\text{lb}}$ have different implications for the observed variables than those from $\mathbf{d}_t^{\text{fg}}$. For example, output and inflation might fall after a negative shock that makes the ZLB constraint bind for longer. But output and inflation might increase after a 'lower-for-longer' announcement extending the fixed-interest-rate regime beyond what the ZLB constraint implies.

[3] Also see Guerrieri and Iacoviello (2015) on solving rational expectations models with occasionally-binding constraints.

[4] In addition to explicit central bank communications about durations, the forward-guidance duration could be influenced by other unconventional policies, such as large scale asset purchases, or by public expectations that the central bank might deviate from its policy rule, such as by raising the policy rate sooner than implied by the ZLB constraint. For example, markets reassessed the likely timing of liftoff as the Fed was tapering the rate of bond purchases in 2013 and this can be thought of as having decreased the forward-guidance duration.

2.2 Constructing the Structural Shadow Rate

To construct the structural shadow rate, we find shocks under the unconstrained-policy-rule regime $\mathbb{I}_t = 0$ that would replicate outcomes in the data under the fixed-interest-rate regime $\mathbb{I}_t = 1$. Specifically, we augment the vector of estimated structural shocks ε_t with a shadow rate shock, $\varepsilon^*_{m,t}$, to a hypothetical policy rate for the periods when $\mathbb{I}_t = 1$. The shadow rate shocks are chosen so that outcomes in a shadow economy based on the structure in (3)

$$x^*_t = \mathbf{J} + \mathbf{Q}x^*_{t-1} + \mathbf{G}\varepsilon^*_t, \tag{7}$$

approximate outcomes for at least some variables in x_t observed in the actual economy (5). When $\mathbb{I}_t = 0$, the structural shocks ε^*_t are the same as ε_t. The structural shadow rate, i^*_t, is defined as the policy interest rate that prevails in the shadow economy (7).

Formally, let ι denote a vector that selects which variables to target in matching the outcomes across systems (5) and (7), with $\Delta_t = \iota(x_t - x^*_t)$ denoting the difference between the observed targeted variables and the same variables as they evolve in the shadow economy.[5] The shadow rate shock $\varepsilon^*_{m,t}$ is chosen for each period to solve

$$\min_{\varepsilon^*_{m,t}} \Delta'_t \mathbf{W} \Delta_t, \tag{8}$$

where $\mathbf{W}$ is a diagonal weighting matrix that reflects the volatility of the targeted variables, with the diagonal elements being the inverse of the variance of each corresponding variable. This weighting scheme effectively standardizes the data and, as a result, implies equal weights in matching each of the targeted variables. The shadow rate i^*_t is then constructed using the unconstrained policy rule with the shadow rate shock that solves the minimization problem (8).

The next subsection presents a simple model to show analytically how changes in the length of the ZLB constraint and forward guidance map into contractionary and expansionary shadow rate shocks and how the constructed shadow rate reflects the stance of monetary policy.

[5]The choice of which variables to target potentially matters because, in principle, shocks to the hypothetical policy rate in a linear system that replicate the dynamics of one variable need not be the same as those that replicate the dynamics of another. However, in our application to the Smets and Wouters (2007) model, we choose to match all of the observed variables other than interest rates and find we can closely match all of the targeted variables simultaneously.

2.3 Analytical Results for a Simple Model

We consider a simple three-equation New Keynesian model that provides analytical solutions for the shadow rate shock and the shadow rate. The model is

$$\hat{y}_t = -\left(i_t - \bar{i} - \mathbb{E}_t \hat{\pi}_{t+1}\right) + \mathbb{E}_t \hat{y}_{t+1} + \varepsilon_{y,t} \tag{9}$$

$$\hat{\pi}_t = \beta \mathbb{E}_t \hat{\pi}_{t+1} + \kappa \hat{y}_t \tag{10}$$

$$i_t = \max(0, \bar{i} + \phi \hat{\pi}_t + \varepsilon_{m,t}), \tag{11}$$

where $\hat{y}_t$ is output in deviation from steady state, i_t is the nominal interest rate in levels with steady-state value $\bar{i} > 0$ and a ZLB constraint $i_t \geq 0$, $\hat{\pi}_t$ is inflation in deviation from steady state, and $\varepsilon_{y,t}$ and $\varepsilon_{m,t}$ are mean-zero serially-uncorrelated shocks, with positive discount factor, $\beta > 0$, positive slope of the Phillips curve, $\kappa > 0$, and a more than one-for-one systematic policy response to inflation, $\phi > 1$. Conditional on $i_t > 0$, mean-zero shocks imply $\mathbb{E}_t \hat{\pi}_{t+1} = \mathbb{E}_t \hat{y}_{t+1} = 0$ and the solution is

$$\hat{y}_t = \frac{1}{1 + \kappa\phi}(\varepsilon_{y,t} - \varepsilon_{m,t}) \tag{12}$$

$$\hat{\pi}_t = \frac{\kappa}{1 + \kappa\phi}(\varepsilon_{y,t} - \varepsilon_{m,t}) \tag{13}$$

$$i_t = \bar{i} + \phi \frac{\kappa}{1 + \kappa\phi}\varepsilon_{y,t} + \frac{1}{1 + \kappa\phi}\varepsilon_{m,t}. \tag{14}$$

Lower Bound. Suppose $\varepsilon_{m,t} = 0$ and the shock $\varepsilon_{y,t}$ is such that the ZLB constraint binds, which from (14) occurs when $\varepsilon_{y,t} \leq -\bar{i}\frac{(1+\kappa\phi)}{\kappa\phi}$. In this case, $i_t = 0$ and the solution is

$$\hat{y}_t^{\text{lb}} = \bar{i} + \varepsilon_{y,t} \tag{15}$$

$$\hat{\pi}_t^{\text{lb}} = \kappa(\bar{i} + \varepsilon_{y,t}). \tag{16}$$

A binding ZLB introduces a kink in the slope of the policy functions for output and inflation, as a comparison of (12) with (15) and (13) with (16) shows.

Our procedure for generating the shadow rate is to find a hypothetical policy shock $\varepsilon_{m,t}^*$ in the no-ZLB solution (12) to (14) that replicates the outcomes $\hat{y}_t^{\text{lb}}$ and $\hat{\pi}_t^{\text{lb}}$. We can do so following our procedure by choosing $\varepsilon_{m,t}^*$ to minimize $\Delta_t' \mathbf{W} \Delta_t$ from (8), which in this case is

$$w_y \left(\hat{y}_t^* - \hat{y}_t^{\text{lb}}\right)^2 + w_\pi \left(\hat{\pi}_t^* - \hat{\pi}_t^{\text{lb}}\right)^2,$$

where w_y and w_π are implicit weights. Because $\hat{\pi}_t = \kappa \hat{y}_t$ and $\hat{\pi}_t^{\text{lb}} = \kappa \hat{y}_t^{\text{lb}}$, minimizing $\Delta_t' \mathbf{W} \Delta_t$ is equivalent to minimizing either $\left(\hat{y}_t^* - \hat{y}_t^{\text{lb}}\right)^2$ or $\left(\hat{\pi}_t^* - \hat{\pi}_t^{\text{lb}}\right)^2$. This can be done exactly with

$\Delta_t = 0$ by setting $\hat{y}_t^* = \hat{y}_t^{\text{lb}}$, or, equivalently,

$$\frac{1}{1+\kappa\phi}(\varepsilon_{y,t} - \varepsilon^*_{m,t}) = \bar{i} + \varepsilon_{y,t},$$

which implies

$$\varepsilon^*_{m,t} = -\bar{i}(1+\kappa\phi) - \kappa\phi\varepsilon_{y,t} \geq 0. \tag{17}$$

The equality in (17) shows the contractionary shadow rate shock required to generate the same equilibrium outcomes as those obtained under the binding ZLB. This shadow rate shock and (14) imply the shadow rate is $i_t^* = 0$, which is higher than the negative level implied by the systematic monetary policy response to economic conditions if the inequality $\varepsilon_{y,t} < -\bar{i}\frac{(1+\kappa\phi)}{\kappa\phi}$ is strict, i.e. $i_{t,\text{systematic}} = \bar{i}(1+\kappa\phi) + \kappa\phi\varepsilon_{y,t} < 0$, where $i_{t,\text{systematic}} \equiv \bar{i} + \phi\hat{\pi}_t$. Meanwhile, as illustrated next, persistence can generate positive shadow rates at the ZLB.

Expected Negative Shock at the Lower Bound. Assume now that $\mathbb{E}_t\varepsilon_{y,t+1} = -\bar{i}\frac{(1+\kappa\phi)}{\kappa\phi}$, so that agents expect a negative shock tomorrow will continue to cause the ZLB to bind beyond the negative shock $\varepsilon_{y,t} = -\bar{i}\frac{(1+\kappa\phi)}{\kappa\phi}$ that hits today. $\mathbb{E}_t\hat{y}_{t+1}$ and $\mathbb{E}_t\hat{\pi}_{t+1}$ are determined from (15) and (16) and, given $i_t = 0$,

$$\hat{y}_t^{\text{lb2}} = -\bar{i}\frac{(2+\kappa)}{\kappa\phi}$$
$$\hat{\pi}_t^{\text{lb2}} = -\bar{i}\frac{(2+\kappa+\beta)}{\phi}.$$

Unlike the previous case, $\hat{\pi}_t^{\text{lb2}} \neq \kappa\hat{y}_t^{\text{lb2}}$, so there is not necessarily a shadow rate shock that would lead to an exact match when targeting both output and inflation. Numerically, we could minimize

$$w_y\left(\hat{y}_t^* - \hat{y}_t^{\text{lb2}}\right)^2 + w_\pi\left(\hat{\pi}_t^* - \hat{\pi}_t^{\text{lb2}}\right)^2. \tag{18}$$

However, for analytical tractability, suppose we choose $\varepsilon^*_{m,t}$ to match inflation only (i.e. let $w_y = 0$).[6] This match can be done exactly by setting $\hat{\pi}_t^* = \hat{\pi}_t^{\text{lb2}}$. In this case, the shadow rate shock is

$$\varepsilon^*_{m,t} = \frac{\bar{i}}{\kappa\phi}(1+\kappa\phi)(1+\kappa+\beta) > 0,$$

and, from (14), the shadow rate is

$$i_t^* = \frac{\bar{i}}{\kappa\phi}(1+\kappa+\beta) > 0.$$

[6]If, instead, we were to match output only, we would get similar, but not identical, analytical expressions for the shadow rate shock and shadow rate, with the same implications in terms of their signs. However, the shadow rate would not be exactly equal to the sum of the implied systematic policy response and the shadow rate shock.

This case illustrates how persistent negative shocks generate contractionary expectations today that can push the shadow rate into positive territory, even though the actual interest rate is at zero, while the level implied by the systematic monetary policy response is negative, i.e. $i_{t,\text{systematic}} = -\bar{i}(1+\kappa+\beta) < 0$. Thus, unlike the SRTSM approach in which the actual short rate is equal to $i_t = \max(i_t^*, 0)$, the shadow rate in our approach can be above zero when $i_t = 0$ because the ZLB constraint being expected to bind in the future acts like a contractionary monetary policy shock in the shadow economy given lower expected future inflation raising the current real interest rate.

Forward Guidance. Suppose, instead, that the ZLB constraint binds in period t because of a negative shock $\varepsilon_{y,t} = -\bar{i}\frac{(1+\kappa\phi)}{\kappa\phi}$, but, in addition to setting $i_t = 0$, the central bank credibly announces it will continue to hold the interest rate at zero in the next period, $\mathbb{E}_t i_{t+1} = 0$. Then, $\mathbb{E}_t \hat{y}_{t+1} = \bar{i}$ and $\mathbb{E}_t \hat{\pi}_{t+1} = \kappa\bar{i}$. In this case,

$$\hat{y}_t^{\text{fg}} = \bar{i}\,(2+\kappa) + \varepsilon_{y,t}$$
$$\hat{\pi}_t^{\text{fg}} = \kappa\bar{i}\,(2+\kappa+\beta) + \kappa\varepsilon_{y,t}.$$

The 'lower-for-longer' announcement boosts output and inflation today relative to the lower-bound solution (15) and (16).[7] In constructing the shadow rate, similar to the case of an expected negative shock, it is analytically convenient to match inflation only, which again can be done exactly with a perfect match $\Delta_t = 0$ by setting $\hat{\pi}_t^* = \hat{\pi}_t^{\text{fg}}$. Rearranging to solve for the shadow rate shock gives

$$\varepsilon_{m,t}^* = -\bar{i}(1+\kappa\phi)(1+\kappa+\beta) < 0.$$

Thus, forward guidance maps into an expansionary shadow rate shock and, from (14), the shadow rate would be less than the positive level implied by the systematic monetary policy response to economic conditions, i.e. $i_{t,\text{systematic}} = \kappa\phi\bar{i}(1+\kappa+\beta) > 0$, as it is strictly negative:

$$i_t^* = -\bar{i}(1+\kappa+\beta) < 0.$$

In this way, the shadow rate is able to reflect the policy stance when there is forward guidance.[8]

[7] We note the differences in the stimulatory effect of forward guidance from the stimulatory effect of a conventional expansionary monetary policy shock, which reflects the fact that forward guidance operates by affecting expectations.

[8] This contrasts with Hills and Nakata (2018), who define a shadow rate in a New Keynesian model as corresponding to the policy rate that would be set according to the policy rule in the absence of the ZLB constraint or monetary policy shocks. Specifically, their shadow rate is linked to the observed rate via $i_t = \max(i_t^*, 0)$. In this case, a negative shadow rate $i_t^* < 0$ simply reflects how much and for how long the constraint binds given the policy rule and other structural shocks, rather than providing a guide to the stance of monetary policy.

3 The Shadow Rate from an Estimated Structural Model

3.1 A Medium-Scale New Keynesian Model

To estimate the structural shadow federal funds rate, we consider the Smets and Wouters (2007) model allowing for the ZLB, as in Kulish et al. (2017). The model is estimated using U.S. data over 1984Q1 to 2019Q4 and we make the following changes to Smets and Wouters (2007): First, similar to Kulish et al. (2017), we expand the set of observables to include the 1-year and 5-year Treasury yields. Second, to capture the trend decline in interest rates over this period, we allow for a decline in trend growth. In particular, motivated by the evidence of structural break in trend growth in the 2000s documented in a number of studies including Fernald (2012), Antolin-Diaz et al. (2017), and Eo and Morley (2020), we allow for a one-time change in trend growth at a magnitude and date to be estimated. This captures the possibility that a decline in trend growth lowers the equilibrium level of the policy rate, which could cause the ZLB to be visited more frequently. Finally, we calibrate the inflation target to a 2% annualized rate to reflect the Fed's inflation objective.

We follow Smets and Wouters (2007) in all other respects, including the remaining observed variables and their construction, the set of estimated parameters, and priors. Motivated by the results in Kulish et al. (2017), we use modal reported values of durations from Blue Chip Financial Forecasts and the New York Fed Survey of Primary Dealers to measure expected durations of zero-interest-rate policy during the ZLB. Full details of the model are in the appendix.

3.2 Estimated Shadow Rate

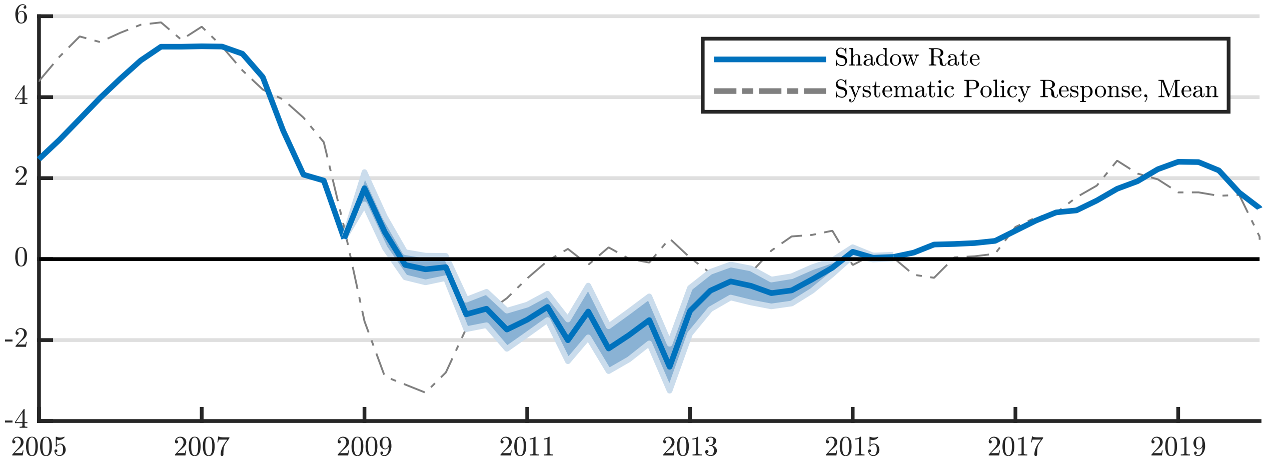
The shadow rate for the Smets and Wouters (2007) model is constructed as described in Section 2.2. We target all of the observed variables except the interest rates.[9]

Figure 1 plots the estimated structural shadow federal funds rate and the shadow rate shocks. The posterior mean of the shadow rate reported in the top panel deviates from the observed federal funds rate when it hit the ZLB in 2009Q1, taking on a value of 1.6% with precise 90% posterior bands. This initial positive value for the shadow rate illustrates the contractionary effects of the ZLB constraint being expected to bind persistently given the large negative

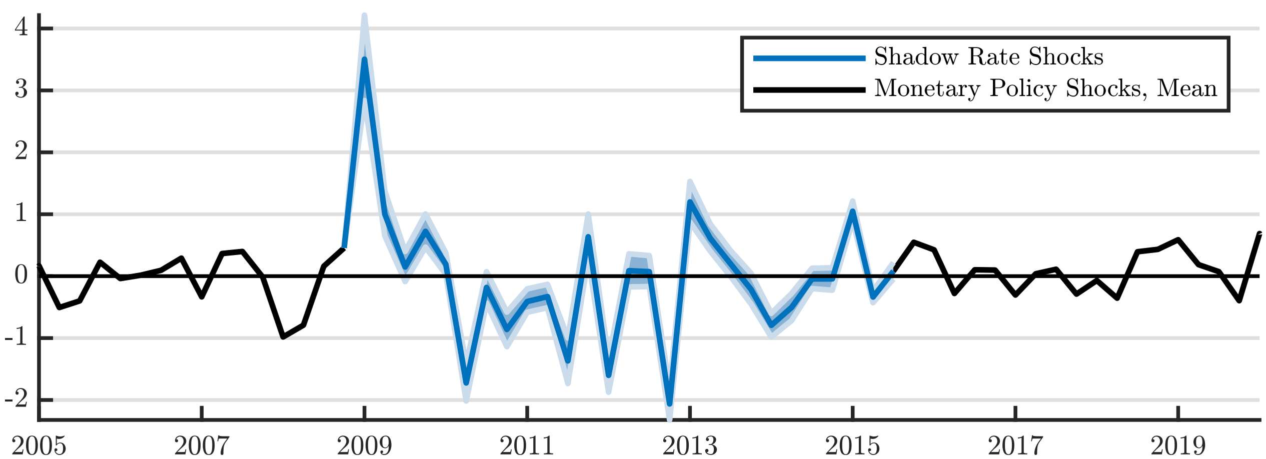
[9]The appendix contains plots of the paths of targeted variables both in the data and in the shadow economy. The plots show that the variables in the shadow economy are very close to the actual data. The shadow rate shocks are thus able to capture, with great accuracy, the dynamics of all of the targeted variables. We find that results are similar if we also target longer-term interest rates.

Figure 1: The Structural Shadow Federal Funds Rate

(a) Shadow Rate, i_t^*, and Systematic Policy Response, % Annualized



(b) Shadow Rate Shocks, $\varepsilon_{m,t}^*$



Notes: Panel (a) plots, in annualized percentage terms, the estimated shadow rate, along with the implied systematic policy response to prevailing economic conditions based on the policy rule without monetary policy or shadow rate shocks but allowing for interest rate smoothing, i.e. $i_{t,\text{systematic}} = \bar{i} + (1-\alpha_i)\alpha_p\hat{\pi}_t + (1-\alpha_i)\,\alpha_y\tilde{y}_t + \alpha_{\Delta y}\Delta\tilde{y}_t + \alpha_i(i_{t-1,\text{systematic}} - \bar{i})$, where the α's are the monetary policy response coefficients and $\tilde{y}_t$ is the output gap from the flexible price equilibrium for the Smets and Wouters (2007) model detailed in the appendix. Panel (b) plots, in annualized percentage point terms, the shadow rate shocks. The lines correspond to posterior means, while the bands show 90 percent equal-tailed posterior intervals for the shadow rate and shadow rate shocks during the ZLB period.

shocks that triggered the Great Recession. The contractionary stance of monetary policy due to the ZLB constraint is especially clear in comparison to the quick decline in the posterior mean of the systematic policy response implied by the policy rule and prevailing economic conditions to values below -3%.[10] However, despite the ZLB constraint, the estimated shadow rate implies relatively expansionary monetary policy compared to the systematic policy response from the beginning of 2011 and declines to about -2.4% by 2012Q4 given implementation of various unconventional policies including forward guidance that allowed the Fed to achieve the equivalent of an unconstrained negative rate in the shadow economy. After bottoming out in 2012, the estimated shadow rate increased back towards the level implied by systematic policy at around -0.5% in 2013Q2 and reaches zero in 2015Q1, just before liftoff. These shifts in the policy stance are reflected in the shadow rate shocks reported in the bottom panel. After the initial contractionary effects of the ZLB constraint being expected to bind for a number of quarters, the shadow rate shocks are typically estimated to be negative throughout the remaining ZLB period and quantify the interest-rate-equivalent effects of forward guidance and other unconventional policies.

There are important differences in the estimates of our shadow rate and other measures. In the appendix, we compare our shadow rate to the one constructed by Wu and Xia (2016). After being slightly more positive in 2009Q1, our shadow rate falls below the Wu-Xia measure in 2010 and has more pronounced fluctuations that appear closely related to forward guidance announcements, as discussed in more detail in Section 4. At its trough in 2012Q4, just before the taper tantrum, our shadow rate is more than a percentage point below the Wu-Xia measure. A stark gap then opens up between our shadow rate and the Wu-Xia shadow rate over 2013 through 2015. Relative to a fairly flat implied systematic policy response to economic conditions at the time, our measure suggests that monetary policy was becoming relatively less accommodative in the lead up to liftoff from the ZLB, while the Wu-Xia measure fell by almost 2 percentage points to a low of -2.9% by 2014Q2, suggesting policy was becoming more expansionary.[11] In Section 4, we compare the performance of both measures in a VAR that controls for economic conditions in terms of inflation and output growth when identifying monetary

[10]The implied systematic policy response is calculated using the prevailing values of the target variables in the policy rule, but setting the monetary policy shocks and shadow rate shocks to zero and allowing for interest rate smoothing according to the estimated policy rule. The policy rule in the Smets and Wouters (2007) model assumes systematic responses to inflation, the output gap, and changes in the output gap.

[11]Sims and Wu (2020) show that the Wu-Xia measure is correlated with the Fed's balance sheet during this period, but acknowledge that the "shadow rate series is based on empirical term structure models that do not have an explicit mapping back into structural economic models or particular unconventional tools".

policy shocks and find that our measure performs better due to the different signals it gives about the stance of policy during the ZLB.

3.3 No Forward Guidance Counterfactual

To understand the effects of unconventional policies, we explore the counterfactual scenario of what would have happened in the aftermath of the Great Recession without forward guidance. We construct this counterfactual using the solution (5) implied by the expected durations $\mathbf{d}_t$ to obtain an estimate of the structural shocks ε_t and then feeding the estimated structural shocks through the occasionally-binding-constraint solution of Jones (2017) to solve for the path of the economy if monetary policy simply followed the prescription of the policy rule constrained by the ZLB. In this case, the durations of the ZLB expected by agents are $\mathbf{d}_t^{\text{lb}}$ in the notation of Section 2.

Figure 2 plots the counterfactual paths of output, inflation, and the shadow rate removing forward guidance. These paths imply that unconventional policies extending expected durations raised the level of output by as much as 4 percent in 2012Q4, while inflation was less affected due to a strong degree of nominal rigidities according to the parameter estimates.[12] Using these variables together with the other non-interest-rate observables as targets in $\Delta_t = \iota(x_t - x_t^*)$, the counterfactual shadow interest rate is positive and not just in the early stages of the ZLB. Instead, it remains non-negative throughout the ZLB and rises somewhat above zero around 2012 when output would have been most depressed without unconventional policies.

4 Applications of the Structural Shadow Rate

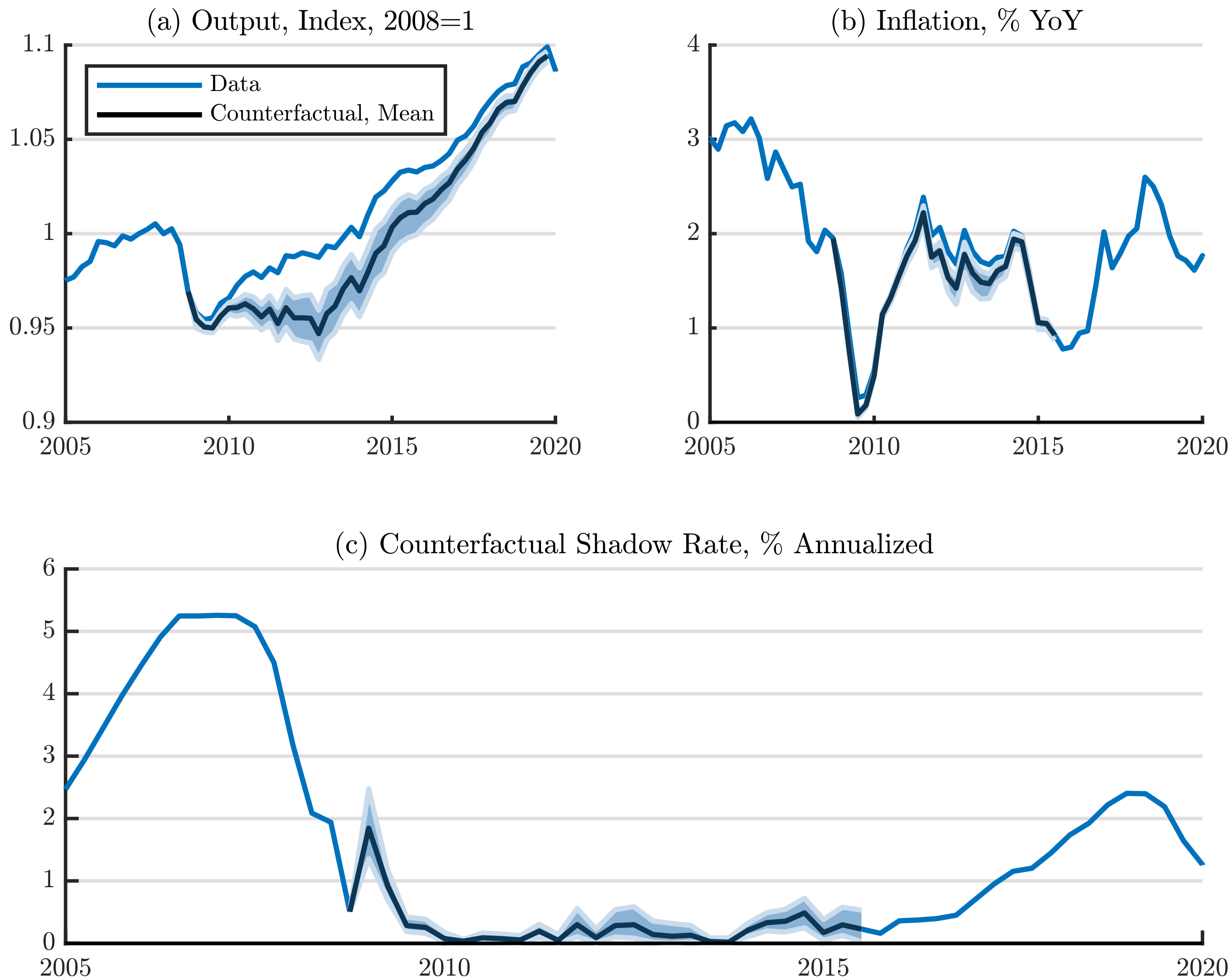
We explore three applications of our structural shadow rate. The first two reflect the typical use of a policy rate in linear setups, while the third considers monetary policy during the recent pandemic.

4.1 Shock Decomposition

The structural shadow rate can be used to perform a shock decomposition in order to assess the contribution of policy shocks to observed variables. Specifically, we obtain smoothed estimates

[12]The larger effects on real activity than inflation are in line with counterfactuals for the unemployment rate and inflation in Eberly et al. (2020) based on a structural VAR with external instruments. We note too that, as shown in the appendix, our shadow rate estimates are robust to an alternative calibration of Calvo parameters motivated by findings in Fitzgerald et al. (2020).

Figure 2: Counterfactual Removing Forward Guidance



Notes: For the data in solid blue and a counterfactual removing the effects of forward guidance in black, panel (a) plots an index of output normalized to 1 in the base year of 2008 and panel (b) plots, in annualized percentage terms, year-on-year inflation. Panel (c) plots, in annualized percentage terms, the shadow rate computed for this counterfactual path. The black lines for the counterfactuals correspond to posterior means, while the bands show 90 percent equal-tailed posterior intervals.

of historical structural shocks, including the shadow rate shock. We then feed each shock one-by-one into the model to calculate the effect each shock has on the observed variables.

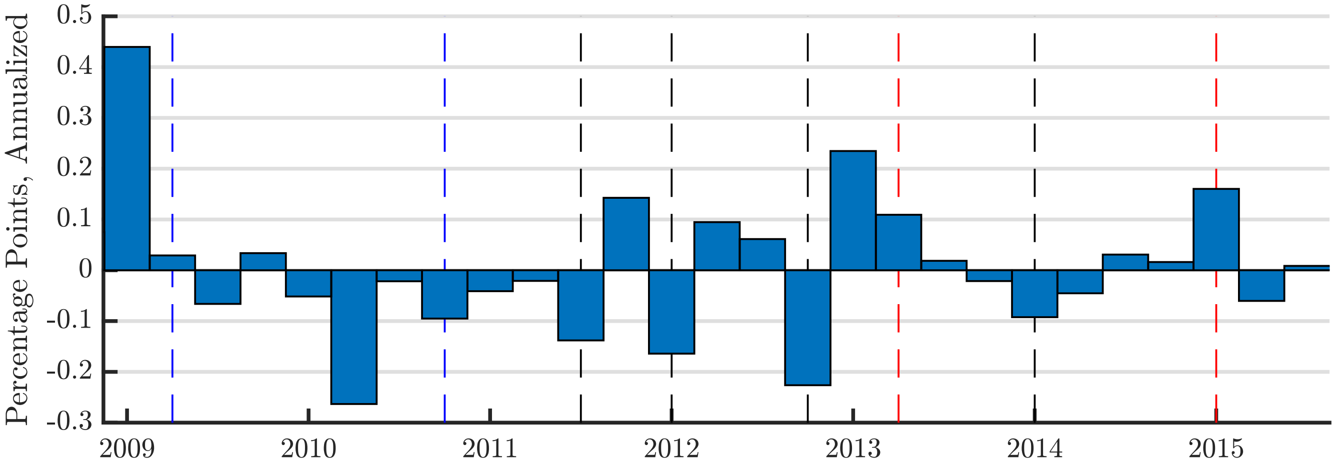
Figure 3 plots the path of the change in the annualized 5-year yield under the shadow rate shock alone, noting some key events related to unconventional policies. After initial contractionary effects from the ZLB constraint at the beginning of 2009, the shadow rate shocks largely act to reduce the long rate during the ZLB. Of particular note, the black dashed vertical lines correspond to quarters in which calendar-based forward guidance announcements were made in FOMC statements. The contribution of the shadow rate shocks to lowering the long rate closely aligns with the quarters of these announcements. For example, calendar-based forward guidance was initiated in 2011Q3 when the FOMC announced that the federal funds rate would be held at zero until “at least through mid-2013”. In 2012Q1, this date was extended to “at least through late-2014”. In 2012Q4, the FOMC introduced threshold-based forward guidance, announcing that the federal funds rate would not be raised until certain values for unemployment and inflation were achieved. These three quarters – 2011Q3, 2012Q1, and 2012Q4 – saw three of the four largest contributions of the shadow rate shock to lowering the 5-year yield. Meanwhile, the red dashed vertical lines correspond to notable events when shadow rate shocks increased the 5-year yield. 2013Q2 covers the taper tantrum, when markets interpreted remarks by the Fed as a signal that it would slow asset purchases. 2015Q1 covers the removal of references by the FOMC in its statement to maintaining the federal funds rate at the lower bound for a “considerable time” following the end of its asset purchase program.

4.2 VAR Analysis

Another use of a shadow rate is in VAR analysis when the sample covers the ZLB. We consider our shadow rate in a three-variable VAR that also includes quarterly inflation and output growth. The VAR is estimated over 2009Q1 to 2015Q3 and we include one lag based on diagnostics suggesting the forecast errors are serially uncorrelated. For comparison, we also consider a version of the VAR with the Wu-Xia shadow rate instead of our shadow rate.

We employ a standard identification of monetary policy shocks by ordering the shadow rate last and using a Cholesky factorization of the forecast-error variance-covariance matrix in order to calculate impulse responses to a one-standard-deviation monetary policy shock under the assumption that the shadow rate can respond to contemporaneous information about inflation and output growth, but only affects them with a lag. Figure 4 plots the impulse response

Figure 3: Contribution of Shadow Rate Shocks to Changes in the 5-Year Yield

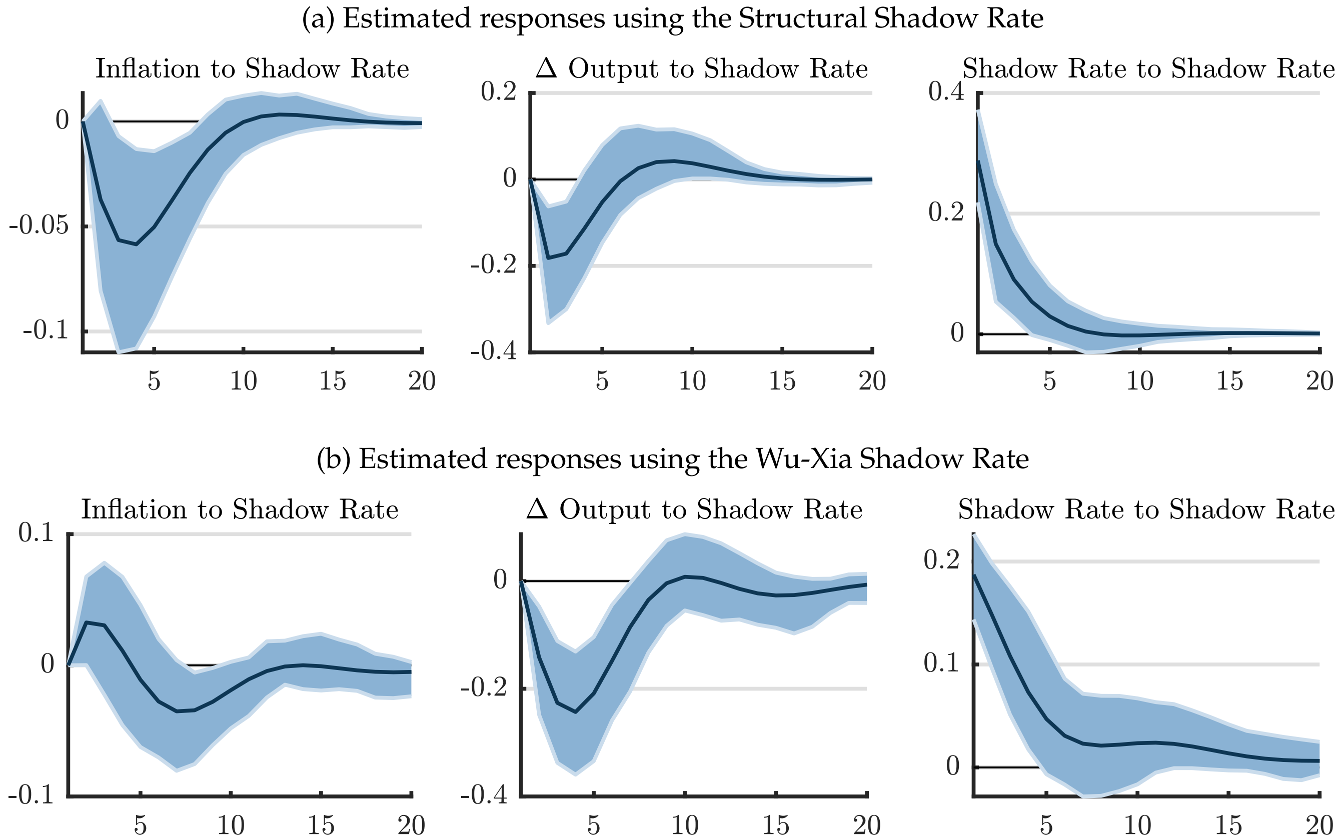


Notes: The bars are annualized percentage point contributions of shadow rate shocks to changes in the 5-year yield based on smoothed estimates. Dashed vertical lines represent the following dates: 2009Q2: QE1; 2010Q4: QE2; 2011Q3: calendar-based forward guidance "at least through mid-2013"; 2012Q1: calendar-based forward guidance "at least through late 2014"; 2012Q3: calendar-based forward guidance "at least through mid-2015"; 2012Q4: threshold-based forward guidance; 2013Q2: taper tantrum; 2014Q1: removal of threshold-based forward guidance; 2015Q1: removal of references to calendar-based forward guidance and to the maintenance of interest rates at the lower bound for a "considerable time" following the end of the asset purchasing program.

functions for a monetary policy shock in the two cases of using our shadow rate and the Wu-Xia shadow rate. Given a contractionary shock, we find significant declines in both inflation and output growth within a one-year horizon when using our shadow rate to identify policy shocks. By contrast, there is a 'price puzzle' in the case of the Wu-Xia shadow rate in the form of initial positive (but insignificant) responses of inflation to a contractionary shock, although they do turn negative at longer horizons.

One interpretation of the price puzzle is that a VAR with Cholesky factorization does not cleanly identify monetary policy shocks, but mixes them with endogenous responses of monetary policy to other shocks with inflationary effects. The Wu-Xia shadow rate tends to fall whenever there is a decline in long-term interest rates, regardless of the reason for the decline. If those declines reflect a deterioration in inflation expectations rather than more expansionary unconventional policies, then it will overstate how accommodative the policy stance has actually become. The identified policy shock in the VAR system will be negative when inflation expectations fall, thus leading to a positive correlation between the identified policy shock and inflation, i.e. the price puzzle. By contrast, our shadow rate should provide a more accurate reading of the policy stance relative to economic conditions and, therefore, can better avoid mixing policy shocks with endogenous responses to other shocks with inflationary effects. Our approach identifies whether a decline in long-term interest rates is due to a deterioration in in-

Figure 4: Impulse Responses in a Three-Variable VAR

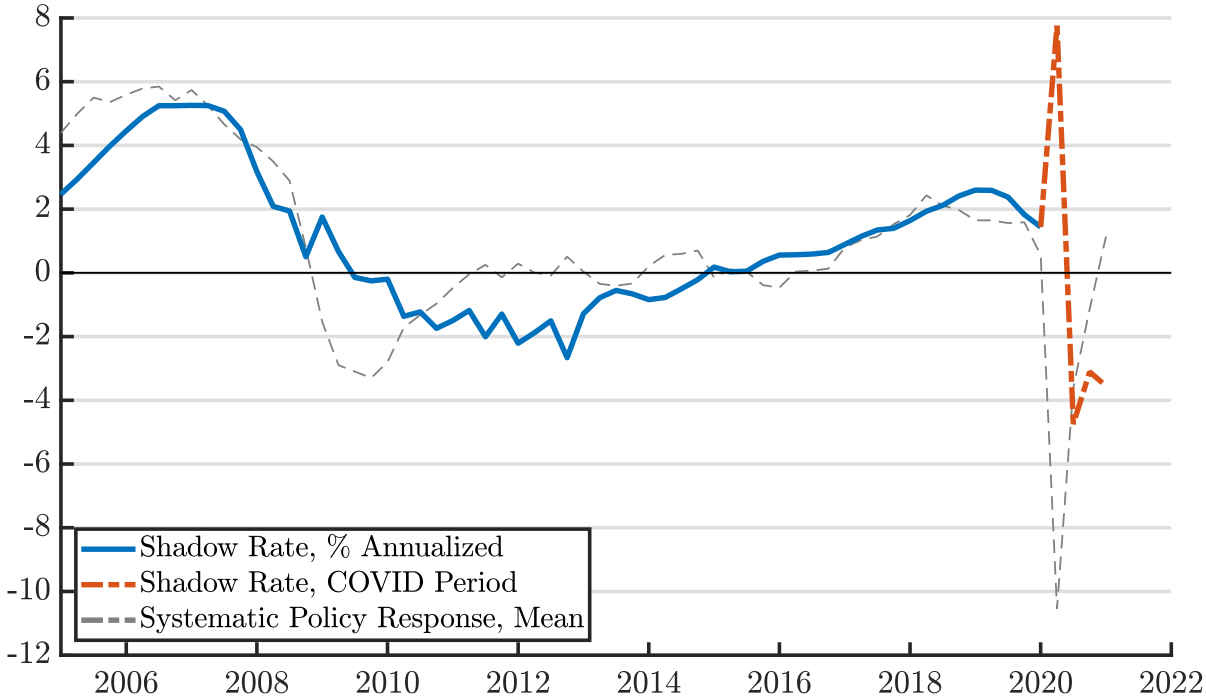


Notes: This figure displays responses of inflation and output growth in quarterly percentage terms and the shadow rate in annualized percentage terms to a one-standard deviation monetary policy shock in a three-variable VAR with quarterly inflation, output growth, and the shadow rate estimated over the ZLB period, 2009Q1 to 2015Q3. The shadow rate is ordered last for identification of monetary policy shocks using a Cholsky factorization of the forecast-error variance-covariance matrix. Panel (a) reports results for the structural shadow rate and panel (b) reports results for the shadow rate measure constructed by Wu and Xia (2016). The impulse responses are computed using a bootstrap procedure drawing residuals with replacement. The black lines correspond to mean responses and the bands show 90 percent equal-tailed bootstrap confidence intervals.

flation expectations increasing the expected duration of the ZLB because of the constraint or a change in forward guidance increasing the expected duration, with a corresponding increase or decrease in the shadow rate, respectively. Unlike with the Wu-Xia shadow rate, the identified policy shock in the VAR using our shadow rate would then be positive when inflation expectations fall, thus leading to a negative correlation between identified policy shock and inflation, i.e. avoiding the price puzzle.

These VAR results support the idea that the structural shadow rate can better reflect the stance of policy than SRTSM measures, suggesting it can be employed in empirical analysis using linear models when the sample period covers the ZLB. While the structural model can be directly used to consider the effects of monetary policy, as in Kulish et al. (2017), it may be useful to have a simple summary of the policy stance during the ZLB when conducting other

Figure 5: The Structural Shadow Federal Funds Rate including the COVID-19 Crisis



Notes: This figure plots, in annualized percentage terms, the estimated shadow rate for a sample that ends in 2021Q1, along with the implied systematic policy response. The red dashed line shows the estimated shadow rate over the second ZLB episode, starting from 2020Q2. The implied systematic policy response to economic conditions is based on the policy rule without monetary policy or shadow rate shocks but allowing for interest rate smoothing, i.e. $i_{t,\text{systematic}} = \bar{i} + (1-\alpha_i)\alpha_p\hat{\pi}_t + (1-\alpha_i)\alpha_y\tilde{y}_t + \alpha_{\Delta y}\Delta\tilde{y}_t + \alpha_i(i_{t-1,\text{systematic}} - \bar{i})$, where the α's are the monetary policy response coefficients and $\tilde{y}_t$ is the output gap from the flexible price equilibrium for the Smets and Wouters (2007) model detailed in the appendix. The lines correspond to posterior means.

empirical analysis. By providing a more accurate measure of monetary policy, our shadow rate can serve as a better control or indicator in such analysis.

4.3 Monetary Policy during the Pandemic

Our last application extends the sample to cover the large economic fluctuations associated with the recent pandemic. Given outliers in some variables, we do not reestimate the model over the extended sample. Instead, we use parameter estimates for data from 1984Q1 to 2019Q4, but extend the sample to 2021Q1 to find structural shocks. During the second ZLB episode that starts in 2020Q2, we again use expected durations based on the modal reported values from the New York Fed Survey of Primary Dealers. The survey implies durations of 8 quarters in 2020Q2, 14 quarters in 2020Q3, 16 quarters in 2020Q4, and 12 quarters in 2021Q1.

Figure 5 plots the estimated structural shadow federal funds rate given the updated data. The red dashed line highlights the shadow rate since 2020Q2. The estimated shadow rate jumped to 7.7% in 2020Q2, reflecting the highly contractionary effects of a persistent expected ZLB constraint when the systematic policy response implied by the policy rule fell to below -10%. However, the estimated shadow rate quickly reversed to almost -5% in 2020Q3, close to the systematic policy response implied by the unconstrained policy rule. This reversal is line with an immediate, albeit partial, recovery in economic conditions, along with the Fed's implementation of a number of extraordinary unconventional policies, including forward guidance related to a shift in the monetary policy framework to consider average inflation in August 2020, with a corresponding doubling of expected durations from 2 years in 2020Q2 to 4 years in 2020Q4.

5 Conclusion

Identifying the stance of monetary policy at the ZLB requires a structural model. Term structure models can produce estimates of the expected duration of zero-interest-rate policy. But a structural model is needed to uncover the reasons behind this expectation. Deteriorating economic conditions that make the ZLB constraint bind for longer are equivalent to tighter monetary policy, while unconventional policies that extend the duration correspond to more expansionary policy. In the SRTSM approach, the short rate follows $i_t = \max(i_t^*, 0)$, which constrains the behavior of the shadow rate to be non-positive when the short rate is at the ZLB. Given persistent large negative shocks that extend the ZLB constraint enough to make our structural shadow rate positive, the SRTSM measure would imply the stance of monetary policy is more expansionary than it actually is. By contrast, our structural shadow rate accurately reflects the stance of policy, as is evident in its strong coherence with announcements by the Fed related to forward guidance, as well as its performance in VAR analysis when the sample covers the ZLB. The structural shadow federal funds rate also suggests a large and quick reversal from an initially high positive level of interest-rate-equivalent policy reflecting a severe binding of the ZLB constraint with the onset of the COVID-19 crisis to an even more negative level than ever occurred during the Great Recession or its aftermath.

The structural shadow rate is, by its nature, dependent on the model used in its estimation. This is no different than the SRTSM approach, which is sensitive to model specification, as highlighted in Bauer and Rudebusch (2016). However, to the extent that expected durations are pinned down by survey data and the component of a duration related to the ZLB constraint is identified by reasonable estimates of structural shocks and the monetary policy rule, results should be fairly robust to a range of related models. Importantly, our approach can be applied to any structural model that accounts for the ZLB. Thus, an interesting extension for future research would be to consider a model in which quantitative easing plays a distinct role, such as in Gertler and Karadi (2011).

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Appendix

For Online Publication

A Description of the Smets and Wouters Model

We list here the linearized equations of the Smets and Wouters (2007) model. We use similar notation for variables and parameters as in their paper, but we substitute i for r when referring to the nominal interest rate and i^k for i when referring to investment. The model variables are presented in terms of deviations from steady state, but with hats suppressed for simplicity. A full description of the model is available in Smets and Wouters (2007) and its accompanying online appendix.

A.1 Sticky Price Economy

Factor prices:

$$mc_t = \alpha r_t^k + (1-\alpha) w_t - \varepsilon_{a,t}$$

$$r_t^k = w_t + l_t - k_t^s$$

$$z_t = \frac{1-\psi}{\psi} r_t^k$$

Investment:

$$i_t^k = \frac{1}{1+\beta\gamma}\left(i_{t-1}^k + \beta\gamma \mathbb{E}_t i_{t+1}^k + \frac{1}{\gamma^2\phi} q_t\right) + \varepsilon_{i,t}$$

$$q_t = \frac{1-\delta}{1-\delta+\bar{R}^k}\mathbb{E}_t q_{t+1} + \frac{\bar{R}^k}{1-\delta+\bar{R}^k}\mathbb{E}_t r_{t+1}^k - i_t + \mathbb{E}_t \pi_{t+1} + \frac{\sigma_c(1+\lambda/\gamma)}{1-\lambda/\gamma}\varepsilon_{b,t}$$

Consumption:

$$c_t = \frac{\lambda/\gamma}{1+\lambda/\gamma} c_{t-1} + \frac{1}{1+\lambda/\gamma}\mathbb{E}_t c_{t+1} + \frac{(\sigma_c - 1)\, W^* L^*/C^*}{\sigma_c(1+\lambda/\gamma)}\ (l_t - \mathbb{E}_t l_{t+1}) - \frac{1-\lambda/\gamma}{\sigma_c(1+\lambda/\gamma)}\ (i_t - \mathbb{E}_t \pi_{t+1}) + \varepsilon_{b,t}$$

Resource constraint:

$$y_t = c_t c_y + i_t^k i_y^k + z_t z_y + \varepsilon_{g,t}$$

Production function:

$$y_t = \phi_p\left(\alpha k_t^s + (1-\alpha)\ l_t + \varepsilon_{a,t}\right)$$

$$k_t^s = z_t + k_{t-1}$$

Monetary policy rule:

$$i_t = (1-\alpha_i)\,\alpha_p \pi_t + (1-\alpha_i)\,\alpha_y\left(y_t - y_t^f\right) + \alpha_{\Delta y}\left(y_t - y_t^f - \left(y_{t-1} - y_{t-1}^f\right)\right) + \alpha_i i_{t-1} + \varepsilon_{m,t}$$

Longer term interest rates:

$$i_{4,t} = \varepsilon_{\eta,t} + \eta_{4,t}$$

$$i_{20,t} = \varepsilon_{\eta,t} + \eta_{20,t}$$

Evolution of capital:

$$k_t = \tfrac{(1-\delta)}{\gamma}k_{t-1} + \tfrac{(\gamma-1+\delta)}{\gamma}i_t^k + \tfrac{(\gamma-1+\delta)}{\gamma}\phi\,\gamma^2\varepsilon_{i,t}$$

Price and wage Philips curves:

$$\pi_t = \tfrac{1}{1+\beta\gamma\,\iota_p}\left(\beta\gamma\,\mathbb{E}_t\pi_{t+1} + \iota_p\,\pi_{t-1} + \tfrac{(1-\xi_p)\,(1-\beta\gamma\,\xi_p)}{\xi_p}\tfrac{1}{1+(\phi_p-1)\,\varepsilon_p}mc_t\right) + \varepsilon_{p,t}$$

$$w_t = w_1 w_{t-1} + (1-w_1)\,\mathbb{E}_t\left(w_{t+1} + \pi_{t+1}\right) - w_2\pi_t + w_3\pi_{t-1} - $$
$$w_4\left(\sigma_l l_t + \frac{1}{1-\lambda/\gamma}c_t - \frac{\lambda/\gamma}{1-\lambda/\gamma}c_{t-1} - w_t\right) + \varepsilon_{w,t},$$

where $w_1 = \frac{1}{1+\beta\gamma}$, $w_2 = w_1\left(1+\beta\gamma\iota_w\right)$, $w_3 = w_1\iota_w$, and $w_4 = w_1\frac{(1-\xi_w)\,(1-\beta\gamma\,\xi_w)}{\xi_w(1+(\phi_w-1)\,\varepsilon_w)}$.

A.2 Flexible Price Economy

The corresponding equations defining the flexible price economy are

$$\alpha r_t^{k,f} + (1-\alpha)w_t^f = \varepsilon_{a,t}$$

$$r_t^{k,f} = w_t^f + l_t^f - k_t^f$$

$$z_t^f = \tfrac{1-\psi}{\psi}r_t^{k,f}$$

$$k_t^f = z_t^f + kp_{t-1}^f$$

$$i_t^{k,f} = \tfrac{1}{1+\beta\gamma}\left(i_{t-1}^{k,f} + \beta\gamma\,\mathbb{E}_t i_{t+1}^{k,f} + \tfrac{1}{\gamma^2\phi}\,q_t^f\right) + \varepsilon_{i,t}$$

$$q_t^f = \tfrac{1-\delta}{1-\delta+\bar{R}^k}\,\mathbb{E}_t q_{t+1}^f + \tfrac{\bar{R}^k}{1-\delta+\bar{R}^k}\,\mathbb{E}_t r_{t+1}^{k,f} - i_t^f + \tfrac{\sigma_c(1+\lambda/\gamma)}{1-\lambda/\gamma}\,\varepsilon_{b,t}$$

$$c_t^f = \tfrac{\lambda/\gamma}{1+\lambda/\gamma}\,c_{t-1}^f + \tfrac{1}{1+\lambda/\gamma}\,\mathbb{E}_t c_{t+1}^f + \tfrac{(\sigma_c-1)\,W^*L^*/C^*}{\sigma_c(1+\lambda/\gamma)}\left(l_t^f - \mathbb{E}_t l_{t+1}^f\right) - \tfrac{1-\lambda/\gamma}{\sigma_c(1+\lambda/\gamma)}i_t^f + \varepsilon_{b,t}$$

$$y_t^f = c_t^f c_y + i_t^{k,f} i_y^{k,f} + z_t^f z_y + \varepsilon_{g,t}$$

$$y_t^f = \phi_p\left(\alpha k_t^f + (1-\alpha)\;l_t^f + \varepsilon_{a,t}\right)$$

$$k_t^{p,f} = \tfrac{(1-\delta)}{\gamma}k_{t-1}^{p,f} + \tfrac{(\gamma-1+\delta)}{\gamma}i_t^{k,f} + \tfrac{(\gamma-1+\delta)}{\gamma}\phi\,\gamma^2\varepsilon_{i,t}$$

$$w_t^f = \sigma_l l_t^f + \tfrac{1}{1-\lambda/\gamma}c_t^f - \tfrac{\lambda/\gamma}{1-\lambda/\gamma}c_{t-1}^f.$$

A.3 Exogenous Processes

Letting ζ denote an i.i.d. standard normal innovation, the exogenous processes are

$$\varepsilon_{a,t} = \rho_a\, \varepsilon_{a,t-1} + \sigma_a\, \zeta_{a,t}$$

$$\varepsilon_{b,t} = \rho_b\, \varepsilon_{b,t-1} + \sigma_b\, \zeta_{b,t}$$

$$\varepsilon_{g,t} = \rho_g\, \varepsilon_{g,t-1} + \sigma_g\, \zeta_{g,t} + \rho_{ga}\sigma_a\zeta_{a,t}$$

$$\varepsilon_{i,t} = \rho_i\, \varepsilon_{i,t-1} + \sigma_i\, \zeta_{i,t}$$

$$\varepsilon_{m,t} = \rho_m\, \varepsilon_{m,t-1} + \sigma_i\, \zeta_{m,t}$$

$$\varepsilon_{p,t} = \rho_p\, \varepsilon_{p,t-1} + \eta_{p,ma,t} - \mu_p\, \eta_{p,ma,t-1}$$

$$\eta_{p,ma,t} = \sigma_p\, \zeta_{p,t}$$

$$\varepsilon_{w,t} = \rho_w\, \varepsilon_{w,t-1} + \eta_{w,ma,t} - \mu_w\, \eta_{w,ma,t-1}$$

$$\eta_{w,ma,t} = \sigma_w\, \zeta_{w,t}$$

$$\varepsilon_{\eta,t} = \rho_\eta \varepsilon_{\eta,t-1} + \sigma_\eta\, \zeta_{\eta,t}$$

$$\eta_{4,t} = \sigma_{i,4}\, \zeta_{4,t}$$

$$\eta_{20,t} = \sigma_{i,20}\, \zeta_{20,t}.$$

A.4 Measurement Equations

Finally, the measurement equations are

$$dy_t = \bar{\gamma} + y_t - y_{t-1}$$

$$dc_t = \bar{\gamma} + c_t - c_{t-1}$$

$$di_t^k = \bar{\gamma} + i_t^k - i_{t-1}^k$$

$$dw_t = \bar{\gamma} + w_t - w_{t-1}$$

$$\pi_t^{obs} = \bar{\pi} + \pi_t$$

$$i_t^{obs} = \bar{i} + i_t$$

$$i_{4,t}^{obs} = \bar{i} + \bar{i}_4 + i_{4,t}$$

$$i_{20,t}^{obs} = \bar{i} + \bar{i}_{20} + i_{20,t}$$

$$l_t^{obs} = \bar{l} + l_t.$$

B Data, Parameter Estimates, and Additional Results

B.1 Data Sources and Mapping to Model

We use the following data series and sources (with FRED mnemonics in parentheses):

- Real Gross Domestic Product (GDPC1)
- Fixed Private Investment (FPI)
- Personal Consumption Expenditures (PCEC)
- Inflation: Gross Domestic Product, Implicit Price Deflator (GDPDEF)
- Nonfarm Business Sector: Average Weekly Hours (PRS85006023)
- Nonfarm Business Sector: Compensation Per Hour (COMPNFB)
- Federal Funds Rate (FEDFUNDS)
- 1-Year Treasury Constant Maturity Rate (GS1)
- 5-Year Treasury Constant Maturity Rate (GS5)
- Population Level (CNP16OV)
- Employment Level (CE16OV)
- ZLB Durations: following Kulish et al. (2017), we use the ZLB durations extracted from the New York Fed Survey of Primary Dealers, conducted eight times a year from 2011Q1 onwards and the Blue Chip Financial Forecasts survey before 2011.[13] For our measure of an expected duration, we take the mode of the distribution implied by these surveys.

We map these series to our observed variables in the following way:

$$\text{CNP16OV\_idx} = \frac{\text{CNP16OV}}{\text{CNP16OV}_{1992Q3}}$$

[13]See the website https://www.newyorkfed.org/markets/primarydealer_survey_questions.html for more information on the Primary Dealers survey. For example, in the survey conducted on January 18 2011, one of the questions asked was: "Of the possible outcomes below, please indicate the percent chance you attach to the timing of the first federal funds target rate increase" (Question 2b). Responses were given in terms of a probability distribution across future quarters.

$$\text{CE16OV\_idx} = \frac{\text{CE16OV}}{\text{CE16OV}_{1992Q3}}$$
$$dc_t = 100 \times \Delta \frac{\text{PCEC}}{\text{GDPDEF}} \times \frac{1}{\text{CNP16OV\_idx}}$$
$$dy_t = 100 \times \Delta \frac{\text{GDPC1}}{\text{CNP16OV\_idx}}$$
$$di_t^k = 100 \times \Delta \frac{\text{FPI}}{\text{GDPDEF}} \times \frac{1}{\text{CNP16OV\_idx}}$$
$$dw_t = 100 \times \Delta \frac{\text{COMPNFB}}{\text{GDPDEF}}$$
$$l_t^{obs} = 100 \times \log \left(\text{PRS85006023} \times \frac{\text{CE16OV\_idx}}{\text{CNP16OV\_idx}} \right).$$

We demean l_t^{obs} over 1984Q1 to 2019Q4.

B.2 Parameter Estimates

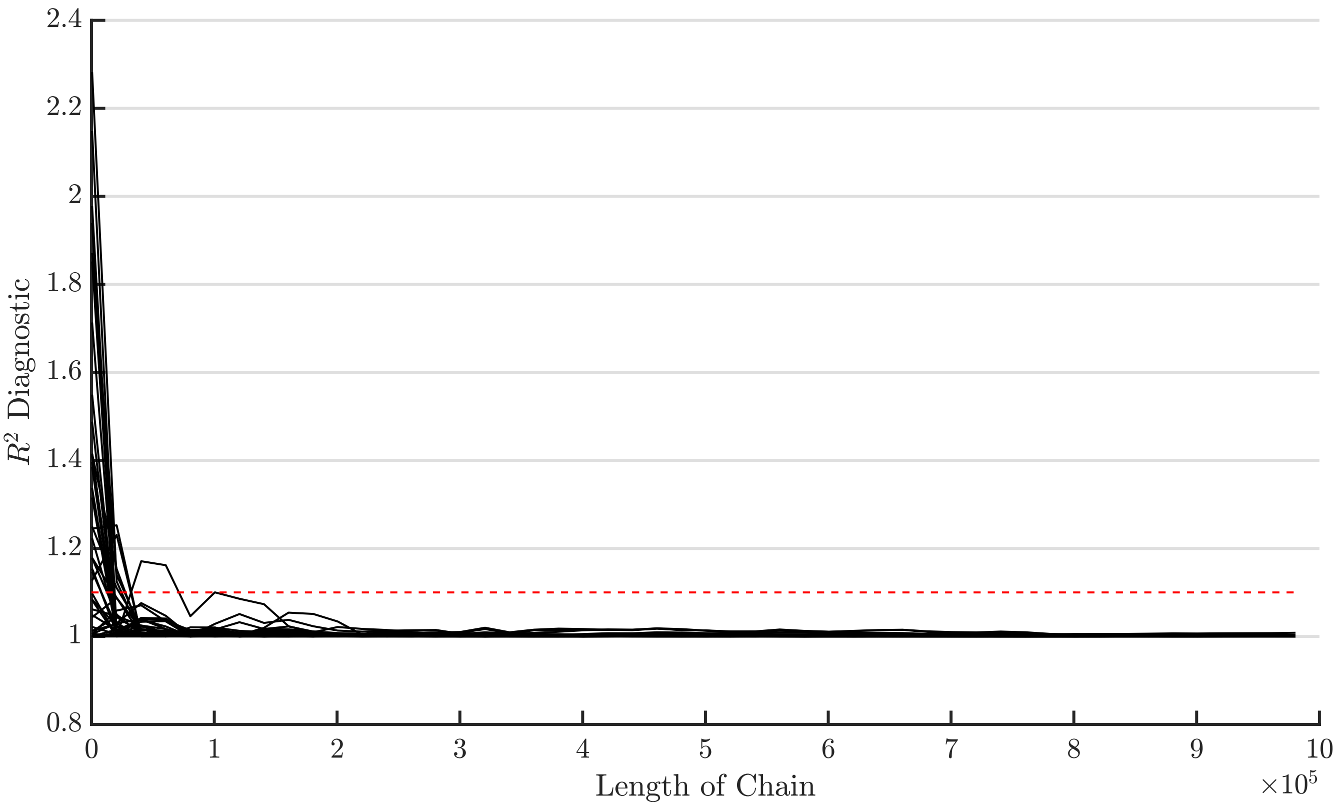
The prior and posterior distributions of the estimated parameters are given in Table B.1. The Calvo price parameter is centered around a value of 0.93, indicating the aggregate data prefers strong nominal rigidities and a relatively flat Phillips curve, which helps to rationalize a relatively stable inflation rate with a large output and employment gap in the post-2009 sample. The Calvo wage parameter is centered around a value of 0.36, in line with the estimates from Fitzgerald et al. (2020) that use relative US state-level data. In Appendix C below, we explore the robustness of our results to calibrating both the Calvo wage and price parameters to their estimates in Fitzgerald et al. (2020), namely a Calvo wage parameter of 0.4 and a Calvo price parameter of 0.6. The results in terms of the shadow rate are highly robust. The posterior estimates of the response of the policy interest rate to inflation and output fluctuations are slightly lower than those reported in Smets and Wouters (2007), noting that their sample ends in 2004. The posterior estimate for trend growth is centered around 0.57% per quarter. We obtain a precisely estimated one-time decline in trend growth of -0.18% in 2003Q4, consistent with other studies noted above that find a decline in trend growth around that time. This decline translates into a difference in the annual rate of trend growth of 2.3% before 2003Q4 to 1.5% thereafter. Given our posterior estimate of the discount factor, the estimated decline in trend growth implies that the annualized steady-state nominal interest rate falls from 5.2% to 4.4%. In simulations, this decline in the steady-state nominal interest rate has the effect of raising the fraction of time spent at the ZLB from about 5% to about 15%.

Table B.1: Parameter Estimates

Parameter	Type	Prior Mean	Prior 5%	Prior 95%	Posterior Mode	Posterior Median	Posterior 5%	Posterior 95%
ϕ	N	4.0	1.5	6.5	6.98	6.79	5.05	8.74
σ_c	N	1.5	0.9	2.1	1.08	1.09	0.90	1.30
λ	B	0.7	0.5	0.9	0.27	0.26	0.19	0.35
ξ_w	B	0.5	0.3	0.7	0.33	0.36	0.25	0.50
σ_l	N	2.0	0.8	3.2	0.89	0.95	0.56	1.49
ξ_p	B	0.5	0.3	0.7	0.93	0.93	0.91	0.95
ι_w	B	0.5	0.3	0.7	0.37	0.40	0.18	0.65
ι_p	B	0.5	0.3	0.7	0.17	0.19	0.09	0.34
ψ	B	0.5	0.3	0.7	0.74	0.74	0.58	0.87
Φ_p	N	1.2	1.0	1.5	1.50	1.50	1.37	1.64
α_p	N	1.5	1.3	1.7	1.64	1.65	1.50	1.80
α_i	B	0.8	0.6	0.9	0.79	0.76	0.57	0.90
α_y	N	0.1	0.0	0.2	0.06	0.06	0.05	0.08
$\alpha_{\Delta y}$	N	0.1	0.0	0.2	0.17	0.17	0.13	0.20
$100\left(\beta^{-1}-1\right)$	G	0.2	0.1	0.4	0.16	0.17	0.08	0.30
γ	N	0.4	0.2	0.6	0.58	0.57	0.53	0.62
α	N	0.3	0.2	0.4	0.15	0.15	0.13	0.18
$\bar{i}_4$	N	0.1	0.0	0.3	0.04	0.04	−0.00	0.08
$\bar{i}_{20}$	N	0.5	0.1	0.9	0.19	0.19	0.12	0.26
$\bar{l}$	N	0.0	−2.5	2.5	2.31	2.25	0.14	3.91
Persistence and Variances of Exogenous Processes								
ρ_a	B	0.5	0.2	0.8	0.93	0.93	0.91	0.95
ρ_b	B	0.5	0.2	0.8	0.98	0.98	0.97	0.99
ρ_g	B	0.5	0.2	0.8	0.99	0.99	0.98	1.00
ρ_i	B	0.5	0.2	0.8	0.82	0.81	0.73	0.88
ρ_p	B	0.5	0.2	0.8	0.80	0.79	0.69	0.87
ρ_w	B	0.5	0.2	0.8	0.99	0.99	0.99	1.00
μ_p	B	0.5	0.2	0.8	0.72	0.69	0.52	0.81
μ_w	B	0.5	0.2	0.8	0.74	0.73	0.56	0.86
ρ_{ga}	N	0.5	0.2	0.8	0.48	0.47	0.35	0.59
ρ_η	B	0.5	0.2	0.8	0.78	0.78	0.69	0.87
σ_a	IG	0.1	0.0	0.3	0.42	0.42	0.38	0.47
σ_b	IG	0.1	0.0	0.3	0.04	0.04	0.03	0.05
σ_g	IG	0.1	0.0	0.3	0.37	0.36	0.33	0.40
σ_i	IG	0.1	0.0	0.3	0.22	0.22	0.19	0.27
σ_m	IG	0.1	0.0	0.3	0.13	0.13	0.11	0.14
σ_p	IG	0.1	0.0	0.3	0.11	0.10	0.09	0.12
σ_w	IG	0.1	0.0	0.3	0.49	0.50	0.42	0.62
σ_η	IG	0.1	0.0	0.3	0.06	0.06	0.05	0.07
$\sigma_{i,4}$	IG	0.1	0.0	0.3	0.01	0.01	0.01	0.02
$\sigma_{i,20}$	IG	0.1	0.0	0.3	0.09	0.09	0.08	0.10
Change in Trend Growth								
$\Delta\bar{\gamma}$	N	0.0	−0.4	0.4	−0.18	−0.18	−0.23	−0.13
Date of $\Delta\bar{\gamma}$	U	2000Q1	1994Q3	2006Q3	2003Q4	2003Q4	2001Q1	2005Q2

Figure B.1 plots the convergence of the two chains along the chain using the Gelman R^2 diagnostic. The R^2 diagnostic lies below the value of 1.1 for all parameters by the end of the chain, indicating convergence, across chains, of the posterior distributions.

Figure B.1: Convergence of MCMC Chains



B.3 Additional Results

Figure B.2 plots the path of the variables targeted in the construction of the shadow interest rate in the first six panels, in the data (in blue), and in the shadow economy (in red) given by the system (7). The plot shows that the paths of these variables under the shadow rate shocks are very close to the observed paths. The numerical matching procedure we use to find the shadow rate shocks is thus able to replicate the data. For completeness, the path of the policy rate and the shadow rate is also given in the last panel.

Figure B.3 plots the ZLB durations decomposed into the lower-bound component, $\mathbf{d}_t^{\text{lb}}$, and the forward-guidance component, $\mathbf{d}_t^{\text{fg}}$. The sum of these two components gives the actual duration expected by agents in the economy, denoted by $\mathbf{d}_t$ in the text. The figure shows how the

Figure B.2: Paths of Targets in the Shadow Economy

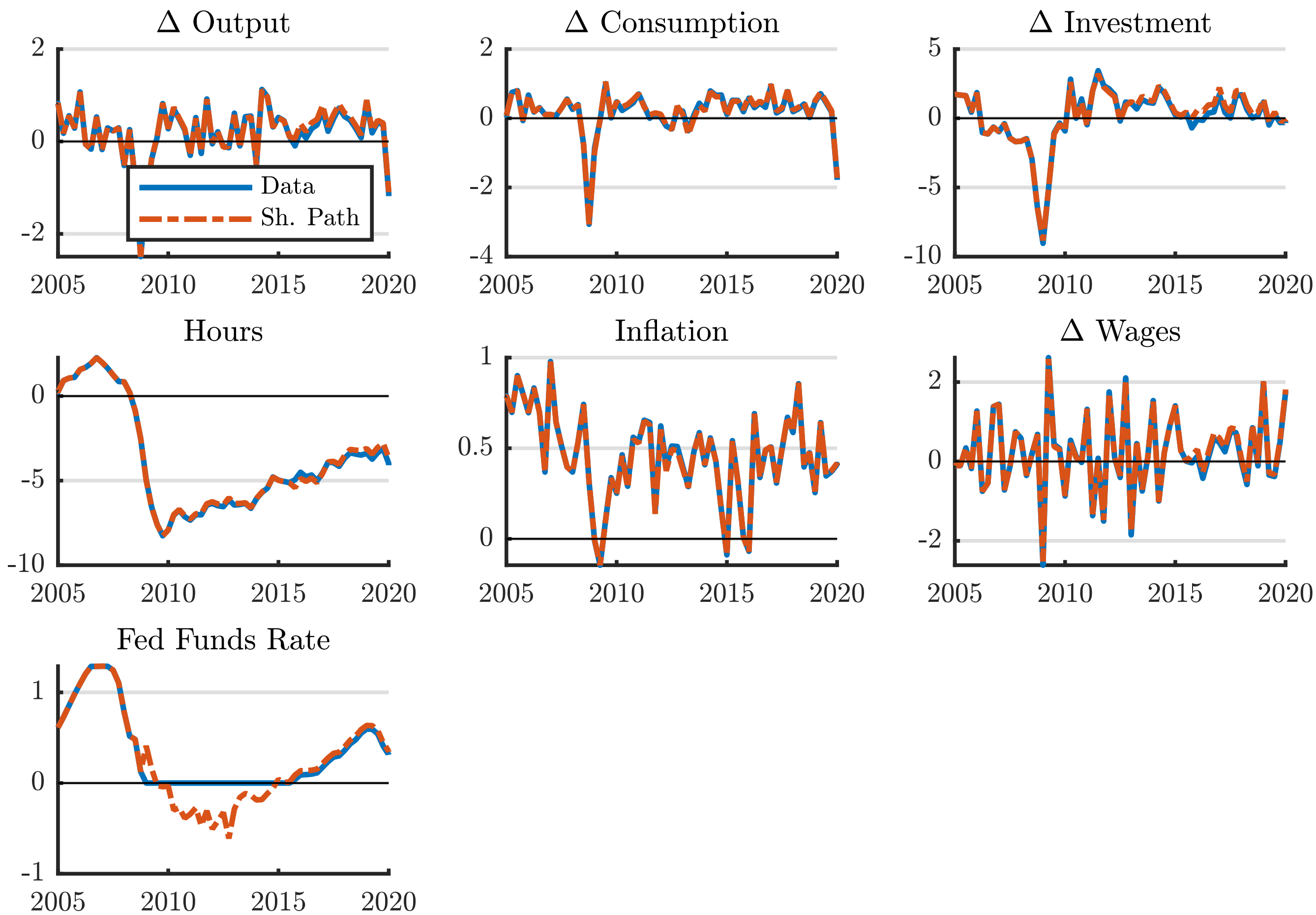
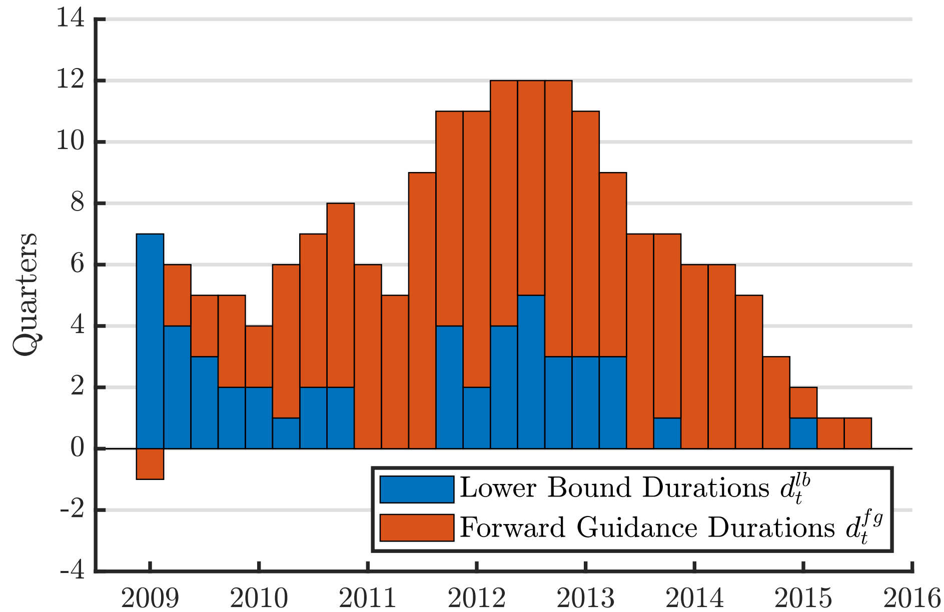


Figure B.3: ZLB Durations at the Posterior Mode



forward-guidance durations were initially quite short, but increased over 2011 and 2012. The forward-guidance duration was even briefly negative in 2009Q1, reflecting an estimated belief by economic agents that the Fed would deviate from the policy rule and begin raising rates one quarter before the ZLB constraint was expected to stop binding for the policy rule. From 2013 to 2015, as the federal funds rate moved closer to liftoff, the actual and forward-guidance durations fell back towards zero.

Figure B.4 plots the paths of all the variables in the data and in the counterfactual scenario where the forward-guidance durations are set to zero.

C Robustness

Figure C.1 plots the shadow rate estimates when the Calvo price and wage parameters are calibrated instead of estimated. The estimates are very similar to those reported in the main text.

Figure C.2 plots the shadow rate constructed when, in addition to the macroeconomic aggregates, we also target the 1-year and 5-year yields. The figure displays the shadow rate estimates including the pandemic and shows that there are almost no differences when compared to the baseline shadow rate we construct using macroeconomic aggregates alone. Figures C.3 and C.4 plot the paths of variables in the shadow economy given the alternative targets including the

Figure B.4: Counterfactual Paths Removing Forward Guidance

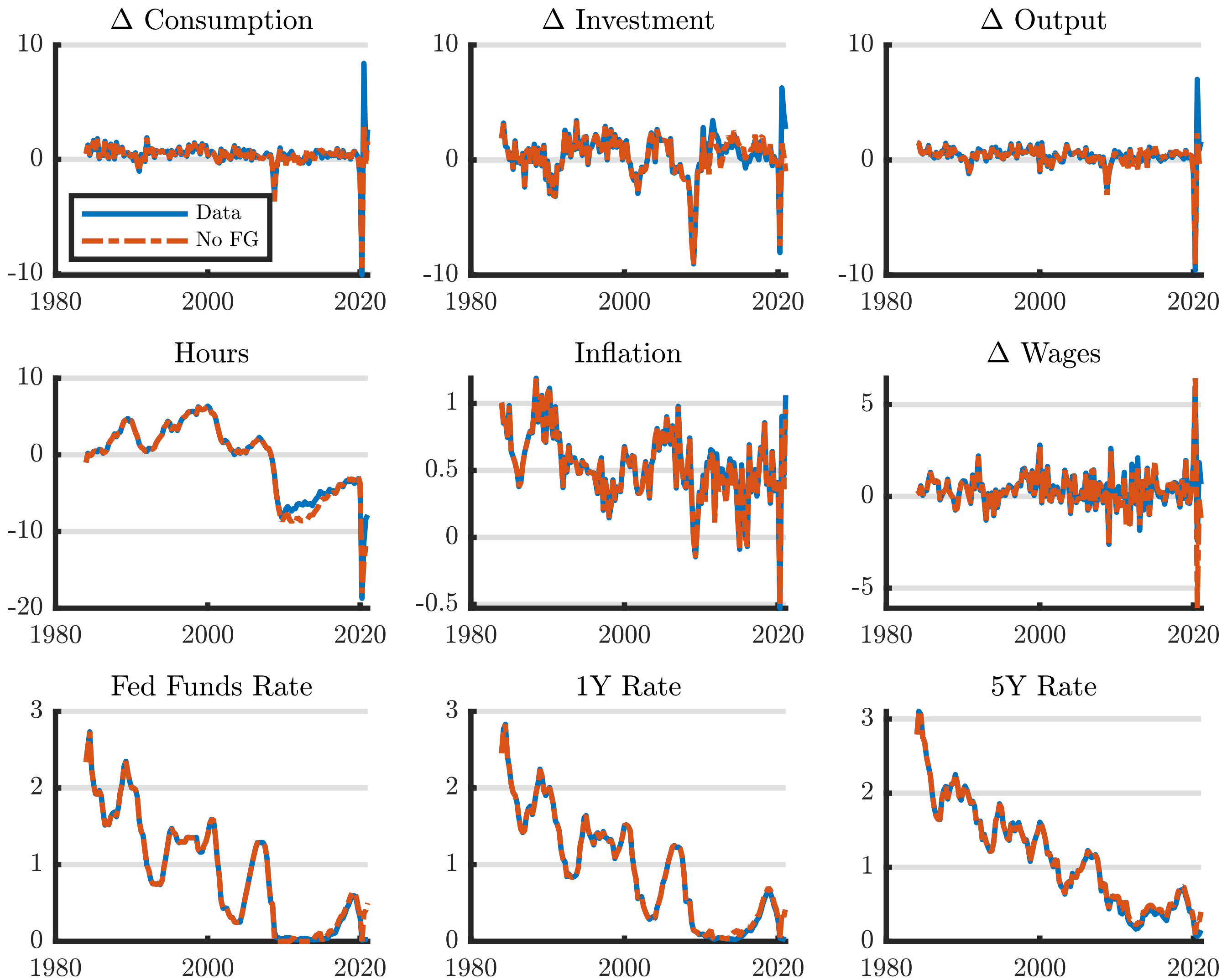
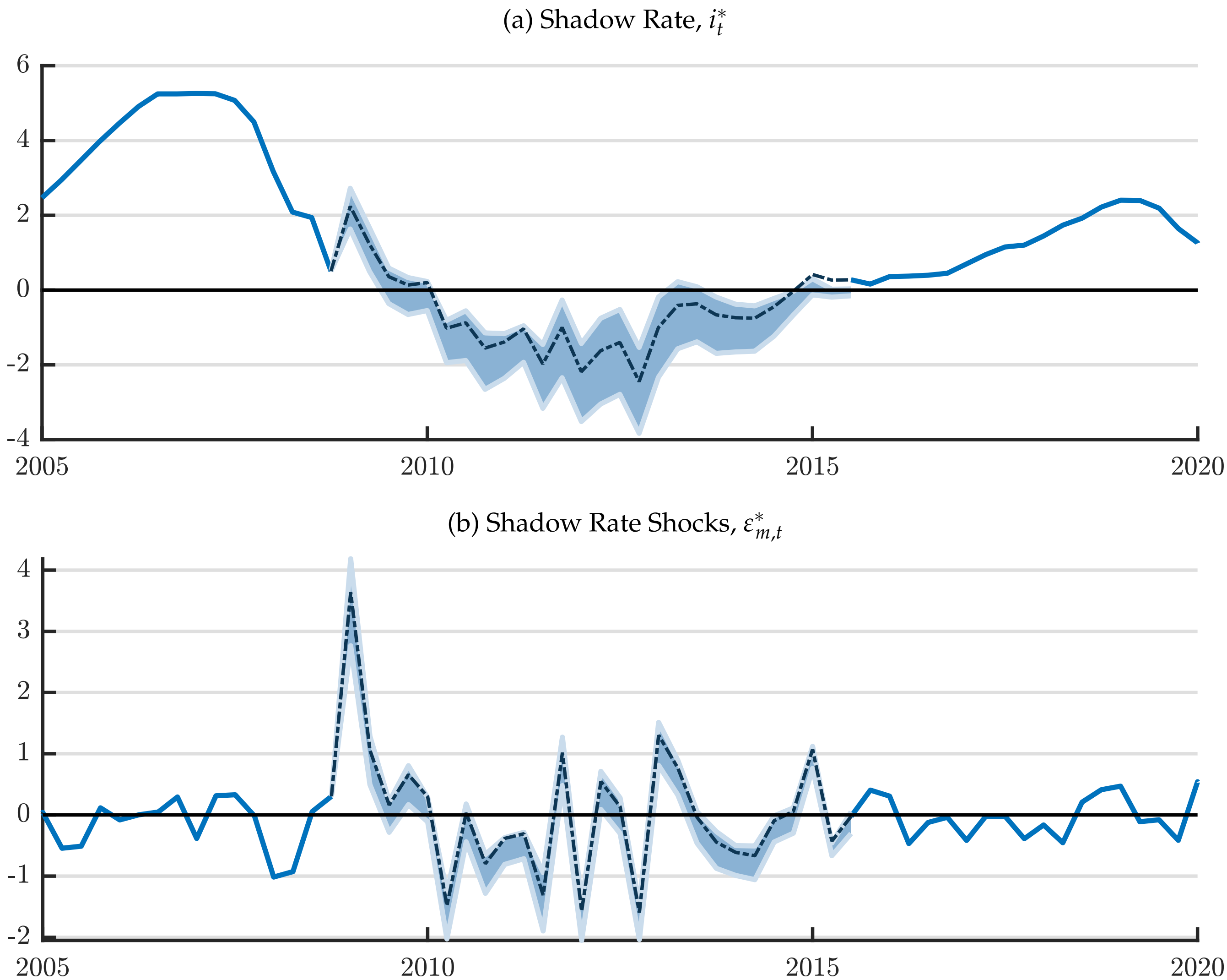
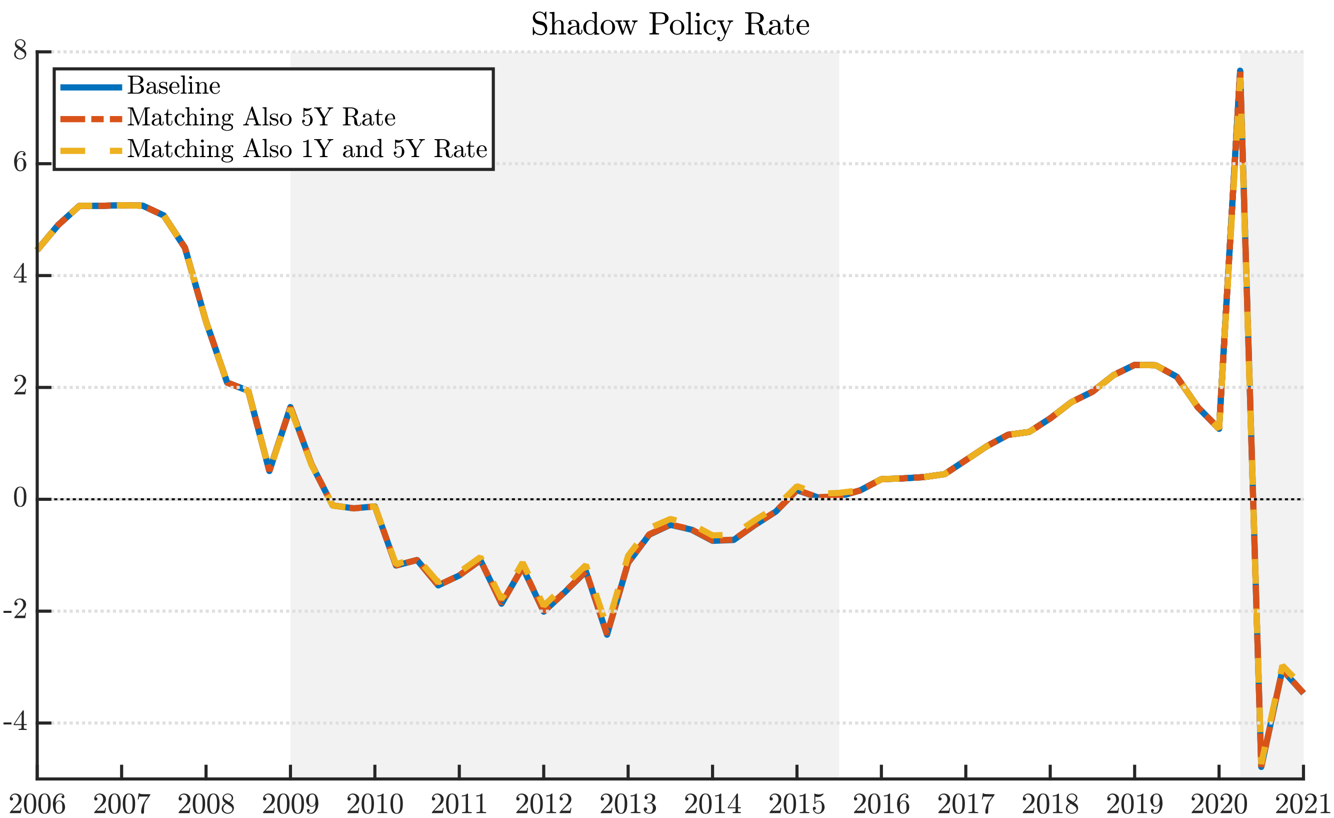


Figure C.1: Shadow Federal Funds Rate given Calibrated Calvo Parameters



Notes: Panel (a) plots, in annualized percentage terms, the estimated shadow rate and panel (b) plots, in annualized percentage point terms, the shadow rate shocks for estimation of the Smets and Wouters (2007) model where the Calvo price and Calvo wage parameters are calibrated to $\xi_p = 0.6$ and $\xi_w = 0.4$, respectively. The dashed black lines correspond to posterior means, while the bands show 90 percent equal-tailed posterior intervals.

Figure C.2: Shadow Federal Funds Rate Additionally Targeting Other Interest Rates



1-year and 5-year yields and show that the fit of the macroeconomic aggregates is virtually unchanged to the baseline, while the interest rates, especially the 1-year yield, behave differently in the unconstrained shadow economy. This robustness exercise makes clear how our identification of the shadow rate is driven by the macroeconomic aggregates, not the term structure of interest rates, which is a key point of contrast to the existing literature on shadow policy rates.

D Comparison with the Wu-Xia Shadow Rate

We compare our structural shadow rate with the shadow rate of Wu and Xia (2016). Figure D.1 plots the mean of our shadow rate series across posterior draws on the same axes as the Wu-Xia measure. Both shadow rates initially start above zero. As explained in the main text, our measure can be above zero when the federal funds rate is at zero because of shocks interacting with the ZLB generate additional nonlinear contractionary effects that map into contractionary shadow rate shocks. In contrast, the Wu-Xia shadow rate can only be above zero when the federal funds rate is at zero if the measure of the short rate used in estimation – the 3-month

Figure C.3: Paths of Targets in the Shadow Economy Additionally Targeting 1Y, 5Y Yields

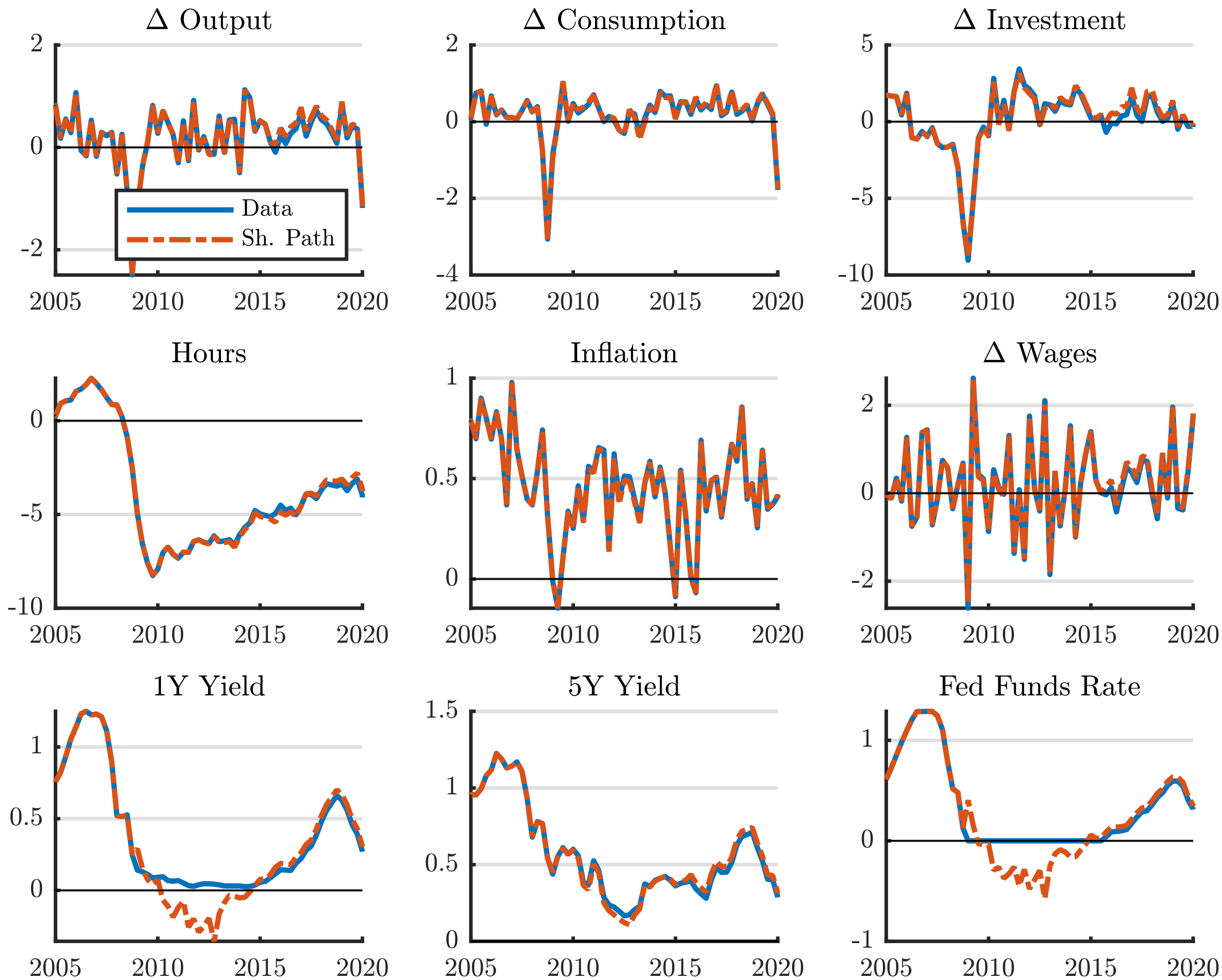


Figure C.4: Paths of Targets in the Shadow Economy Additionally Targeting 5Y Yield

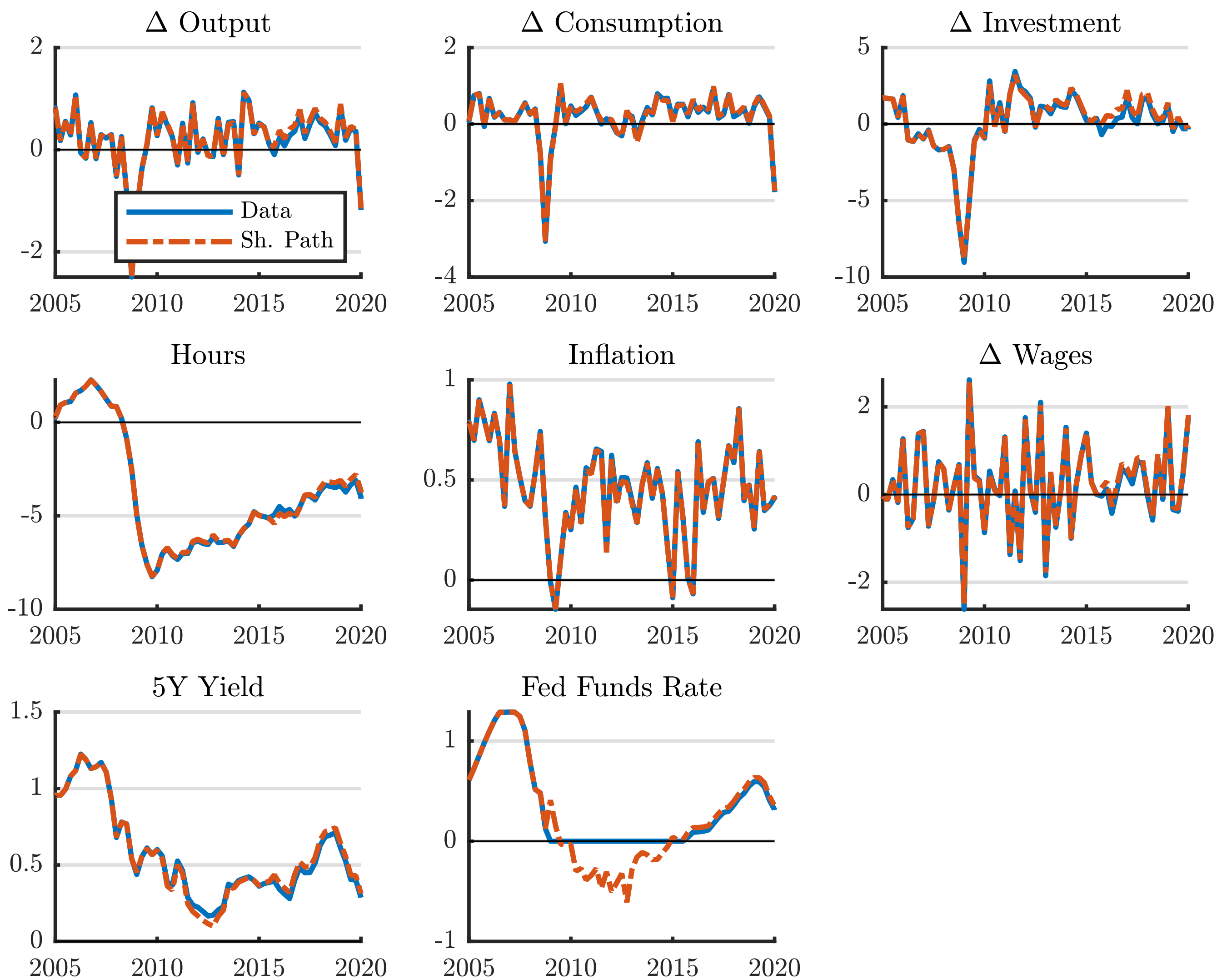
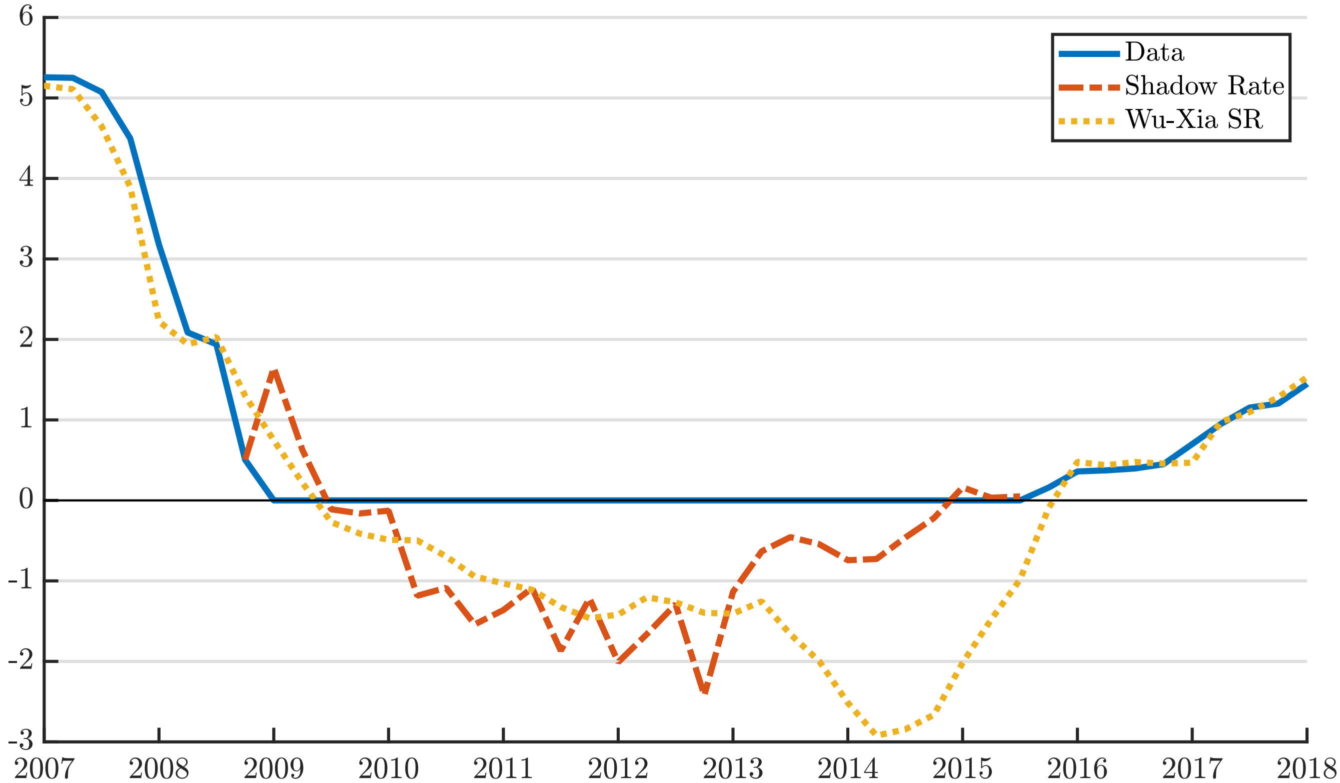


Figure D.1: Comparison of Different Measures of the Shadow Federal Funds Rate

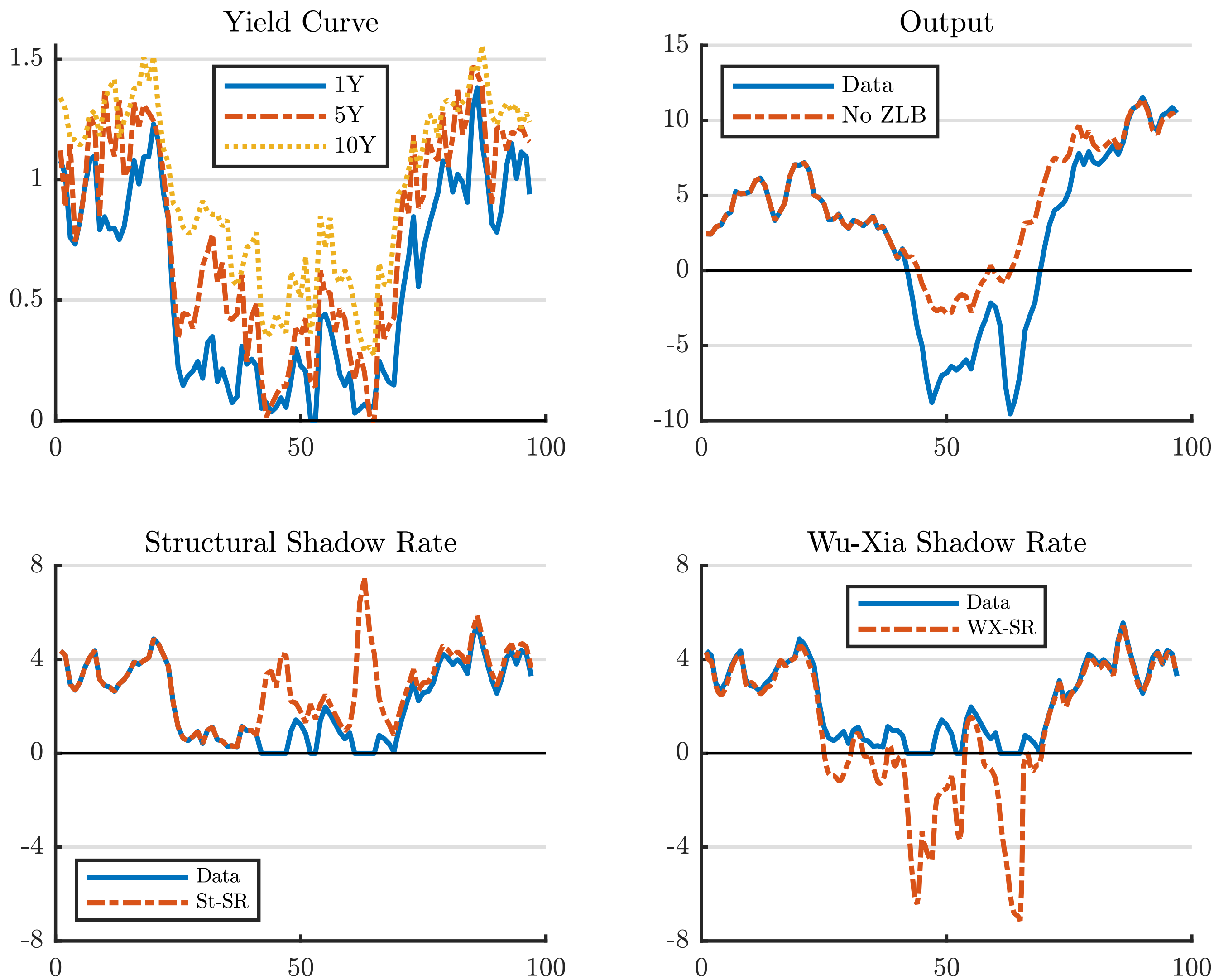


forward rate – is positive. Thus, both measures have similar paths before 2010, but for completely different reasons.

The two measures both decline through to 2013, although our measure is about half a percentage point more negative at the mean across posterior draws. Our measure also displays more volatility. Larger differences become apparent after 2013, when our measure increases from its trough of -2.4% over the course of 2013 to about -0.5%, where it stays before increasing to zero by the time of liftoff in 2015. The Wu-Xia measure shows a notable contrast, falling to almost -3% by the end of 2014, despite the taper tantrum in 2013 and the removal of threshold-based forward guidance in 2014, with a much more sudden rise back to zero in 2015.

One illustrative way to contrast our approach with that of Wu-Xia is to simulate data from a structural model and use the two methods to construct the respective implied shadow interest rates. To do so, we take the Smets and Wouters (2007) model estimated in this paper on the 1984Q1 to 2019Q4 sample and additionally add yield curve variables up to 10-years. This allows us to simulate a yield curve with the following interest rates – 1Q, 2Q, 1Y, 2Y, 5Y, 7Y, 10Y – that we can use to estimate a term structure model. We simulate a long sample of the model

Figure D.2: Simulation to Compare Different Approaches to Measuring the Shadow Rate



and choose a period of the simulation where the ZLB is a significant constraint. For the most contrast between methods, we abstract from forward guidance.

The top two panels of Figure D.2 plot the simulated yield curve and the simulated output series. Also plotted in the output panel is the output series if the ZLB were removed as a constraint on monetary policy. Comparing this with output under the ZLB illustrates the contractionary forces that the ZLB can induce on the economy. The simulated federal funds rate is shown in the bottom two panels in blue – the ZLB binds on and off over the sample.

The shadow rate constructed using our approach is shown in the dashed red series in the bottom left panel. In order to replicate the contractionary effects of the ZLB, our procedure finds contractionary shadow rate shocks, which work to push the shadow interest rate well above zero in the periods that the ZLB binds.

The constructed Wu-Xia shadow rate, presented in the final panel, provides a stark contrast, with the shadow rate measure falling well below zero and almost reaching -8%. Clearly, the inferences that would be made from this measure of the shadow rate would not line up with the conceptual basis of the shadow rate, which is that it reflects the policy stance of the central bank. In this simulation, the central bank does not react at all to shocks that occur at the ZLB. The Wu-Xia shadow rate instead simply reflects the behavior of the yield curve, but the yield curve can decline either because of negative shocks (which is the case here) or because of policy actions (which is not the case here). The Wu-Xia measure is not able to discriminate between the two sources of decline.